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## IfA Fachpraktikum - Experiment 2.6 :

### Control of a Helicopter Model

#### Solutions

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Helicopters have six degrees of freedom (DoF), which makes them very hard to control. The IfA laboratory contains a helicopter model that stands on a stick, which is a lot easier to control as it only has two DoFs. The objective of this experiment is to design a controller that can stabilize both states of the helicopter model. The latter can be described as a non-linear MIMO system, since the two DoFs are coupled by the reaction movements of the propellers. In order to design a controller for this system, using the notions learned in the lecture Control Systems, the system's equations must first be linearized. Also, a closer look at the model will reveal that the system can be easily controlled by means of two SISO-controllers. A closer inspection of the model also reveals that the system can be properly controlled by means of two SISO controllers, constructed by means of compensation methods. The implementation of the controller is done by means of an Ethernet-based control system made by "B&R".

The aim of this experiment is to design and implement the notions learned during the lecture Control Systems.

Part of this experiment has to be carried out at home. If further clarifications are required, please do not hesitate to contact the supervisor.

The Simulink models and Matlab files to conduct the experiment can be downloaded at [http://people.ee.ethz.ch/~ifa-fp/wikimedia/images/b/b9/IfA\\_2-6\\_matlab.zip](http://people.ee.ethz.ch/~ifa-fp/wikimedia/images/b/b9/IfA_2-6_matlab.zip). These are:

<code>Helipar.m</code>	Contains all helicopter parameters
<code>Heli_Sim.mdl</code>	Non-linear helicopter Simulink model
<code>HeliTest.m</code>	Test of non-linear helicopter model
<code>reduce.m</code>	Helper script for simplification of transfer functions.

The following files will only be used during the lab:

<code>Heli_data_log.m</code>	Uploads the designed controller to the Matlab workspace
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# Chapter 1

## Problem Presentation and Modelling

### 1.1 Task Description

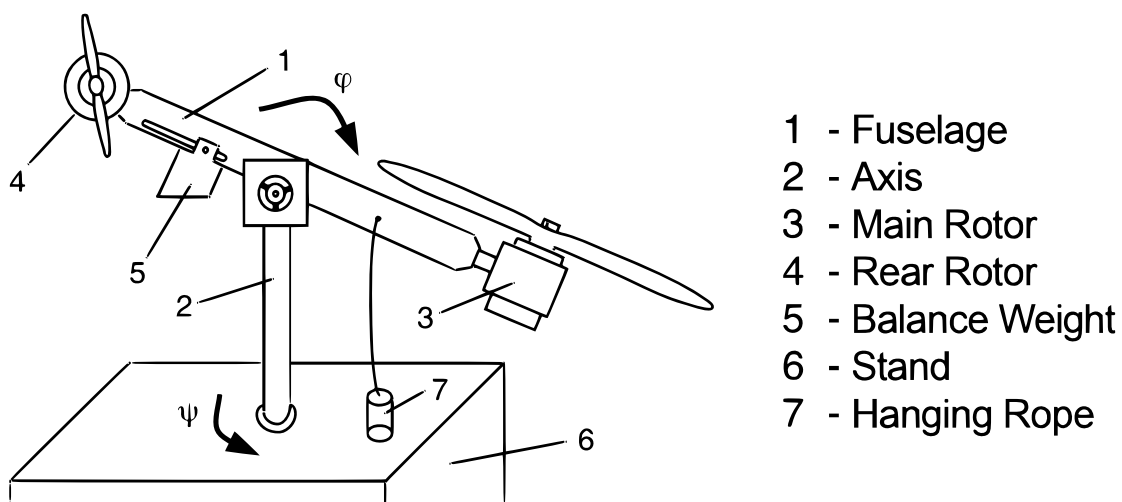


Figure 1.1: Helicopter Model.

During the experiment the following angle definitions are adopted:

$\phi_H = \phi$	Angle w.r.t. the horizontal axis (positive direction clockwise (see fig. 1.1))
$\phi_V = \psi$	Angle w.r.t. the vertical axis (positive direction counter-clockwise (see fig. 1.1))

The equipment used in this experiment consists of a mechanical part, the base of which contains the system to actuate the two motors, and a PC needed to design the controller and communicate with the actuator.

*The cage is there for your safety, do not operate the model without it !*

### 1.2 Experiment Layout

This chapter lists the experiment layout, including a description of the model and a list of sensors and actuators, including their properties and limitations.

### 1.2.1 The Helicopter Model

The objective of this experiment is the design of a controller for the horizontal and vertical positions of the fuselage (see Figure 1.1). The Helicopter model is a MIMO system with two inputs and two outputs. The control variables are the voltages of the main and rear rotors. The output, the we measure, are the angles  $\phi$  and  $\psi$ . A mathematical model of the system is first required to design a controller. Said description can be obtained by means of two methods:

1. Analyzing physical laws (modelling).
2. Fitting a system description through experimental measurements (identification).

This experiment makes use of the first method. Appendix A and B present a detailed description of the parameters and the modelling. The most important parameters used by the sensors and actuators can be found in tables 1.1 and 1.2.

### 1.2.2 The Control System

The angle measurement and the control of the motors are both carried out via a compact control system, configured via an ethernet interface. The control system (X20 CP1484) consists of an Intel Celeron based CPU with different interfaces and the following module (listed from left to right of the control system):

- Analog input module (AI2622) for recording the movements of manual control.
- Motor bridge module (MM4456) for the actuation of both motors via PWM-Signals.
- Analog input module (Ai2622) for angle measurement.
- Module (PS4951) for potentiometer actuation.

### 1.2.3 Sensors and Actuators

The main and rear motors are actuated via the motor bridge module through PWM signals. The module can be operated with a maximum current of 6 A, however it can deliver short bursts of 10 A for short periods of time (2 s).

Gain	$\pm 1$
Output voltage (main motor)	$\pm 16 \text{ V}$
Output voltage (rear motor)	$\pm 8 \text{ V}$
Current limit	$\pm 6 \text{ A}$
Current limit (short time < 2 s)	$\pm 10 \text{ A}$

Table 1.1: Parameters of the motor bridge module

Note these are relatively small amounts and that they should be taken into account during the control design. Other restrictions are listed in Table 1.1. Under the assumption that the period of the PWM-signal is chosen very small, the motor bridge module can be modelled as a dynamic-free, saturating amplifier with  $\pm 16 \text{ V}$  (main motor) resp.  $\pm 8 \text{ V}$  (rear motor) and  $\pm 6 \text{ A}$  (short time  $\pm 10 \text{ A}$ ).

Angles are measured on both axis by means of a potentiometer. The angles also have constraints, for instance the helicopter cannot freely rotate around the vertical axis. Table 1.2 shows sensor specifications and angle constraints.

Horizontal Axis:	$-60^\circ < \phi < 60^\circ$ $U_\phi = k_\phi \phi$ $k_\phi = 3.29 \text{ V/rad}$
Vertical Axis:	$-170^\circ < \psi < 170^\circ$ $U_\psi = k_\psi \psi$ $k_\psi = 3.36 \text{ V/rad}$

Table 1.2: Sensor parameters

### 1.3 Differential Equations of the Model

In the course of this experiment, we use the following definitions:

$\omega_R$	Angular velocity of the main rotor
$\phi_H$	Angle of the fuselage wrt horizontal axis
$\omega_H$	Angular velocity of the fuselage wrt horizontal plane
$\omega_S$	Angular velocity of the rear rotor
$\phi_V$	Angle of the fuselage wrt vertical axis
$\omega_V$	Angular velocity of the fuselage wrt vertical plane

The input values are defined as follows:

$U_R$	Voltage of main rotor
$U_S$	Voltage of rear rotor

The meaning and the approximate numerical values of the above parameters are defined in table A.3. The modelling in Appendix B derives the following differential equations for the helicopter model:

$$\frac{d\omega_R}{dt} = \frac{1}{\Theta_R} \left( \frac{c_{MR}}{R_R} \tilde{U}_R - \frac{c_{MRCGR}}{R_R} \omega_R - k_{MR} \text{sign}(\omega_R) \omega_R^2 - c_{\mu R} \omega_R \right) \quad (1.1)$$

$$\frac{d\phi_H}{dt} = \omega_H \quad (1.2)$$

$$\begin{aligned} \frac{d\omega_H}{dt} = \frac{1}{\Theta_H} \left[ -k_{FR} \text{sign}(\omega_R) \omega_R^2 r_R + \frac{c_{MS}}{R_S} \tilde{U}_S - \frac{c_{MSCGS}}{R_S} \omega_S \right. \\ \left. + m_G g (r_H \cos \phi_H + r_V \sin \phi_H) - c_{\mu H} \omega_H \right. \\ \left. - m_G \omega_V^2 (r_H \cos \phi_H + r_V \sin \phi_H) (r_V \cos \phi_H - r_H \sin \phi_H) \right. \\ \left. + \omega_R \omega_V \Theta_R \sin \phi_H \right] \quad (1.3) \end{aligned}$$

$$\frac{d\omega_S}{dt} = \frac{1}{\Theta_S} \left( \frac{c_{MS}}{R_S} \tilde{U}_S - \frac{c_{MSCGS}}{R_S} \omega_S - k_{MS} \text{sign}(\omega_S) \omega_S^2 - c_{\mu S} \omega_S \right) \quad (1.4)$$

$$\frac{d\phi_V}{dt} = \omega_V \quad (1.5)$$

$$\begin{aligned} \frac{d\omega_V}{dt} = \frac{1}{\Theta_V} \left[ k_{FS} \text{sign}(\omega_S) \omega_S^2 (r_S \cos \phi_H - r_V \sin \phi_H) \right. \\ \left. - \frac{c_{MR}}{R_R} \tilde{U}_R \cos \phi_H + \frac{c_{MRCGR}}{R_R} \omega_R \cos \phi_H - c_{\mu V} \omega_V + \omega_H \omega_R \Theta_R \sin \phi_H \right. \\ \left. + 2m_G \omega_V \omega_H (r_H \cos \phi_H + r_V \sin \phi_H) (r_V \cos \phi_H - r_H \sin \phi_H) \right] \quad (1.6) \end{aligned}$$

where

$$\Theta_V = \Theta_{zyl} + \Theta_{V_1} \cos^2 \phi_H + \Theta_{V_2} \sin^2 \phi_H + m_G (r_H \cos \phi_H + r_V \sin \phi_H)^2 \quad (1.7)$$

$$\tilde{U}_S = \text{sat}(U_S) \quad (1.8)$$

$$\tilde{U}_R = \text{sat}(U_R) , \quad (1.9)$$

and

$$\text{sat}(x) = \begin{cases} x & |x| \leq u_{max} \\ u_{max} \cdot \text{sign}(x) & |x| > u_{max} \end{cases} \quad (1.10)$$

These equations are coded in the Simulink model `Heli_Sim.mdl` and are available in electronic form.

# Chapter 2

## Theoretical Tasks

This chapter serves as preparation for the experiment and must be completed thoroughly. The following exercises will be needed on the day of the experiment and must be solved before coming to the lab.

### 2.1 Linearisation of the Differential Equations

Please note that the following bar ( $\bar{\cdot}$ ) defines a parameter at its equilibrium point. Any deviation from said equilibrium is noted with a  $\Delta$ .

The equilibrium point  $\bar{\phi}_H = \bar{\phi}_V = 0$  is found by means of Equations (1.1), (1.3), (1.4) and (1.6):

$$\begin{aligned} 0 &= \frac{c_{MR}}{R_R} \bar{U}_R - \frac{c_{MR}c_{GR}}{R_R} \bar{\omega}_R - k_{MR} \text{sign}(\bar{\omega}_R) \bar{\omega}_R^2 - c_{\mu R} \bar{\omega}_R \\ 0 &= -k_{FR} \text{sign}(\bar{\omega}_R) \bar{\omega}_R^2 r_R + \frac{c_{MS}}{R_S} \bar{U}_S - \frac{c_{MS}c_{GS}}{R_S} \bar{\omega}_S + m_G g r_H \\ 0 &= \frac{c_{MS}}{R_S} \bar{U}_S - \frac{c_{MS}c_{GS}}{R_S} \bar{\omega}_S - k_{MS} \text{sign}(\bar{\omega}_S) \bar{\omega}_S^2 - c_{\mu S} \bar{\omega}_S \\ 0 &= k_{FS} \text{sign}(\bar{\omega}_S) \bar{\omega}_S^2 r_S - \frac{c_{MR}}{R_R} \bar{U}_R + \frac{c_{MR}c_{GR}}{R_R} \bar{\omega}_R \end{aligned} \tag{2.1}$$

The equilibrium values found by solving equations (2.1) are:

$$\begin{aligned} \bar{U}_R &= 7.033 \text{ V} \\ \bar{U}_S &= 4.343 \text{ V} \\ \bar{\omega}_R &= 202 \text{ rad/s} \\ \bar{\omega}_S &= 301.7 \text{ rad/s.} \end{aligned} \tag{2.2}$$

The following voltage and current values are measured when the helicopter is in a stationary horizontal position:

$$\begin{aligned} \bar{U}_R &= 6.8 \text{ V} \\ \bar{U}_S &= 3.6 \text{ V} \\ \bar{i}_R &= 1.5 \text{ A} \\ \bar{i}_S &= 0.8 \text{ A} \end{aligned} \tag{2.3}$$

These values slightly differ to the calculated ones, since only approximate system parameters are known. The other equilibrium values are:



$$\begin{aligned}
\overline{\omega_R} &= 195 \text{ rad/s} \\
\overline{\phi_H} &= 0 \\
\overline{\omega_H} &= 0 \\
\overline{\omega_S} &= 290 \text{ rad/s} \\
\overline{\phi_V} &= 0 \\
\overline{\omega_V} &= 0
\end{aligned} \tag{2.4}$$

where:

$$\begin{aligned}
U_R &= \overline{U_R} + \Delta U_R \\
U_S &= \overline{U_S} + \Delta U_S \\
\omega_R &= \overline{\omega_R} + \Delta \omega_R \\
\omega_S &= \overline{\omega_S} + \Delta \omega_S
\end{aligned} \tag{2.5}$$

The system is then linearized using the following state-space representation:

$$\begin{aligned}
\dot{\underline{x}} &= \mathbf{A}\underline{x} + \mathbf{B}\underline{u} \\
\underline{y} &= \mathbf{C}\underline{x}
\end{aligned} \tag{2.6}$$

The state and input vectors are as follows:

$$\underline{x} = \begin{bmatrix} \Delta \omega_R \\ \phi_H \\ \omega_H \\ \Delta \omega_S \\ \phi_V \\ \omega_V \end{bmatrix} \quad \underline{u} = \begin{bmatrix} \Delta U_R \\ \Delta U_S \end{bmatrix}. \tag{2.7}$$

The physical description of the states is as follows:

$x_1$	$=$	$\Delta \omega_R$	Angular velocity deviation of main rotor from calculated equilibrium
$x_2$	$=$	$\phi_H$	Angle between fuselage and horizontal axis
$x_3$	$=$	$\omega_H$	Angular velocity of the fuselage on the horizontal axis
$x_4$	$=$	$\Delta \omega_S$	Angular velocity deviation of rear rotor from calculated equilibrium
$x_5$	$=$	$\phi_V$	Angle between fuselage and vertical axis
$x_6$	$=$	$\omega_V$	Angular velocity of the fuselage on the vertical axis

The physical description of the inputs is as follows:

$u_1$	$=$	$\Delta U_R$	Voltage deviation of the main rotor motor from calculated equilibrium
$u_2$	$=$	$\Delta U_S$	Voltage deviation of rear rotor motor from calculated equilibrium

Firstly, the linearization of the matrices is carried out:

$$\mathbf{A}_0 = \begin{bmatrix} -\frac{c_{MR}c_{GR}}{R_R} - 2k_{MR}\overline{\omega_R} - c_{\mu R} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -2k_{FR}r_R\overline{\omega_R} & m_Ggr_V & -c_{\mu H} & -\frac{c_{MS}c_{GS}}{R_S} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{c_{MS}c_{GS}}{R_S} - 2k_{MS}\overline{\omega_S} - c_{\mu S} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{c_{MR}c_{GR}}{R_R} & -k_{FS}\overline{\omega_S}^2r_V & 0 & 2k_{FS}r_S\overline{\omega_S} & 0 & -c_{\mu V} & 0 \end{bmatrix}$$

$$\mathbf{B}_0 = \begin{bmatrix} \frac{c_{MR}}{R_R} & 0 \\ 0 & 0 \\ 0 & \frac{c_{MS}}{R_S} \\ 0 & \frac{c_{MS}}{R_S} \\ 0 & 0 \\ -\frac{c_{MR}}{R_R} & 0 \end{bmatrix},$$

which are then multiplied by the diagonal matrix of the moments of inertia,

$$\Theta = \begin{bmatrix} \Theta_R & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Theta_H & 0 & 0 & 0 \\ 0 & 0 & 0 & \Theta_S & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Theta_{zyl} + \Theta_{V1} + m_Gr_H^2 \end{bmatrix}$$

so as to obtain the dynamics and input matrices:

$$\mathbf{A} = \Theta^{-1}\mathbf{A}_0$$

$$\mathbf{B} = \Theta^{-1}\mathbf{B}_0.$$

The output matrix is the following,

$$\mathbf{C} = \begin{bmatrix} 0 & k_{messH} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & k_{messV} & 0 \end{bmatrix},$$

where the outputs have the following physical meaning:

$$\begin{array}{l|l} y_1 & \text{Voltage for horizontal angle sensor } \phi_H \\ y_2 & \text{Voltage for vertical angle sensor } \phi_V \end{array}$$

## 2.2 Controller Design

The following exercises lead to the design of a stable controller, designed by means of the compensation principle. Since the program files will be modified during this exercise, we kindly ask students to bring their own files to the lab.

### Task 1: Modelling

Study the modelling process and summarize the simplifications made.

#### Solution 1

- Ground effect not considered
- Cage effect not considered
- Helicopter axis and propeller considered homogeneous models

### Task 2: Linearization

Verify the expressions of the linear model from page 10, which makes use of the differential equations from page 6, linearized around the equilibrium points  $\bar{\phi}_H = 0$  and  $\bar{\phi}_V = 0$ .

### Solution 2

As given in text.

### Task 3: Non-linear Model

In the non-linear helicopter model `Helicopter.mdl` apply first an input of 10 V to the rear rotor. Then, after 4 seconds, apply an input of 8 V to the rear rotor.

- 3.1) Explain the behavior of the response graph along the horizontal and vertical axis. What influence do the inputs to the main and rear rotors have?
- 3.2) Try other step sizes and study the behavior of the helicopter. Please note what influence the movement of the horizontal axis has on the vertical one and vice-versa.

**Suggestion:** Initialize the helicopter parameters using the file `Helicopter.m` and use the file `HelicopterTest.m` to start the simulation after configuring the steps in the model `Helicopter.mdl`.

### Solution 3

- 3.1) the graph of both angles is seen in Figure 2.1. First the helicopter falls downwards due to the force of gravity. At  $t = 1$  s the main rotor starts and the helicopter moves upwards. The main rotor also causes the fuselage to move around clockwise along the vertical axis. The helicopter reaches the horizontal limit at approximately ( $t \approx 2.9$  s), causing the angular velocity and the spin around the horizontal axis to be zero. As a result, the change in the spin and thus the torque (see B.8 in Appendix B.1) becomes briefly very large and accelerates the helicopter counter-clockwise along the vertical axis. The rear rotor is started at  $t = 5$  s, making the helicopter spin in the counter-clockwise direction, thus breaking free from the reaction moment of the main rotor.
- 3.2) The angular rotation around the vertical axis doesn't cause any movement around the horizontal axis. Instead, when forcing a rotation around the horizontal axes we also observe a rotation around the vertical axis.

**Warning:** The model parameters are only known approximately. As a consequence, the main rotor may have a significantly larger effect on the vertical axis than expected.

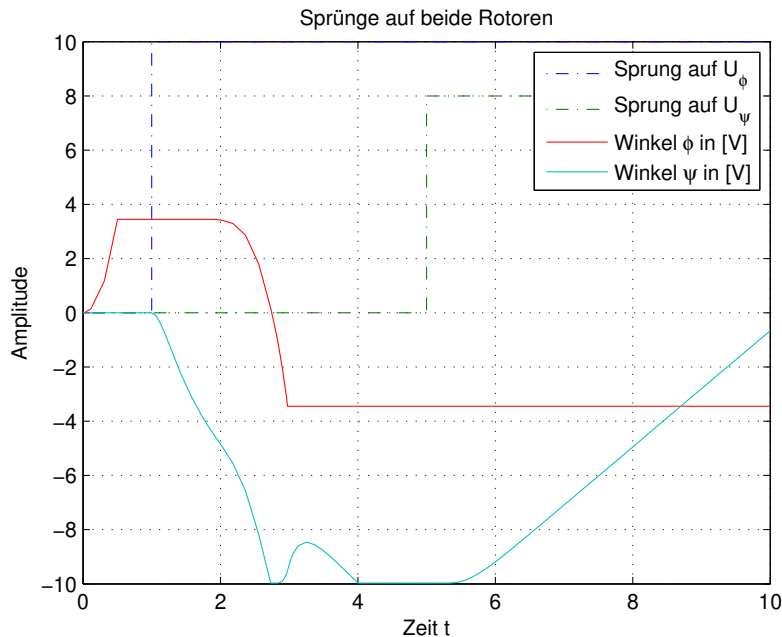


Figure 2.1: Solution of task 3; At time  $t = 1$  a step of 10 V is given to the main rotor, at  $t = 5$  a step of 8 V is given to the rear rotor.

#### Task 4: Study of the Linearized Model

Examine the linearized model by using Matlab to solve the following points. Please note that the system matrices on page 10 are defined in the file `helipar.m`.

- 4.1) Find the eigenvalues of the linearized system.
- 4.2) Find the transfer function matrix  $G(s)$  of the linearized system.

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \underbrace{\begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}}_{G(s)} \cdot \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}$$

#### Suggestions:

- All 4 blocks should contain transfer functions of the 3. order. For this purpose, use the Built-In Matlab functions `ss2tf` and `minreal`, as well as the function `reduce` contained in the zip-file downloaded on the Fachpraktikums website.
- Use the command `minreal`. Increase the tolerance until you get a 3. order transfer functions.
- The controller implementation makes use of the transfer functions  $G_{11}(s), G_{22}(s)$ , which always have positive gain. Therefore, you may have to multiply these transfer functions by a factor of  $-1$  for some calculations. This factor must be removed towards the end of the experiment.

**Warning:** It's important to reduce the transfer functions  $G_{11}(s)$  and  $G_{22}(s)$  to the third order, so as to remove the unstable pole in  $G_{22}(s)$ , which arose from simplified modeling assumptions.

#### Solution 4

For the calculations look at the Matlab m-File `theoaufg.m`.

4.1) 0   -1.83   -3.68   3.50   -2.68   -4.35

4.2) After pole-zero cancellations, the upgraded transfer functions, calculated in the Matlab m-File `theoaufg.m`, are noted with the subscript *r* `G12r` (reduced). The reduced transfer functions are as follows:

$$G_{11}(s) = \frac{-15.28}{s^3 + 2.86s^2 - 12.39s - 34.5}$$

$$G_{12}(s) = \frac{-3.83s - 3.65}{s^3 + 4.51s^2 + 4.90s}$$

$$G_{21}(s) = \frac{0.92s + 2.03}{s^3 + 4.54s^2 - 12.1s - 56.11}$$

$$G_{22}(s) = \frac{8.28}{s^3 + 6.19s^2 + 7.97s}$$

**Suggestion:**  $G_{11}(s)$  must be multiplied by a factor of  $-1$  in order for the gain to be positive, which is necessary for controller design. This factor must be taken away at the end of the experiment in the final controller implementation

#### Task 5: Controllability and Observability

5.1) Is the linearized system controllable?

5.2) Is the linearized system observable?

#### Solution 5

The following Matlab commands are used to compute controllability and observability:

5.1) `rank(ctrb(A,B)) = 6`, the controllability matrix is full-rank  
→ system is controllable.

5.2) `rank(ctrb(A',C')) = 6`, the observability matrix is full rank  
→ system is observable.

#### Task 6: Lead Compensator for the Vertical Axis

Put a lead compensator on the  $\psi$  axis to limit the overshoot (high phase margin) whilst increasing the rise time (lower crossover frequency). Proceed as follows:

6.1) Plot the bode diagram of  $G_{22}(s)$ .

6.2) Design a lead compensator

$$Gr_\psi = \frac{k_{R\psi} \left( \frac{1}{\omega_{E\psi}} s + 1 \right)}{\left( \frac{\alpha_\psi}{\omega_{E\psi}} s + 1 \right)} \quad (2.8)$$

with the following specifications:

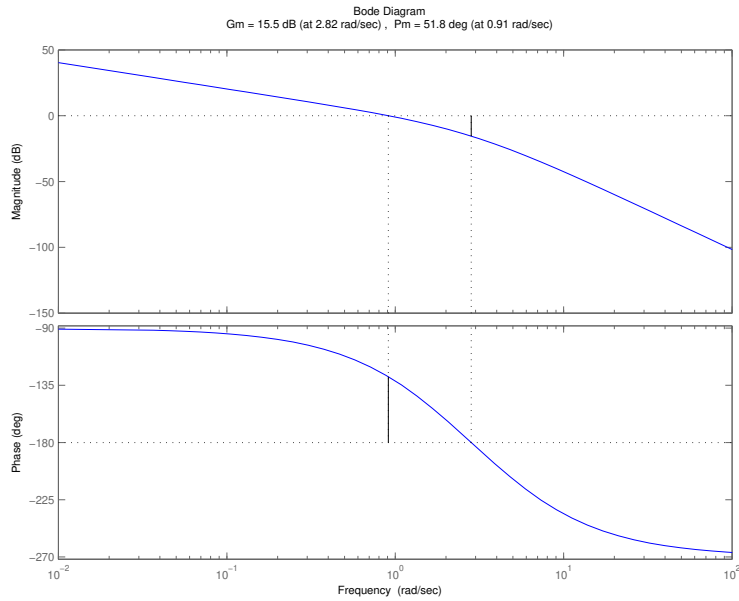


Figure 2.2: Bode diagram of  $G_{22}(s)$

- Crossover frequency  $\leq 5 \frac{rad}{s}$
- Phase margin  $\geq 40^\circ$
- $\alpha_\psi \geq 0.1$

Plot the Bode diagram of the compensator.

**Tip:** Write a Matlab m-file with all the compensator parameters and iterate over all possible combinations. The phase margin can be read via the command `margin`.

- 6.3) Calculate the pole of the compensator.
- 6.4) Plot the step response of the compensator.

### Solution 6

For the calculations use the m-file `theoaufg.m`.

- 6.1) The Bode diagram of  $G_{22}(s)$  is found in Fig 2.2.
- 6.2) A possible lead compensator is:

$$Gr_\psi = \frac{1(\frac{1}{3}s + 1)}{(\frac{0.1}{3}s + 1)}$$

The bode diagram of the compensator can be found in Figure 2.3.

- 6.3) By using the above compensator the closed loop transfer function is as follows:

$$G_{22cl} = \frac{2.76s + 8.28}{0.03s^4 + 1.21s^3 + 6.45s^2 + 10.73s + 8.28}$$

The closed loop poles locations are: -30.10 -3.99 -1.05 + 0.99i -1.05 - 0.99i

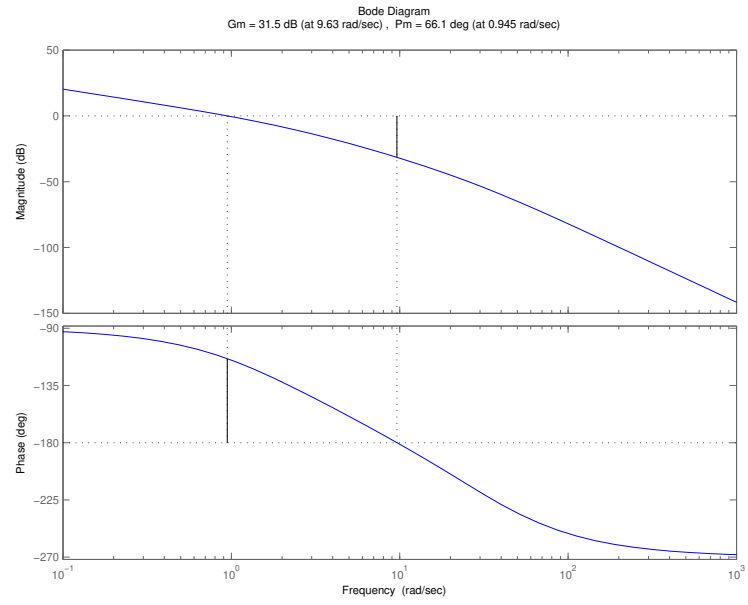


Figure 2.3: Bode-Diagram of  $G_{22}(s)Gr_\psi$

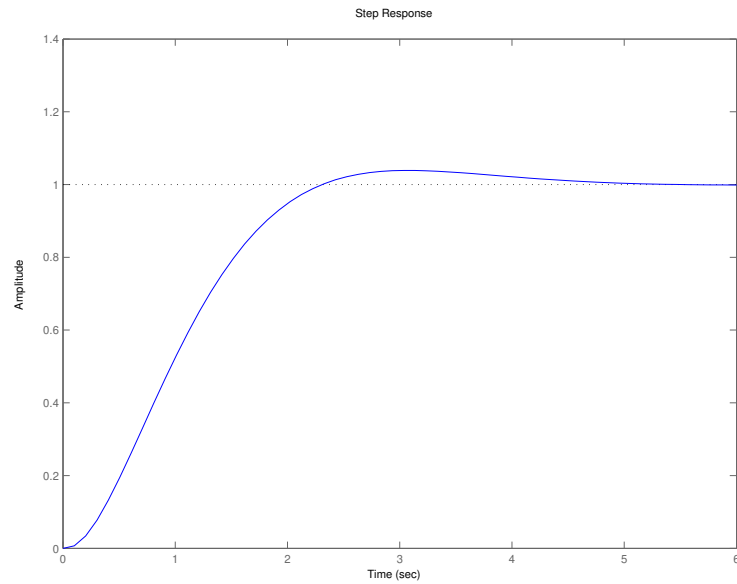


Figure 2.4: Step response of the closed loop system  $G_{22cl}(s)$

6.4) The step response is given in Figure 2.4.

### Task 7: Robustness

A stable system is defined *robust* if small variations in the plant do not affect its stability. Evaluate the robustness of the closed loop system computed in the previous point by considering the Bode diagram of the open loop system.

### Solution 7

The closed loop system has a phase margin of  $66.1^\circ$  and is therefore very robust. Choosing a too high filter gain ( $k_{R\psi} \geq 20$ ) leads to a very small phase margin. This increases the sensitivity of the system to uncertainties and unmodeled dynamics.

### Task 8: Lead Compensator for the Horizontal Axis

Place a double lead compensator for the  $\phi$  axis. Please go through the following steps:

8.1) Plot the Nyquist diagram of  $G_{11}(s)$ , which shows unit feedback. Is the closed loop system stable?

**Suggestion:** Since  $G_{11}(s)$  is unstable, the control design must not be carried out on the Bode diagram.

8.2) Find the parameters of the double lead compensator

$$Gr_\phi = \frac{k_{R\phi} \left( \frac{1}{\omega_{E\phi}} s + 1 \right)^2}{\left( \frac{\alpha_\phi}{\omega_{E\phi}} s + 1 \right)^2}, \quad (2.9)$$

leading to the following UTF specifications:

- Crossover frequency between  $2 \frac{rad}{s}$  and  $15 \frac{rad}{s}$
- Phase margin between  $20^\circ$  and  $40^\circ$
- $\alpha_\phi \geq 0.1$

**Tips:**

- Please note that the current and voltages of the motor bridge module are limited. This must be considered when choosing the gain parameter  $k_{R\phi}$ . (Remember:  $k_{R\phi} \leq 30$ )

8.3) Calculate the pole of the closed loop system.

8.4) Plot the step response of the closed loop system.



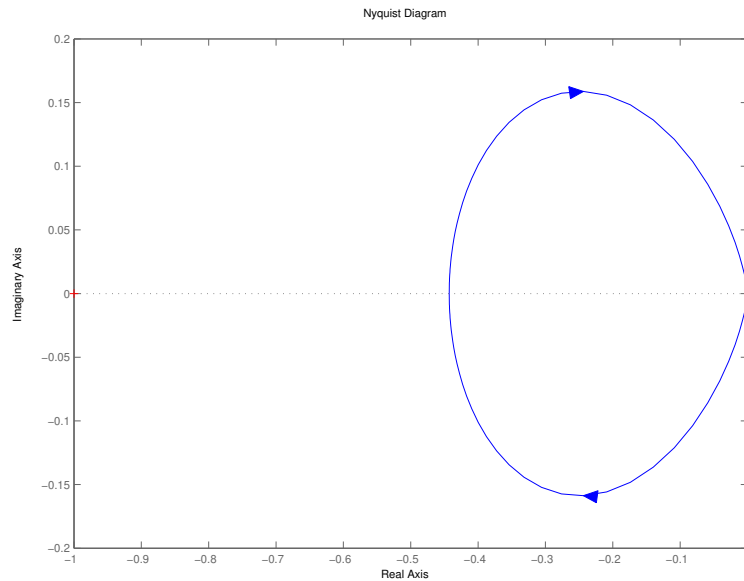


Figure 2.5: Nyquist-Diagramm von  $G_{11}(s)$

### Solution 8

The calculations can be found in the Matlab m-file `theoaufg.m`.

8.1) Since the open loop system is unstable, the Bode criterion must be used with caution. It's therefore better to work with the Nyquist diagram in order to assess the stability of the closed loop system with the Nyquist criterion.

With unit feedback we have the same Nyquist diagram as for  $G_{11}(s)$  (see Figure 2.5). It can be seen that the graph never encircles the point  $-1$ . In order to ensure stability of the system, the point  $-1$  must be encircled once in *counter-clockwise* direction (once for every unstable pole - see Nyquist Criterion). It can therefore be concluded that the system with unit feedback is unstable.

#### Nyquist Criterion:

The closed loop curve  $T(s)$  is asymptotically stable for  $\gamma = \rho + \sigma/2$ .

$\rho$ : Number of poles of the open loop function  $L(s)$  with positive real part

$\sigma$ : Number of poles of the open loop function  $L(s)$  that are purely imaginary

$\gamma$ : Number of times the open loop transfer function  $L(j\omega)$  encircles the point  $-1$  for  $\omega \in [-\infty, \infty]$ .

8.2) A possible double lead compensator is:

$$Gr_\phi = \frac{8(\frac{1}{4}s + 1)^2}{(\frac{0.1}{4}s + 1)^2}$$

The Nyquist diagram of the open loop system is seen in Figure 2.6. Now the point  $-1$  is encircled once in the CCW direction leading to stability of the closed loop system.

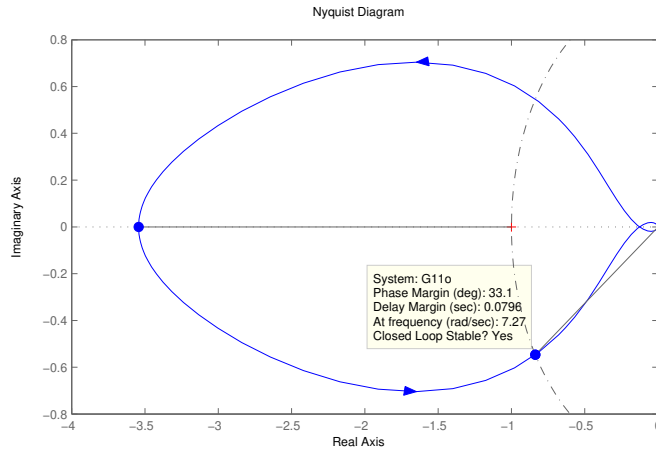


Figure 2.6: Nyquist diagram for  $G_{11}(s)Gr_\phi$

8.3) The closed loop transfer function using the above lead compensator is:

$$G_{22cl} = \frac{7.64s^2 + 61.11s + 122.2}{0.00063s^5 + 0.052s^4 + 1.14s^3 + 9.86s^2 + 47s + 87.73}$$

Leading to the following closed loop poles:

$$-54.31 \quad -17.03 \quad -3.89 + 5.03i \quad -3.89 - 5.03i \quad -3.75$$

8.4) The step response can be seen in Figure 2.7. Please note that there is a control error on the vertical axis step response (Figure 2.7), due to the fact that  $G_{11}(s)$  doesn't have an integrator.

### Task 9: Simulink Simulation

Simulate both controllers in Simulink and plot the angles  $\phi$  and  $\psi$ .

**Suggestion:** Do not forget to re-insert the factor  $-1$ , which was taken away from UTF  $G_{11}(s)$  (see Point 4) which was then added to  $Gr_\phi$ .

9.1) Simulate the controller on the linear helicopter model. Use a block “state space” from the library “linear” making use of the system matrices defined in `Helipar.m` and close the control loop. Simulate the behavior of the closed loop system for a reference 0 and a small initial deflection (i.e.  $\phi_0 = 10^\circ$ ). How does the system behavior change when considering the real system's constraints?

9.2) Simulate the controller on the nonlinear helicopter model. The nonlinear model can be found in the file `Heli_Sim.mdl`. You can find this file on the link stated on the first page of this manual. Simulate the system behavior for 0 reference value and initial condition  $\phi_0 = 10^\circ$ .

**Suggestion:** Please note the outputs of the controller  $\Delta U_R, \Delta U_S$  must be substituted with the equilibrium points  $\bar{U}_R, \bar{U}_S$  found in (2.5).

**Warning:** The equilibrium points in (2.3) are the values *measured* on the helicopter. The equilibrium points found by solving equation (2.1) are critical for the *simulation* with the non-linear helicopter model in `Heli_Sim.mdl`, since they slightly deviate from the measured ones given on Page 8.

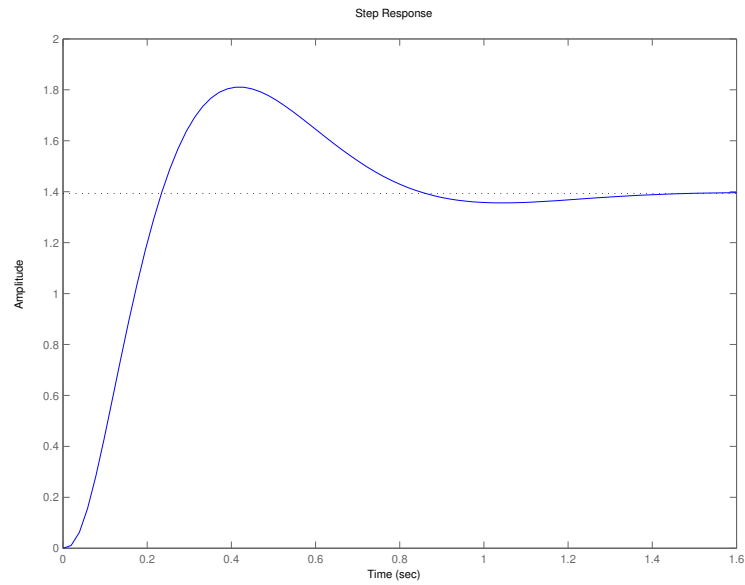


Figure 2.7: Step response of the closed loop system  $G_{11cl}(s)$

### Solution 9

- 9.1) The system behavior can be seen in Figure 2.8. The simulation is carried out using file `linear_control_loop.mdl` and the plot is from `linear_control_loop_run.m`. Due to the limits of the real system, even a small deviation of the initial values causes a significant worsening of the system's behavior, as seen in Figure 2.9.
- 9.2) The simulation is carried out by means of the file `nonlinear_control_loop.mdl`, while the results are plotted with `nonlinear_control_loop_run.m`. The system behavior is worse compared to the linear one, as seen in Figure 2.10.

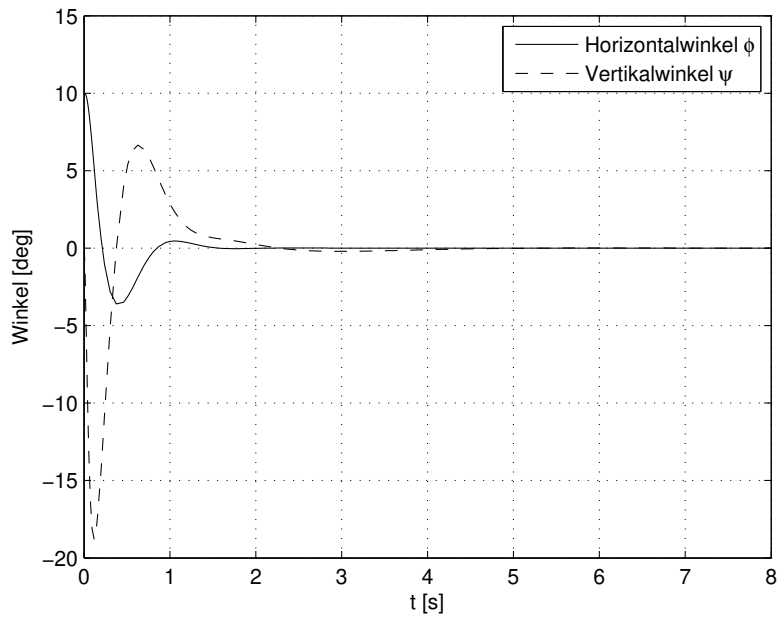


Figure 2.8: Behavior of closed system using the linear helicopter model with initial value  $\phi_0 = 10^\circ$ .

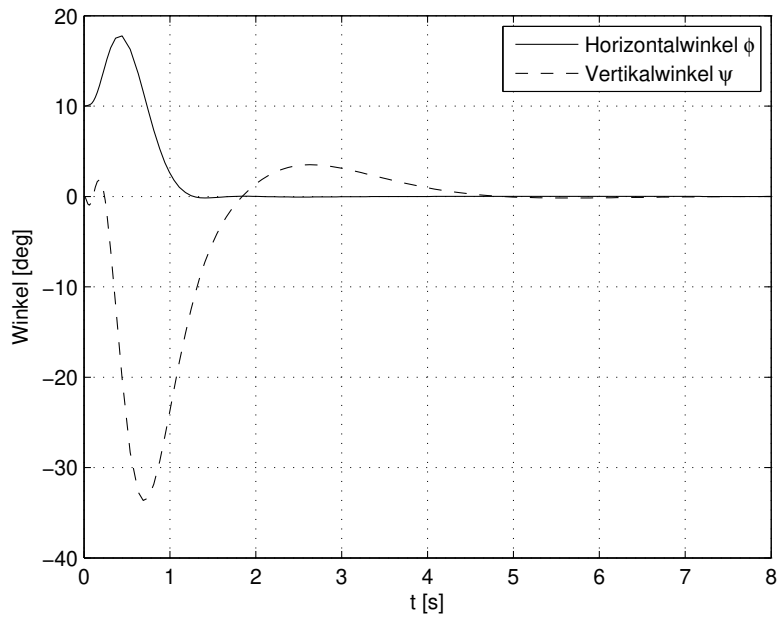


Figure 2.9: Behavior of the closed system using the linear helicopter model with initial value  $\phi_0 = 10^\circ$  (with limitations).

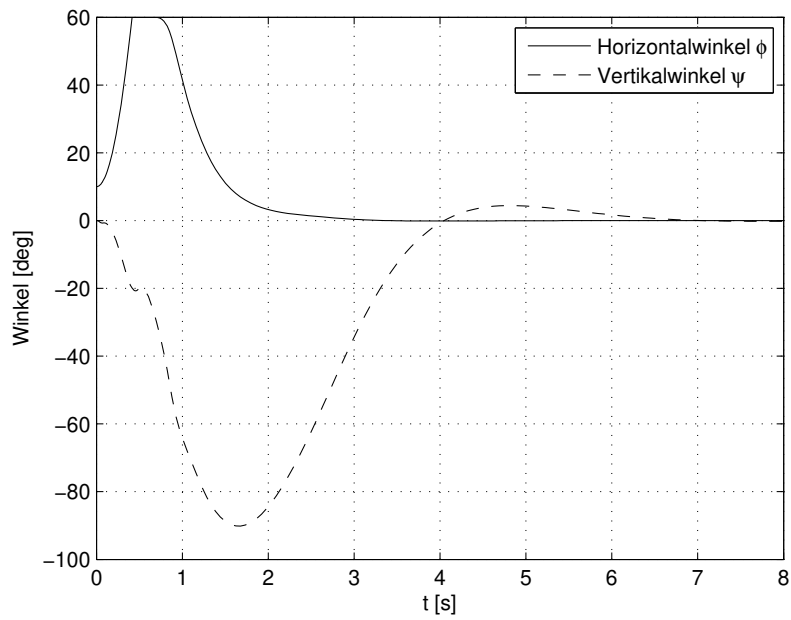


Figure 2.10: Behavior of closed system using the non-linear helicopter model with initial value  $\phi_0 = 10^\circ$  (with limitations).

# Chapter 3

## Lab Session Tasks

### 3.1 Setup of Lab Equipment

The following tasks must be solved during the lab. The control system can be configured via VNC from the host PC.

#### Task 10: Setup of the Control System

Follow the next points for system start up.

- 10.1) Start the host PC and turn on the helicopter model (the switch is behind the helicopter structure).
- 10.2) Open the VNC-Viewer and use the following credentials for login:

Server: autx20-02  
Password: control

- 10.3) Click on the button **START** to start the experiment.

#### Solution 10

No solutions necessary.

**Important:** The experiment can be stopped at any time by means of the button **STOP** or by pressing the spacebar on the keyboard.

#### Task 11: GUI Accustomization

Get familiar with the GUI.

- 11.1) Navigate through the different pages by means of the dark blue buttons **Run**, **Controller** and **Settings**. By using the file **Run** you always return to the exit page.

- 11.2) Go to the page **Settings**. Here the motor bridge module can be reset by using the button **Clear Error**. This must be kept in mind when a motor outputs the message **Over Current Error**. The other buttons in this page are not important for this experiment.
- 11.3) You will later configure your controller in the page **Controller**.
- 11.4) Press **Run** to switch to the exit page. In the page **Control Mode** you can find buttons **Automatic** and **Manual**. Here you can choose whether to control the helicopter model manually (by means of the joystick) or automatically (by means of the control system). Since none of the modes are active, the helicopter is in idle state. Please note that pressing the **STOP** button or the spacebar automatically sets the helicopter to idle.

### Solution 11

No solutions necessary.

### Task 12: Manual Mode

- 12.1) Please note the controller parameters **K Phi**, **K Psi**, **U FF,Phi** and **U FF,Psi** are all set to zero. Press the button **Automatic** and go to the page **Controller**, where you can set the motor voltages by changing the parameters **U FF,Phi** and **U FF,Psi**. Check the effects this has on the helicopter. Finally set all the values to zero and press the button **STOP**.
- 12.2) Change to manual mode by pressing the button **Manual** in the page **Run**. Try stabilizing the helicopter on the horizontal axis manual.
- 12.3) Finish by pressing **STOP** or the spacebar.

### Solution 12

No solutions necessary.

## 3.2 Implementation and Fine Tuning

In the next part of the experiment you will implement and fine tune your controller.

### Task 13: Automatic Mode

Implement the controller you designed at home.

- 13.1) Put your controller parameters in the required fields.  
**Suggestion:** The factor -1 must be taken into account.
- 13.2) Put the equilibrium voltages (from page 8) in the fields **U FF,Phi** and **U FF,Psi**.
- 13.3) Test your controller by activating automatic mode. If the system doesn't behave as expected stop the execution by means of the spacebar or the button **STOP**.
- 13.4) Note down eventual observations.

### Solution 13

Possible controller parameters:

$K_\phi = 8$	$\omega_\phi = 4$	$\alpha_\phi = 0.1$	$U_{FF,\phi} = 6.8$
$K_\psi = 1$	$\omega_\psi = 3$	$\alpha_\psi = 0.1$	$U_{FF,\psi} = 3.6$

**Important:** One of the motors may cut out because it reached the maximum available current. In this eventuality the message **Over Current Error** is outputted. To solve this issue go to the **Settings** page and click on the button **Clear Error**.

### Task 14: Controller Fine Tuning

As you have seen in the previous task, the system behavior is slightly different compared to the model's. The reason for this is the use of approximated parameters. Try manually fine tuning your controller to account for these approximations.

- 14.1) Change the controller parameters slightly and see what happens.
- 14.2) Tune your controller until it can stay in horizontal equilibrium without issues.

**Tips:**

- Use the input fields **Phi** and **Psi** in the block **Reference Tracking** on the page **Run** to change the reference tracking behavior.
- You can also change the control outputs (system inputs) in the block **Control Variables**. The use of too large voltages can cause a too high gain, leading to an **Over Current Error**.

### Solution 14

No solutions necessary.

## 3.3 System Evaluation

A final study of the system behavior with the implemented controller will now be carried out.

### Task 15: System Evaluation and Conclusion

Trace a helicopter trajectory with the joystick and plot it in Matlab. In order to do so, follow the next instructions:

- 15.1) Place the helicopter in the idle position and press the button **Record** on the page **Run**.
- 15.2) Press both **Joystick** buttons in the block **Reference Tracking** and trace a reference signal with the joystick.
- 15.3) Open Matlab and run the code `Heli_data_log.m`. The trajectories of  $\Phi_{soll}$ ,  $\Phi_{mess}$ ,  $\Psi_{soll}$  and  $\Psi_{mess}$  will be uploaded to the control system directly via FTP from Matlab.  
**Suggestion:** Before running the code, wait until the control system has finished uploading the data. The latter occurs when the button **Record** gets released.
- 15.4) Plot the trajectories.

**Suggestions:**



- Data is sampled with a  $T_S = 1$  ms time step.
- The reference signal of the joystick is filtered with a low-pass filter to prevent a too aggressive system response, which would lead to an **Over Current Error** and general instabilities of the system.

### **Solution 15**

No solutions necessary.

We kindly ask you, once finished with the tasks in this part of the experiment, to shut off the lab equipment used and to put your working space in order. You can then proceed to the chapter 4 of the assignment.

# Chapter 4

## Lessons Learned

This chapter summarizes the most important learning objectives of the experiment. Note down your thoughts and discuss the results obtained with the TA.

### Lessons Learned 1: Decoupled Control

- 1.1) What did you understand about decoupled control?
- 1.2) How did this strategy work to control the helicopter?
- 1.3) When did it not work? What other control methods can you suggest?

### Answer 1

- 1.1) Decoupled control is a special strategy used to control MIMO-systems with the same amount  $m$  of inputs and outputs, where each input is associated to the output on which it has the biggest influence. A SISO control strategy is associated to each I/O pair, resulting in a system of  $m$  SISO controllers. This strategy works well when the cross-couplings in the system are small. The latter can be evaluated with several tools, one example of which is the RGA-matrix.
- 1.2) In the helicopter model the cross couplings  $G_{12}(s)$  and  $G_{21}(s)$  are small and both inputs  $U_R$  and  $U_S$  have a very strong influence on the horizontal response of the vertical axis.
- 1.3) Decoupled control fails when cross-couplings cannot be neglected when they are large and their influence is as strong as the main couplings. In this situation other control strategies must be implemented, examples of which are LQR,  $H_\infty$ ,  $H_2$  or MPC. Another possibility is using a decoupling compensator.

### Lessons Learned 2: Input Limitations

- 2.1) What effect do the voltage and current limitations have on the helicopter? What needs to be modified to improve the system?
- 2.2) How are the input limitations considered in other control strategies (i.e. PID, Lead, Lag, etc.)?

### Answer 2

- 2.1) The voltage constraints limit the angular velocity of both motors and therefore the lift forces of the rotors. The current is proportional to the torque which in turn is proportional to the acceleration of the motor. This means that a current constraint limits the motor acceleration.

A possible solution to improve the system would be increasing the current limit, enlarging rotor acceleration. Increasing the voltage limit would instead increase the maximum rotor speed.

- 2.2) When implementing other control strategies the input limitations aren't considered. Since the gain  $K$  leads to an output proportional to the input, it can be tuned so the control inputs do not reach the limitations.

### Lessons Learned 3: Lead Compensation applied to Helicopter Model

- 3.1) What influence do the parameters  $K, \omega$  and  $\alpha$  have on the control behavior?
- 3.2) Why is there a constant control error when controlling the horizontal axis but not when controlling the vertical axis?

### Answer 3

- 3.1) See slides from **Control Systems I**.
- 3.2) The control errors on the horizontal axis can be seen in Figure 2.7. This shows that the open loop transfer function  $G_{11}(s)$  also requires an integrator and not just the lead compensator  $G_{r\phi}(s)$ . The open loop system  $G_{22}(s)$  contains an integrator and no steady-state error is observed on the vertical axis (see Figure 2.4).

### Lessons Learned 4: Completion of the Experiment

- 15.1) Please, fill out the online feedback form on the registration page under **MyExperiments**. Each student/participant has to fill out his/her own feedback form. This will help us to improve the experiment.
- 15.2) Thank you for your help. You can now discuss the lab session with the TA to get your testate.

### Answer 4

- 15.1) No solution necessary.
- 15.2) No solution necessary.

# Appendix A

## Parameter

This chapter presents how the helicopter parameters were derived. In the end you can find a list of all the parameters used in the equations. The full understanding of this chapter is not necessary for the completion of the experiment. You can continue reading chapter B and come back to this section for a deeper explanation regarding the unclear parameters. The parameters in this list have been coded in the file `Helipar.m`.

### A.1 Motor Bridge Module

The parameters of the motor bridge module are listed in Table 1.1 on page 5. Under the assumption that the PWM-signal of the motor bridge module is very small, it can be modeled as a dynamic-free model with voltage limit of  $\pm 16$  V (main rotor) and  $\pm 8$  V (rear rotor) and current limit  $\pm 10$  A (bursts of 2 s).

### A.2 Propeller

	Main Rotor (Index $R$ )	Rear Rotor (Index $S$ )
Mass	0.0592 kg	0.0159 kg
Diameter	0.30 m	0.15 m
Pitch	4.5°	4.5°

Table A.1: Propeller Parameters

### A.3 Moment of Inertia of the Main Rotor

The moment of inertia of the rotor was obtained by dividing the system in simpler parts, calculating the moments of inertia of the single pieces and subsequently summing them up. The formulas for the simplified moments of inertia are as follows. The moment of inertia of a rod on its transverse axis is:

$$\Theta = \frac{1}{12}ml^2 . \quad (\text{A.1})$$

Moment of Inertia of cylinder along axis of symmetry:

$$\Theta = \frac{1}{2}mr^2 . \quad (\text{A.2})$$

Density of the rod:

$$\rho_{Stahl} = 7900 \text{ kg/m}^3 \quad (\text{A.3})$$

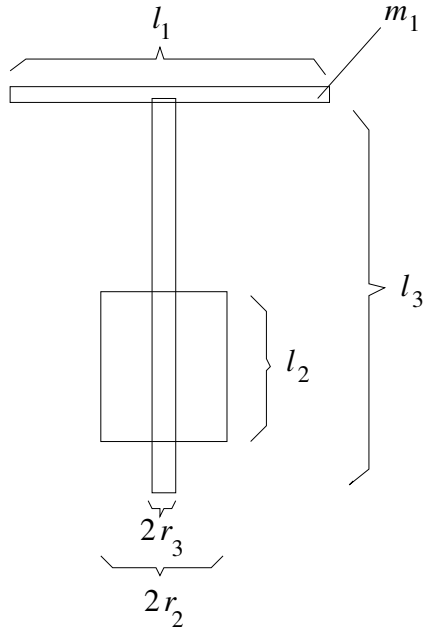


Figure A.1: Rotor dimension

The dimensions of the rotor  $R$  according to A.1 are

$$\begin{aligned}
 m_1 &= 0.0592 \text{ kg} \\
 l_1 &= 0.30 \text{ m} \\
 r_2 &= 0.016 \text{ m} \\
 l_2 &= 0.045 \text{ m} \\
 r_3 &= 0.002 \text{ m} \\
 l_3 &= 0.09 \text{ m} .
 \end{aligned}
 \tag{A.4}$$

The value of  $m_1$  hasn't been proven by the authors; the values were measured without actually disassembling the rotors, which means they are only approximate values. The moment of inertia of the main rotor  $\Theta_R$  is obtained via

$$\Theta_R = \underbrace{\frac{1}{12}m_1l_1^2}_{\text{Propeller}} + \underbrace{\frac{1}{2}\rho\pi l_2r_2^4}_{\text{Rotor angle}} + \underbrace{\frac{1}{2}\rho\pi(l_3 - l_2)r_3^4}_{\text{Rotor axis}} = 4.8 \cdot 10^{-4} \text{ kg m}^2 .
 \tag{A.5}$$

## A.4 Moment of Inertia of the Rear Rotor

The procedure for finding the moment of inertia of the rear rotor is similar to the one used for the main rotor, found in A.3. The necessary formulas can also be found there. With the

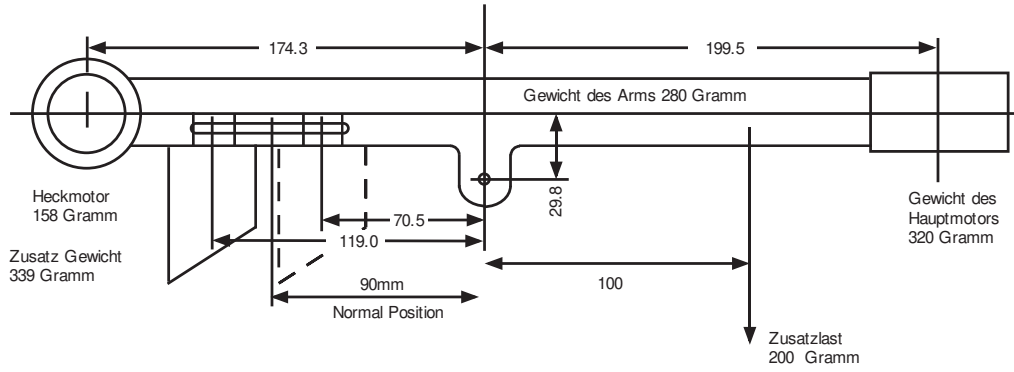


Figure A.2: Fuselage Dimensions.

parameters listed in Appendix A.1 the rear rotor is specified as follows

$$\begin{aligned}
 m_1 &= 0.0159 \text{ kg} \\
 l_1 &= 0.150 \text{ m} \\
 r_2 &= 0.010 \text{ m} \\
 l_2 &= 0.020 \text{ m} \\
 r_3 &= 0.0015 \text{ m} \\
 l_3 &= 0.070 \text{ m} .
 \end{aligned} \tag{A.6}$$

The value of  $m_1$  hasn't been proven by the authors; the values were measured without actually disassembling the rotor, which means they are only approximate values. The moment of inertia of the main rotor  $\Theta_S$  is obtained via

$$\Theta_S = \frac{1}{12} m_1 l_1^2 + \frac{1}{2} \rho \pi l_2 r_2^4 + \frac{1}{2} \rho \pi (l_3 - l_2) r_3^4 = 3.2 \cdot 10^{-5} \text{ kg m}^2 . \tag{A.7}$$

## A.5 The Motors

The rotors make use of DC-motors, described in App. A.10. The values considered come from similar models and shouldn't be considered precise.

## A.6 Moment of Inertia of the Fuselage on the Horizontal Axis ( $\phi$ -Axis)

The moment of inertia of the fuselage  $\Theta_H$  is calculated by disassembling the fuselage in geometrically simpler pieces. The dimensions and weights of the fuselage pieces can be found in App. A.2.

Values are given in Table A.2. The moment of inertia of the rod is

$$\Theta_{beam_H} = \frac{1}{12} m_1 (r_R + r_S)^2 + m_1 \left( \frac{r_R - r_S}{2} \right)^2 . \tag{A.8}$$

The second term of the sum is due to the translation of the rotation axis from the middle point. The motor and propeller assembly is approximated as a point mass and the value of  $r_V$  is neglected, leading to

$$\Theta_H = \Theta_{beam_H} + (m_2 + m_3) r_R^2 + (m_4 + m_5) r_S^2 + m_6 r_G^2 = 0.027 \text{ kg m}^2 . \tag{A.9}$$

Mass of the bar	$m_1$	= 0.280 kg
Mass of the main rotor	$m_2$	= 0.330 kg
Mass of the main propeller	$m_3$	= 0.0592 kg
Mass of the rear rotor	$m_4$	= 0.158 kg
Mass of the rear propeller	$m_5$	= 0.0159 kg
Mass of the balance	$m_6$	= 0.339 kg
Distance of the main motor	$r_R$	= 0.1995 m
Distance of the rear motor	$r_S$	= 0.1743 m
Distance of the balance	$r_G$	= 0.0953 m
Height of the rod	$r_V$	= 0.0298 m

Table A.2: Fuselage parameters

## A.7 Centre of Gravity

The next equations make use of the following total mass:

$$m_G = \sum_{i=1}^6 m_i = 1.182 \text{ kg} . \quad (\text{A.10})$$

The centre of gravity is considered the same as the pivot point (On the main rotor side). The horizontal difference from the pivot point is given by

$$r_H = \frac{1}{m_G} \left( m_1 \frac{r_R - r_S}{2} + (m_2 + m_3)r_R - (m_4 + m_5)r_S - m_6 r_G \right) = 1.57 \cdot 10^{-2} \text{ m} . \quad (\text{A.11})$$

## A.8 Moment of Inertia of the Fuselage on the Vertical Axis ( $\psi$ -Achse)

The moment of inertia of the fuselage  $\Theta_V$  is calculated by disassembling the fuselage in geometrically simpler pieces. The dimensions and weights of the fuselage pieces can be found in App. A.2. This moment of inertia is calculated from the position  $\phi$ .

$$\Theta_V = \Theta_{zyl} + \Theta_{V_1} \cos^2 \phi + \Theta_{V_2} \sin^2 \phi + m_G (r_H \cos \phi + r_V \sin \phi)^2 , \quad (\text{A.12})$$

where

$$\Theta_{V_1} \approx \Theta_H = 0.027 \text{ kg m}^2 \quad (\text{A.13})$$

is the moment of inertia on the perpendicular axis, and

$$\Theta_{V_2} \approx 2.0 \cdot 10^{-4} \text{ kg m}^2 \quad (\text{A.14})$$

is the moment of inertia of the rod. The cylindrical stand can be approximated to

$$\Theta_{zyl} \approx 10^{-5} \text{ kg m}^2. \quad (\text{A.15})$$

The final component of the sum comes from shifting the rotation point from the centre of mass. The values  $\Theta_{V_2}$  and  $\Theta_{zyl}$  were approximated from those of similar objects.

## A.9 Sensors

Every axis is equipped with a potentiometer to measure the angle of rotation. The angles have constraints, since for instance the helicopter cannot move freely around the vertical axis. The sensors with relative constraints can be found in table 1.2 from page 6.

## A.10 List of Parameters

The following is a list of all the parameters that can be found in the differential equations in chapter B, which are already initialized in the matlab file `Helipar.m`.



Model Parameters

$r_V$	=	0.0298	m	Pivot point – Horizontal axis
$r_H$	=	0.0157	m	Pivot point – Centre of gravity
$r_G$	=	0.0953	m	Pivot point – Balance
$r_R$	=	0.1995	m	Pivot point – Main rotor
$r_S$	=	0.1743	m	Pivot point – Rear rotor
$m_G$	=	1.19	kg	Total weight of the helicopter
$g$	=	9.81	m/s <sup>2</sup>	

Parameters of the Main Rotor

$\Theta_R$	=	$4.8 \cdot 10^{-4}$	kg m <sup>2</sup>	Moment of inertia
$c_{\mu R}$	=	$10^{-5}$	N m s/rad	Friction coefficient
$k_{MR}$	=	$10^{-6}$	N s <sup>2</sup> /rad <sup>2</sup>	Propeller coefficient (torque)
$c_{MR}$	=	0.028	N m/A	Motor constant
$c_{GR}$	=	0.028	V s/rad	Generator constant
$R_R$	=	0.9	$\Omega$	Motor friction
$i_{max}$	=	10	A	Maximum rotor current
$u_{max}$	=	16	V	Maximum rotor voltage

Parameters of the Main Rotor

$\Theta_S$	=	$3.2 \cdot 10^{-5}$	kg m <sup>2</sup>	Moment of inertia
$c_{\mu S}$	=	$10^{-5}$	N m s/rad	Friction coefficient
$k_{MS}$	=	$10^{-7}$	N s <sup>2</sup> /rad <sup>2</sup>	Propeller coefficient (torque)
$c_{MS}$	=	0.0091	N m/A	Motor constant
$c_{GS}$	=	0.0091	V s/rad	Generator constant
$R_S$	=	1.2	$\Omega$	Motor friction
$i_{max}$	=	10	A	Maximum rotor current
$u_{max}$	=	8	V	Maximum rotor voltage

Horizontal Axis Parameters

$\Theta_H$	=	0.027	kg m <sup>2</sup>	Moment of inertia
$c_{\mu H}$	=	$5 \cdot 10^{-3}$	N m s/rad	Friction coefficient (bearing)
$k_{FR}$	=	$2.4 \cdot 10^{-5}$	N s <sup>2</sup> /rad <sup>2</sup>	Propeller constant (power)
$\phi_{maxH}$	=	$60^\circ \cdot \pi/180^\circ$	rad	Maximum angle horizontal axis
$\phi_{minH}$	=	$-\phi_{maxH}$	rad	Minimum angle horizontal axis

Vertical Axis Parameters

$\Theta_{V1}$	=	0.027	kg m <sup>2</sup>	Moment of inertia
$\Theta_{V2}$	=	$2.0 \cdot 10^{-4}$	kg m <sup>2</sup>	Moment of inertia
$\Theta_{zyl}$	=	$10^{-5}$	kg m <sup>2</sup>	Moment of inertia
$c_{\mu V}$	=	$5 \cdot 10^{-2}$	N m s/rad	Friction coefficient (bearing)
$k_{FS}$	=	$2.7 \cdot 10^{-6}$	N s <sup>2</sup> /rad <sup>2</sup>	Propeller constant (power)
$\phi_{maxV}$	=	$170 \cdot \pi/180$	rad	Maximum angle vertical axis
$\phi_{minV}$	=	$-\phi_{maxV}$	rad	Minimum angle vertical axis

Sensor Parameters

$k_{messH}$	=	3.29	V/rad
$k_{messV}$	=	3.36	V/rad

Table A.3: Model parameters used in equations

# Appendix B

## Modelling

This chapter explains the helicopter model. We encourage you to read the following chapter to understand the explanations, even if the full comprehension of this section isn't necessary for the completion of the experiment.

From App. B.1 to B.3 the less obvious effects are explained, while App. B.4 and B.5 show the derivation of the differential equations of motion.

### B.1 Spin

This section shows what effects the change in rotation direction of the propeller has. It can be easily shown that the rear rotor doesn't have any influence on the movable axis. It is a lot more complicated to understand the influence of the main rotor. As shown in App. B.1, the spin  $L$  of the main rotor is

$$\vec{L}_{abs} = \Psi \Phi \vec{L}_{rel} = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ L \end{bmatrix}, \quad (\text{B.1})$$

where

$$L = \Theta_R \omega_R. \quad (\text{B.2})$$

From

$$\frac{d}{dt} \vec{L}_{abs} = \Psi \Phi \frac{d}{dt} \vec{L}_{rel} + \frac{d\Psi}{dt} \Phi \vec{L}_{rel} + \Psi \frac{d\Phi}{dt} \vec{L}_{rel} \quad (\text{B.3})$$

we can highlight the first term (spin change), shown in (B.34), and the second term can be expressed as

$$\vec{M}_{D_{abs}} = \frac{d\Psi}{dt} \Phi \vec{L}_{rel} + \Psi \frac{d\Phi}{dt} \vec{L}_{rel} \quad (\text{B.4})$$

where

$$\frac{d}{dt} \Psi = \omega_\psi \begin{bmatrix} -\sin \psi & -\cos \psi & 0 \\ \cos \psi & -\sin \psi & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (\text{B.5})$$

and

$$\frac{d}{dt} \Phi = \omega_\phi \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sin \phi & \cos \phi \\ 0 & -\cos \phi & -\sin \phi \end{bmatrix} \quad (\text{B.6})$$

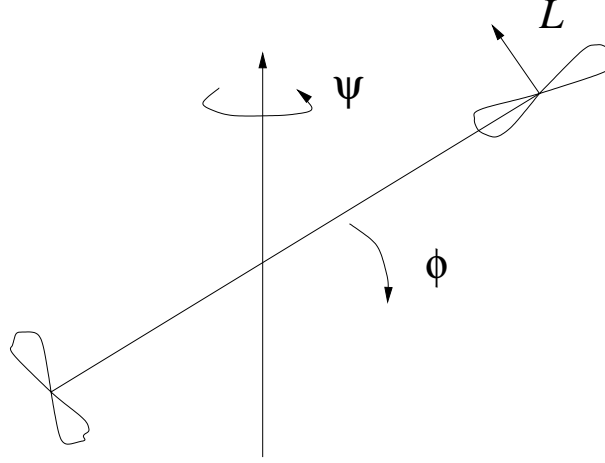


Figure B.1: Spin vector

follow (with the abbreviation  $s\alpha := \sin(\alpha)$  und  $c\alpha := \cos(\alpha)$ )

$$\begin{aligned} \vec{M}_{D_{abs}} &= \omega_\psi \begin{bmatrix} -s\psi & -c\psi c\phi & -c\psi s\phi \\ c\psi & -s\psi c\phi & -s\psi s\phi \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ L \end{bmatrix} + \omega_\phi \begin{bmatrix} 0 & s\psi s\phi & -s\psi c\phi \\ 0 & -c\psi s\phi & c\psi c\phi \\ 0 & -c\phi & -s\phi \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ L \end{bmatrix} \\ &= \omega_\psi \begin{bmatrix} -c\psi s\phi \\ -s\psi s\phi \\ 0 \end{bmatrix} L + \omega_\phi \begin{bmatrix} -s\psi c\phi \\ c\psi c\phi \\ -s\phi \end{bmatrix} L . \end{aligned} \quad (\text{B.7})$$

The equation for the spin moment on the vertical axis is as follows:

$$M_{DV} = [0 \ 0 \ 1] \vec{M}_{D_{abs}} = -\Theta_R \omega_R \omega_\phi \sin \phi . \quad (\text{B.8})$$

The torque on the horizontal axis is

$$\begin{aligned} \Psi^{-1} \vec{M}_{D_{abs}} &= \Psi^{-1} \omega_\psi \begin{bmatrix} -c\psi s\phi \\ -s\psi s\phi \\ 0 \end{bmatrix} L + \Psi^{-1} \omega_\phi \begin{bmatrix} -s\psi c\phi \\ c\psi c\phi \\ -s\phi \end{bmatrix} L \\ &= \begin{bmatrix} c\psi & s\psi & 0 \\ -s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \omega_\psi \begin{bmatrix} -c\psi s\phi \\ -s\psi s\phi \\ 0 \end{bmatrix} L + \Psi^{-1} \omega_\phi \begin{bmatrix} 0 \\ c\phi \\ -s\phi \end{bmatrix} L \\ &= \begin{bmatrix} -s\phi \\ 0 \\ 0 \end{bmatrix} \omega_\psi L + \begin{bmatrix} 0 \\ c\phi \\ -s\phi \end{bmatrix} \omega_\phi L \end{aligned} \quad (\text{B.9})$$

$$M_{DH} = [1 \ 0 \ 0] \Psi^{-1} \vec{M}_{D_{abs}} = -\Theta_R \omega_R \omega_\psi \sin \phi . \quad (\text{B.10})$$

## B.2 Centripetal and Coriolis Forces

When considering speed  $\omega_V$  on the vertical axis we must also take into account the rotation on the horizontal axis, since it introduces centripetal and Coriolis forces on the fuselage. App. B.2

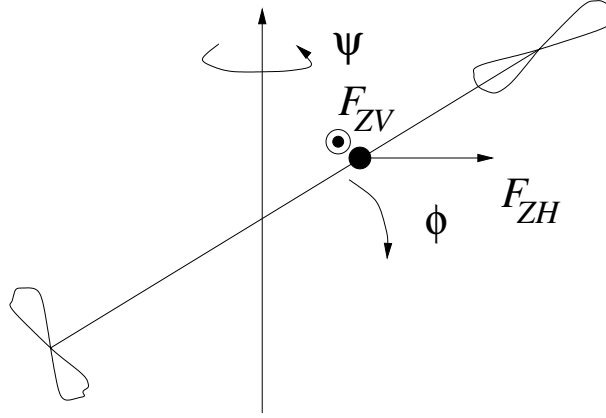


Figure B.2: Centripetal and Coriolis forces

presents the centripetal and Coriolis forces. Considering rotation around the center of gravity, calculations are as follows:

$$F_{ZH} = m_G \omega_V^2 (r_H \cos \phi_H + r_V \sin \phi_H) , \quad (\text{B.11})$$

resp.

$$F_{ZV} = 2m_G \omega_V \omega_H (r_H \cos \phi_H + r_V \sin \phi_H) . \quad (\text{B.12})$$

### B.3 Aerodynamic Effects

The air resistance on the propeller can be given as

$$M_L = k_M \text{sign}(\omega) \omega^2 \quad (\text{B.13})$$

while the lift is

$$F = k_F \text{sign}(\omega) \omega^2 . \quad (\text{B.14})$$

These quadratic formulas are simplifications of the actual equations. In this experiment we also neglect ground effects.

### B.4 Equations derivation

The helicopter can move around two axis of rotation, meaning we have to construct torque equations around both axis. Next we will state what the torque equations for the rotors are. Since we are dealing with four second order equations, we expect to have 8 possible solutions. Since only the rotor speed affects the system, we can ignore the rotor position, leading to a system of the 6<sup>th</sup> order.

1. Motor:

$$i_R = \frac{U_R - c_{GR} \omega_R}{R_R} \quad (\text{B.15})$$

$$i_S = \frac{U_S - c_{GS} \omega_S}{R_S} \quad (\text{B.16})$$

2. Torque of the main rotor (Index R):

$$\sum_i M_{iR} = \underbrace{M_{RR}}_{\text{Mainrotor}} - \underbrace{M_{LR}}_{\text{Airresistance}} - \underbrace{M_{\mu R}}_{\text{Friction}} \quad (\text{B.17})$$

$$M_{RR} = c_{MR} \dot{r}_R = c_{MR} \frac{U_R - c_{GR} \omega_R}{R_R} \quad (\text{B.18})$$

$$M_{LR} = k_{MR} \text{sign}(\omega_R) \omega_R^2 \quad (\text{B.19})$$

$$M_{\mu R} = c_{\mu R} \omega_R \quad (\text{B.20})$$

3. Torque on the horizontal axis (Index H):

$$\sum_i M_{iH} = - \underbrace{M_{RH}}_{\text{Mainrotor}} + \underbrace{M_{SH}}_{\text{rearrotor}} - \underbrace{M_{\mu H}}_{\text{Friction}} + \underbrace{M_{GH}}_{\text{Speed}} + \underbrace{M_{ZH}}_{\text{centripetal}} - \underbrace{M_{DH}}_{\text{Spin}} \quad (\text{B.21})$$

$$M_{RH} = k_{FR} \text{sign}(\omega_R) \omega_R^2 r_R \quad (\text{B.22})$$

$$M_{SH} = c_{MS} \dot{r}_S \quad (\text{B.23})$$

$$M_{\mu H} = c_{\mu H} \omega_H \quad (\text{B.24})$$

$$M_{GH} = m_G g (r_H \cos \phi_H + r_V \sin \phi_H) \quad (\text{B.25})$$

$$M_{ZH} = m_G \omega_V^2 (r_H \cos \phi_H + r_V \sin \phi_H) \\ (r_V \cos \phi_H - r_H \sin \phi_H) \quad (\text{B.26})$$

$$M_{DH} = -\omega_R \omega_V \Theta_R \sin \phi_H \quad (\text{B.27})$$

4. Torque of the rear rotor (Index S):

$$\sum_i M_{iS} = \underbrace{M_{SS}}_{\text{Rearrotor}} - \underbrace{M_{LS}}_{\text{Airresistance}} - \underbrace{M_{\mu S}}_{\text{Friction}} \quad (\text{B.28})$$

$$M_{SS} = c_{MS} \dot{r}_S = c_{MS} \frac{U_S - c_{GS} \omega_S}{R_S} \quad (\text{B.29})$$

$$M_{LS} = k_{MS} \text{sign}(\omega_S) \omega_S^2 \quad (\text{B.30})$$

$$M_{\mu S} = c_{\mu S} \omega_S \quad (\text{B.31})$$

5. Torque on the vertical axis (Index V):

$$\sum_i M_{iV} = \underbrace{M_{SV}}_{\text{Rearrotor}} - \underbrace{M_{RV}}_{\text{Mainrotor}} - \underbrace{M_{\mu V}}_{\text{Friction}} + \underbrace{M_{ZV}}_{\text{Coriolis}} + \underbrace{M_{DV}}_{\text{Spin}} \quad (\text{B.32})$$

$$M_{SV} = k_{FS} \text{sign}(\omega_S) \omega_S^2 (r_S \cos \phi_H - r_V \sin \phi_H) \quad (\text{B.33})$$

$$M_{RV} = c_{MR} \dot{r}_R \cos \phi_H \quad (\text{B.34})$$

$$M_{\mu V} = c_{\mu V} \omega_V \quad (\text{B.35})$$

$$M_{ZV} = 2m_G \omega_V \omega_H (r_H \cos \phi_H + r_V \sin \phi_H) \\ (r_V \cos \phi_H - r_H \sin \phi_H) \quad (\text{B.36})$$

$$M_{DV} = \omega_H \omega_R \Theta_R \sin \phi_R \quad (\text{B.37})$$

## B.5 Differential Equations

The following part lists all equations of the system. The torque equations are as follows:

$$\Theta \frac{d\omega}{dt} = \sum_i M_i \quad (\text{B.38})$$

$$\frac{d\phi}{dt} = \omega \quad (\text{B.39})$$

Full system:

$$\frac{d\omega_R}{dt} = \frac{1}{\Theta_R} \left( \frac{c_{MR}}{R_R} \tilde{U}_R - \frac{c_{MR}c_{GR}}{R_R} \omega_R - k_{MR} \text{sign}(\omega_R) \omega_R^2 - c_{\mu R} \omega_R \right) \quad (\text{B.40})$$

$$\frac{d\phi_H}{dt} = \omega_H \quad (\text{B.41})$$

$$\begin{aligned} \frac{d\omega_H}{dt} = \frac{1}{\Theta_H} & \left[ -k_{FR} \text{sign}(\omega_R) \omega_R^2 r_R + \frac{c_{MS}}{R_S} \tilde{U}_S - \frac{c_{MS}c_{GS}}{R_S} \omega_S \right. \\ & + m_G g (r_H \cos \phi_H + r_V \sin \phi_H) - c_{\mu H} \omega_H \\ & - m_G \omega_V^2 (r_H \cos \phi_H + r_V \sin \phi_H) (r_V \cos \phi_H - r_H \sin \phi_H) \\ & \left. + \omega_R \omega_V \Theta_R \sin \phi_H \right] \quad (\text{B.42}) \end{aligned}$$

$$\frac{d\omega_S}{dt} = \frac{1}{\Theta_S} \left( \frac{c_{MS}}{R_S} \tilde{U}_S - \frac{c_{MS}c_{GS}}{R_S} \omega_S - k_{MS} \text{sign}(\omega_S) \omega_S^2 - c_{\mu S} \omega_S \right) \quad (\text{B.43})$$

$$\frac{d\phi_V}{dt} = \omega_V \quad (\text{B.44})$$

$$\begin{aligned} \frac{d\omega_V}{dt} = \frac{1}{\Theta_V} & \left[ k_{FS} \text{sign}(\omega_S) \omega_S^2 (r_S \cos \phi_H - r_V \sin \phi_H) \right. \\ & - \frac{c_{MR}}{R_R} \tilde{U}_R \cos \phi_H + \frac{c_{MR}c_{GR}}{R_R} \omega_R \cos \phi_H - c_{\mu V} \omega_V + \omega_H \omega_R \Theta_R \sin \phi_H \\ & \left. + 2m_G \omega_V \omega_H (r_H \cos \phi_H + r_V \sin \phi_H) (r_V \cos \phi_H - r_H \sin \phi_H) \right] \quad (\text{B.45}) \end{aligned}$$

where

$$\Theta_V = \Theta_{zyl} + \Theta_{V_1} \cos^2 \phi_H + \Theta_{V_2} \sin^2 \phi_H + m_G (r_H \cos \phi_H + r_V \sin \phi_H)^2 \quad (\text{B.46})$$

$$\tilde{U}_S = \text{sat}(U_S) \quad (\text{B.47})$$

$$\tilde{U}_R = \text{sat}(U_R) , \quad (\text{B.48})$$

and

$$\text{sat}(x) = \begin{cases} x & |x| \leq u_{max} \\ u_{max} \cdot \text{sign}(x) & |x| > u_{max} \end{cases} \quad (\text{B.49})$$

These equations are coded in the Simulink model `Heli_Sim.mdl`, where they can be viewed.

# Appendix C

## Matlab m-Files

### C.1 Helipar.m

```
% Allgemeine Modellparameter
r_V = 0.0298; % Abstand zur horizontalen Achse
r_H = 0.0157; % Abstand zum Schwerpunkt
r_G = 0.0953; % Abstand des Gegengewichts
r_R = 0.1995; % Abstand zum Tragrotor
r_S = 0.1743; % Abstand zum Heckrotor
m_G = 1.19; % Gesamtgewicht des Helikopters
g = 9.81;

% Parameter des Tragrotors
theta_R = 4.8e-4; % Traegheitsmoment
c_muR = 1e-5; % Reibungskonstante
k_MR = 1e-6; % Propellerkonstante
c_MR = 0.028; % Motorkonstante
c_GR = 0.028; % Generatorkonstante
R_R = 0.9; % Motorwiderstand
i_max = 10; % Maximaler Rotorstrom
u_max_phi = 16; % Maximale Rotorspannung

% Parameter des Heckrotors
theta_S = 3.2e-5; % Traegheitsmoment
c_muS = 1e-5; % Reibungskonstante
k_MS = 1e-7; % Propellerkonstante
c_MS = 0.0091; % Motorkonstante
c_GS = 0.0091; % Generatorkonstante
R_S = 1.2; % Motorwiderstand
i_max = 10; % Maximaler Rotorstrom
u_max_psi = 8; % Maximale Rotorspannung

% Parameter der horizontalen Achse
theta_H = 0.027; % Traegheitsmoment
c_muH = 5e-3; % Reibungskonstante (Lagerreibung und Kabel)
k_FR = 2.4e-5; % Propellerkonstante
phi_maxH= 60*pi/180; % Maximaler Winkel der horizontalen Achse
phi_minH= -phi_maxH; % Minimaler Winkel der horizontalen Achse

% Parameter der vertikalen Achse
theta_V1 = 0.027; % Traegheitsmoment
theta_V2 = 2.0e-4; % Traegheitsmoment
theta_zyl= 1e-5; % Traegheitsmoment
c_muV = 5e-2; % Reibungskonstante (Lagerreibung und Kabel)
k_FS = 2.7e-6; % Propellerkonstante
phi_maxV = 170*pi/180;% Maximaler Winkel der vertikalen Achse
phi_minV = -phi_maxV; % Minimaler Winkel der vertikalen Achse

% Sensoren
k_messH = 3.29;
k_messV = 3.36;

% Gleichgewichtslage
omega_R_b = 202.0147;
omega_S_b = 301.6782;
phi_FF = 7.0331;
psi_FF = 4.3432;
```

```

% Lineares Modell
a11 = -c_MR*c_GR/R_R - 2*k_MR*omega_R_b-c_muR;
a31 = -2*k_FR*r_R*omega_R_b;
a32 = m_G*g*r_V;
a33 = -c_muH;
a34 = -c_MS*c_GS/R_S;
a44 = -c_MS*c_GS/R_S - 2*k_MS*omega_S_b - c_muS;
a61 = c_MR*c_GR/R_R;
a62 = -k_FS*omega_S_b^2*r_V;
a64 = 2*k_FS*r_S*omega_S_b;
a66 = -c_muV;

A0 = [ a11  0  0  0  0  0  0 ;
        0  0  1  0  0  0  0 ;
        a31 a32 a33 a34  0  0 ;
        0  0  0 a44  0  0 ;
        0  0  0  0  0  0  1 ;
        a61 a62  0 a64  0 a66 ];

B0 = [c_MR/R_R  0 ;
        0  0 ;
        0 c_MS/R_S ;
        0 c_MS/R_S ;
        0  0 ;
        -c_MR/R_R  0 ];

THETA = diag([theta_R, 1, theta_H, theta_S, 1, theta_zyl+theta_V1+m_G*r_H^2]);

A = inv(THETA)*A0;
B = inv(THETA)*B0;
C = [0 k_messH 0 0 0 0;
      0 0 0 0 k_messV 0];
D = [0 0;
      0 0];

% -----
% Simulation Parameters
% -----

phistart_H = 0;
phistart_V = 0;

```

## C.2 HeliTest.m

```

%Run the helicopter simulation
if(~exist('phistart_H') | ~exist('phistart_V') )
    error('Please specify phistart_H and phistart_V before running HeliTest');
    return;
end

disp('Simulation wird gestartet!');
figure(1);
sim('Heli_Sim');

plot(t,Usphi, '-.', t, Uspsi, '-.', t, Uphi, '-', t, Upsi, '-');
xlabel('Zeit t');
ylabel('Amplitude');
title('Sprünge auf beide Rotoren');
legend('Sprung auf U_{\phi}', 'Sprung auf U_{\psi}', ...
       'Winkel \phi in [V]', 'Winkel \psi in [V]', 1);

```

## C.3 reduce.m

```

function [out] = reduce(in)
% cuts away entries in the vector that are smaller than some relative
% accuracy.
n = length(in);
maxin = max(abs(in));
epsilon = 1e-8;
for i=1:n
    if (abs(in(i)) < epsilon*maxin)
        in(i) = 0;
    end
end
out = in;
return;

```



## C.4 Heli\_data\_log.m

```
% Heli_data_log.m
% Lädt die Messungen auf den Host-PC via FTP.
%
% (c) Marc Osswald, Sept 2009

file = 'heli_record.csv';           % define file name
ftp_object = ftp('autx20-02');     % set ftp server address
mget(ftp_object,file);             % get the data
data = csvread(file,1,0);          % read the data
PHI_desired = data(:,1);           % desired phi
PHI_actual = data(:,2);            % actual phi
PSI_desired = data(:,3);           % desired psi
PSI_actual = data(:,4);            % actual psi

% TODO: Add your code here
```

## C.5 theoaufg.m

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% theoaufg.m
% Lösungen zu den theoretischen Aufgaben (Versuch 2.6).
%
% Revision (after adding new hardware): Marc Osswald, Aug. 2009
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

format compact

s= tf('s');

% Kompensator fuer phi Achse: Grphi
% -----

kRphi = 8;
w_phi = 4;
alpha_phi = 0.1;

Grphi = kRphi*(1/w_phi*s+1)^2/(alpha_phi/w_phi*s+1)^2;

% Kompensator fuer psi Achse: Grpsi
% -----

kRpsi = 1;
w_psi = 3;
alpha_psi = 0.1;

Grpsi = kRpsi*(1/w_psi*s+1)/(alpha_psi/w_psi*s+1);

% Modell
%-----

Helipar
eps = 1e-1;

% Anfangsbedingungen
%-----
phistart_H = phi_maxH;
phistart_V = 0;
x0          = [0 phistart_H 0 0 phistart_V 0];

disp('Eigenvalues of the System')
disp('-----')
ewA = eig(A)
P   = ss(A,B,C,D);

[n1,d1] = ss2tf(A,B,C,D,1);
[n2,d2] = ss2tf(A,B,C,D,2);

n11 = n1(1,:)*(-1); % NOTE: sign considered here
n12 = n1(2,:);

n21 = n2(1,:);
n22 = n2(2,:);

d11 = d1;
d12 = d1;
```

```

d21 = d2;
d22 = d2;

% Cancels common pole/zeros
G11m = minreal(tf(n11,d11),eps);
G12m = minreal(tf(n12,d12),eps);
G21m = minreal(tf(n21,d21),eps);
G22m = minreal(tf(n22,d22),eps);

[n11m,d11m] = tfdata(G11m,'v');
[n12m,d12m] = tfdata(G12m,'v');
[n21m,d21m] = tfdata(G21m,'v');
[n22m,d22m] = tfdata(G22m,'v');

n11r = reduce(n11m);
n12r = reduce(n12m);
n21r = reduce(n21m);
n22r = reduce(n22m);

G11r = tf(n11r,d11m);
G12r = tf(n12r,d12m);
G21r = tf(n21r,d21m);
G22r = tf(n22r,d22m);

% Bode Plots der Strecken
%-----

figure(1)
margin(G11r)
%title('G11')

figure(2)
margin(G22r)
%title('G22')

% Offene Kreise
% -----

G11o = series(Grphi,G11r);
G22o = series(Grpsi,G22r);

figure(3)
margin(G11o);
%title('G11o');

figure(4)
margin(G22o);
%title('G22o');

% Geschlossene Kreise
%-----

Gphi2 = feedback(G11o,1);
Gpsi2 = feedback(G22o,1);

poles_Gphi2 = pole(Gphi2);
poles_Gpsi2 = pole(Gpsi2);

% Schrittantworten
%-----

figure(5)
step(Gphi2)
title('closed loop \phi: Step response')

figure(6)
step(Gpsi2)
title('closed loop \psi: Step response')

format loose

```

## C.6 steady\_state\_calc.m

```

% solving the nonlinear system of equations in 3.1, i.e. computing the steady
% state values of the heli
%
% (c) by Stefan Richter, Sept 08

```

```

clear all

global c_MR R_R c_GR R_R k_MR c_muR
global k_FR r_R c_MS R_S c_GS m_G g r_H
global k_MS c_muS
global k_FS r_S

% load model parameters
Helipar

% do fminsearch
start_point = [6 200 3 300]; % from measurements in the lab
options.TolFun = 1e-10;
opt_sol = fminsearch(@ss_residuals, start_point, options)

ss_residuals(opt_sol)

```

## C.7 ss\_residuals.m

```

function res = ss_residuals(x)

global c_MR R_R c_GR R_R k_MR c_muR
global k_FR r_R c_MS R_S c_GS m_G g r_H
global k_MS c_muS
global k_FS r_S

U_R = x(1);
w_R = x(2);
U_S = x(3);
w_S = x(4);

f1 = c_MR*U_R/R_R - c_MR*c_GR*w_R/R_R - k_MR * sign(w_R) * w_R^2 - c_muR * w_R;
f2 = -k_FR * sign(w_R) * w_R^2 * r_R + c_MS * U_S / R_S - c_MS * c_GS / R_S * w_S + m_G * g * r_H;
f3 = c_MS * U_S / R_S - c_MS * c_GS * w_S/R_S - k_MS * sign(w_S) * w_S^2 - c_muS * w_S;
f4 = k_FS * sign(w_S) * w_S^2 * r_S - c_MR * U_R / R_R + c_MR * c_GR * w_R / R_R;

res = f1^2 + f2^2 + f3^2 + f4^2;

```

## C.8 linear\_control\_loop\_run.m

```

% run-file for linear_control_loop.mdl
% Uses the linearized model of the helicopter and controls with
% Lead/Double-Lead Compensator
%
% (c) by Stefan Richter, modified by Marc Osswald in September 2009

%% 1st case: Everything linear, i.e. no actuator saturations, no output
% saturations
clear all
close all

% load heli parameters from helipar.m
Helipar

% compute the controllers from theoaufg.m
theoaufg
close all

% used config
act_sat = 0;
output_sat = 0;
x0 = [0; 10*pi/180; 0; 0; 0; 0]; % initial state of plant

sim_time_span = 5;

[T,X,Y] = sim('linear_control_loop',sim_time_span,[])
plot(T,Y(:,1),'k-', T,Y(:,2),'k--')
grid on
legend({'Horizontalwinkel \phi', 'Vertikalwinkel \psi'})
% title(['Eigenverhalten des geschlossenen Kreises für Anfangszustand ', mat2str(x0 .* 180/pi), ' des Helis'])
ylabel('Winkel [deg]')
xlabel('t [s]')
print(gcf, '-deps2', 'eigen_geschl_kreis.eps')

%% 2nd case: Everything linear, BUT actuator saturations, output

```

```

% saturations
clear all
close all

% load heli parameters from helipar.m
helipar

% compute the controllers from theoaufg.m
theoaufg
close all

% used config
act_sat = 1;
output_sat = 1;
x0 = [0; 10*pi/180; 0; 0; 0; 0]; % initial state of plant

sim_time_span = 5;

[T,X,Y] = sim('linear_control_loop',sim_time_span,[])
plot(T,Y(:,1),'k-', T,Y(:,2),'k--')
grid on
legend({'Horizontalwinkel \phi','Vertikalwinkel \psi'})
% title(['Eigenverhalten des geschlossenen Kreises für Anfangszustand ', mat2str(x0 .* 180/pi), ' des Helis '])
ylabel('Winkel [deg]')
xlabel('t [s]')
print(gcf, '-deps2', 'eigen_geschl_kreis_sat.eps')

```

## C.9 nonlinear\_control\_loop\_run.m

```

% run-file for non_linear_control_loop.mdl
% Uses the nonlinear model of the helicopter and controls with
% Lead/Double-Lead Compensator around the setpoint
%
% (c) by Stefan Richter, modified by Marc Osswald in September 2009

%% 1st case: Nonlinear with actuator saturations
% saturations
clear all
close all

% load heli parameters from helipar.m
Helipar

% compute the controllers from theoaufg.m
theoaufg
close all

% used config
phistart_H = 10*pi/180; % initial state of plant (horizontal axis)
phistart_V = 0; % initial state of plant (vertical axis)

sim_time_span = 10;

[T,X,Y] = sim('nonlinear_control_loop',sim_time_span,[]);
plot(T,Y(:,1),'k-', T,Y(:,2),'k--')
grid on
legend({'Horizontalwinkel \phi','Vertikalwinkel \psi'})
ylabel('Winkel [deg]')
xlabel('t [s]')
print(gcf, '-deps2', 'eigen_geschl_kreis_nonlinear.eps')

```