1 Mass Balance and Glacier Geometry

We consider an Alpine glacier with a constant surface slope of β, and the highest elevation at z_0. To simplify calculations, a unit glacier width can be assumed. The glacier is in its steady state and in this problem set we assume that for mass balance calculations $z_s = z_b$ (surface elevation equals bed elevation), i.e. we only consider a position-dependent specific mass balance rate. For problem parts a) - c) work with symbols, only (no numeric values).

a) Balance flux The volumetric specific mass balance rate has the form $\dot{b}_i(x) = a_1 x + a_0$. What are the signs of a_1 and a_0? Calculate and plot the balance flux with distance x along a flow line.

b) Glacier length How long is the glacier? What is the length L_{acc} of the accumulation area? Note that the glacier length is independent of the ice flow law. (This calculation is only possible if the elevation of the glacier surface is not taken into account for the mass balance calculation).

c) Glacier thickness Use the *shallow ice approximation* ice flow formula to determine the maximum thickness of the glacier. How does this thickness depend on surface slope β?

Hint: calculate the maximum balance flux, and set that equal to the ice flux formula (2.12), and where $\frac{dH}{dx}$ should be replaced by $\tan\alpha$ (why?).

d) Balance gradient A relationship between specific mass balance rate and glacier flow line distance has little meaning in terms of climate and climate change. Rather, one uses an elevation-dependent volumetric specific mass balance rate such as $\dot{b}_i(z) = \dot{g}(z_s(x) - z_{ELA})$, where z_{ELA} is the elevation of the equilibrium line.

First, make sure you understand the units of $\dot{g}$. We talk about the volume of ice that is added (or removed) per unit area of the glacier during a year, and $\dot{g}$ is the vertical gradient thereof.

Express parameters a_1 and a_0 in terms of given quantities. For $z_{ELA} = 3000\,\mathrm{m}$, $z_0 = 3500\,\mathrm{m}$, $\beta = 5°$ and $\dot{g} = 0.007\,\mathrm{a}^{-1}$ calculate numerical values for the length of the accumulation area L_{acc}, the whole glacier's length L, and the balance flux.

e) Glacier change By how much does the flux change if the equilibrium line altitude (ELA) is shifted by $\Delta z_{ELA} = \pm 100\,\mathrm{m}$? Where are the changes biggest, and why?

2 Derivation of Ice Sheet Profile

Consider again the differential equation of the surface profile of a viscous ice sheet (as derived in the course script)

$$\dot{b}x = \frac{2A}{n+2}\left(\rho g \left|\frac{dH}{dx}\right|\right)^n H^{n+2}\,. \tag{1}$$

Show that

$$H^{2+2/n} = K\left(L^{1+1/n} - x^{1+1/n}\right) \tag{2}$$

is indeed a solution to Equation 1 with

$$K = \frac{2(n+2)^{1/n}}{\rho g}\left(\frac{\dot{b}}{2A}\right)^{1/n}\,. \tag{3}$$

Hint: You can differentiate Equation 2 directly using the chain rule. Alternatively, you can solve Equation 2 first for H and then differentiate. In this case express terms involving L and x in terms of H to reduce the algebra.

3 Questions about the 2015 IPCC Report

The IPCC report is the best synthesis of climate science. The 2007 issue received wide attention, and won the authors a Nobel price. These questions are about Sections 4.3 and 4.4 (pages 335-357) of Chapter 4 ("Observation: Cryosphere") of the "Physical Science Basis" of the 2015 IPCC report. Off course you are invited to read all of it!

You can download the report at http://www.ipcc.ch/report/ar5/wg1/

Places It is instructive to visit the glaciers and ice sheets mentioned in the report with a mapping software such as Google Earth (http://earth.google.com) or Worldwind.

Rapid changes Consider the dynamic reactions of glaciers and ice sheets as discussed in class and as documented in the IPCC report. Explain to what extent these elements of the cryosphere can be expected to react to recent and ongoing climate changes such as documented in Neukom et al. (2019), *No evidence for globally coherent warm and cold periods over the preindustrial Common Era* (Nature, 571(7766), 550-554). It is important that your answer to this question is concise (1/2 page).