Computation of Radiation Leakage in Photonic Crystal Waveguides

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ABSTRACT

We have performed accurate analysis of radiation leakage in the propagation plane of photonic crystal (PhC) waveguides using a modification of the classical supercell approach. We have opened the classical supercell in the direction perpendicular to the propagation direction in order to take the radiation losses into account. Using this so-called "open supercell", the waveguide eigenvalue analysis becomes complex valued as well as the constant for the periodic boundary condition. Quite surprising results have been achieved when analyzing the radiation loss along PhC waveguide channels for various numbers of confining crystal layers. We can show, that for a certain range of frequencies PhC waveguides surrounded by only one crystal layer may still support efficient waveguiding operation.

INTRODUCTION

Photonic crystal (PhC) structures [1, 2] are very attractive because they offer a new way to design and fabricate devices in high-density integrated optics [3]. Here, the photonic bandgap [4] is essential for obtaining ultra-compact optical waveguiding [5, 6] providing sharp bends [7] and functional device like multiplexers [8] and power splitters [9].

The computation of band diagrams [1, 2] that describe perfect PhCs is well known and can be performed with all kind of numerical methods. Essentially, the periodic symmetry is taken into account wile solving a corresponding eigenvalue problem. Waveguides in PhCs mostly consist of linear defects, i.e., modifications of the geometry as well as of the material properties along a line in the lattice [5, 6]. These defects break the periodic symmetry of the perfect crystal in the direction perpendicular to the waveguide. The resulting problem is still periodic in the direction of the waveguide. Since the photonic crystal structure, i.e., the domains and boundaries, extend to infinity in the directions perpendicular to the waveguide, one encounters severe numerical problems for all numerical methods. In the following, we outline the most widely used supercell approach [1, 2] and we present a new, direct approach that has several advantages. For reasons of simplicity, we focus on 2D PhCs.

SUPERCELL APPROACH

The main idea of the supercell approach is to re-establish some periodic symmetry in the direction perpendicular to the waveguide by considering an infinite array of identical, parallel waveguides at a constant distance of $n \cdot d$ between neighboring waveguides, where d denotes the lattice period of the underlying PhC in the direction perpendicular to the waveguide. As a consequence, one again obtains periodic symmetry in the direction perpendicular to the waveguide that is comprised by a single "supercell" encompassing now n unit cells of the original crystal. This considerably increases the numerical effort, but it allows one to use the same codes as for the analysis of the perfect photonic crystal. The method may obviously only provide good results when the coupling between the parallel waveguides is weak enough, i.e., when n is large enough. With increasing n, the computation time for the band diagram is obviously increased because of the increasing size of the supercell, but it is additionally increased due to the appearance of various extended modes given by the manifold symmetry transformations associated with the formation of the supercell's 1st Brillouin zone. The latter modes are easily retrievable when comparing Figures 1 and 2. For this reason, the supercell approach turns out to be numerically inefficient.

Another drawback of the supercell approach stems from the fact that it is not realistic at all. Realistic waveguides must have a finite size, i.e., they have only a limited number of crystal layers on both sides in order to confine the PhC waveguide mode. As a consequence, realistic waveguides inevitably exhibit radiation loss that cannot be computed with the classical super-cell approach.

In order to illustrate the super-cell approach, we have selected a simple perfect crystal based on circular dielectric rods arranged on a rectangular lattice. The band structure is computed using the Multiple Multiple Program (MMP) [10] that is implemented in MaX-1 [11]. As one can see from Figure 1, bandgaps appear only for E-polarization. The first, relatively wide bandgap usually specifies the frequency range where waveguides and more sophisticated structures within the PhC may operate. By introducing line defects, defect modes with an electromagnetic field localized in the proper area of the defect channel may be obtained for wavelengths well within the bandgap. According to Figure 1 the first bandgap covers a relatively wide frequency range: $\omega \cdot a/(2 \cdot \pi \cdot c) \in [0.3, 0.44]$. The simplest way to produce a PhC waveguide is to introduce vacancies of rods along a line. The vacancies form a guiding channel. The surrounding crystal provides total reflection. As mentioned before, this reflection is not perfect when the widths of the crystals on both sides of the channel are finite. As a result, the modes in the neighboring channels of the supercell model are coupled. Beside this, the number of defect modes that propagate in the channel and their properties depend on the original PhC, on the defect itself, and on the frequency. The dispersion diagram of the defect waveguide based on a classical supercell approach is shown in Figure 2. As one can see, the waveguide supports single-mode propagation within almost the entire bandgap. There is a portion of the bandgap where no waveguide mode exists. By proper modifications of the defect one may obtain additional modes and one may even increase the size of the bandgap.



Figure 1. Basic properties of a perfect crystal and its band structure; **Top:** Original lattice, 1st (irreducible) Brillouin zone, lattice constant and material properties. **Bottom:** Band diagram for E-polarization (left-hand side) and H-polarization. (right-hand side). Bandgaps exist only in the case of E-polarization (gray areas).



Figure 2. Line defect waveguide (vacancies) and detail of the corresponding classical supercell. **Top:** Section of the band diagram around the first bandgap (left-hand side) and supercell (right-hand side). **Bottom:** Detail of the irreducible 1st Brillouin zone. Many extended modes appear above and below the bandgap (gray area). They are not interesting for practical applications. A single defect mode (i.e. the defect waveguide mode) covers almost the entire bandgap.

DIRECT APPROACH

In order to evaluate the radiation loss, we have considered a more realistic model – we also call it the open-supercell approach. - that is periodic and infinite along the waveguide, but consists of a finite number of cells on each side of the waveguide channel. Outside these cells, there is a constant dielectric material that extends to infinity. In this dielectric, radiation is observed because the finite PhC does not provide perfect light confinement (see Figure 3). As a consequence, the "guided" waves propagating along the waveguide are lossy and must be characterized by a complex-valued propagation constant even when all dielectrics are assumed to be lossless. This finally leads to a complex eigenvalue problem that turns out to be much more demanding than the real eigenvalue problem associated with the classical supercell approach. Since we do not re-establish the broken symmetry perpendicular to the direction of the waveguide, we only have periodic symmetry in one direction. Thus, there is only a one-dimensional (complex-valued) reciprocal lattice space left. Furthermore, one can limit the eigenvalue search to the first bandgap, which leads to a considerable simplification and reduction of the numerical costs. In PhC waveguides the periodicity constant that spans the one-dimensional reciprocal lattice space plays the same role as the propagation constant of a cylindrical waveguide. Therefore, we associate this constant to the propagation constant. In our notation, the real part of the propagation constant describes the field propagation along the waveguide and its imaginary part accounts for the wave attenuation.

General waveguide analysis [10] either assigns the frequency (at a fixed propagation constant) or the propagation constant (for a predefined frequency) as the eigenvalue that has to be retrieved. In case of the classical supercell approach the former procedure is commonly used whereas for lossy waveguides with complex propagation constants, the latter is reasonable. When radiation loss becomes sufficiently small, one obviously can ignore the attenuation constant and hence the imaginary part of the propagation constant, leading to a real eigenvalue search for real-valued propagation constants. The attenuation of the complex propagation constant is efficient, but limited to waveguides with small loss. From the practical point of view these are the waveguides we are actually looking for. Its worth mentioning that based on our experience, even the direct complex eigenvalue search (making use of the proposed open-supercell scheme) is numerically more efficient than a corresponding real eigenvalue search based on the classical supercell approach.



Figure 3. Model of the direct, "open-supercell" approach.

From the proposed eigenvalue analysis (see Figure 4), we obtain interesting results for the practical design of PhC waveguides: Modes that are sufficiently far away from cutoff may experience small attenuation loss even when only a single crystal layer is present on each side of the waveguide channel. Therefore, one might use sections of a PhC waveguide with such "thin walls" and appropriate absorbers outside the walls as mode strippers. As one can see from Figure 5, even sharp 90°-bends [7] are feasible with such kind of waveguide scheme having rather small radiation loss and small reflection coefficients. As we have shown, the direct approach does not only provide interesting information on the radiation loss of realistic PhC waveguides. It also is more efficient and much more reliable than the classical supercell approach, and also more akin to the computation of cylindrical waveguides.



Figure 4. Numerical results of complex eigenvalue search for different waveguides. **Top:** Poynting vector field for waveguides with different widths of the PhC walls at relative frequency $f \cdot a/c = 0.33$. **Bottom:** Real part (left-hand side) and imaginary part (right-hand side) of the propagation constant.



Figure 5. 90°-bend based on the single-crystal-layer-waveguide scheme. **Left:** The Poynting vector (linear scale). **Right:** The Poynting vector (logarithmic scale, inverted grayscale). Numerical results: $R \approx 7\%$ (power reflection), $T \approx 60\%$ (power transmission), $A_{rad} \approx 33\%$ (radiation).

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