



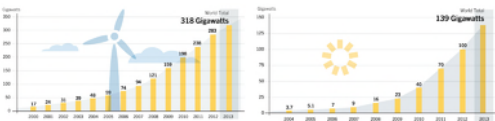
# Control of Low-Inertia Power Systems

Florian Dörfler, ETH Zürich

DFG Autumn School @ KIT 2019

# (recent) power systems control challenges

→ integration of renewable sources



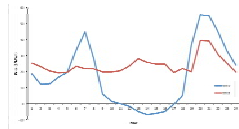
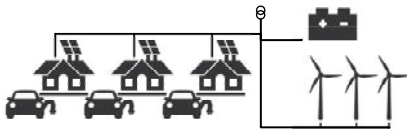
→ changing generation technology



→ scaling



→ distributed generation & prosumption → liberalized markets



## opportunities:

- converter-interfaced sources
- fast / modular / flexible actuation
- technological advances
- sensing / actuation / communication
- scientific advances
- control / optimization / learning
- ⇒ **end-to-end & real-time automation** of cyber-socio-technical power system

# Replacing the system foundation



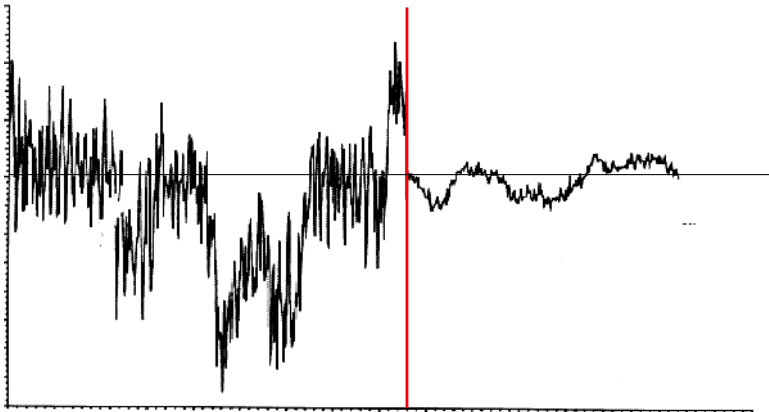
## fuel & synchronous machines

- not sustainable
- + central & dispatchable generation
- + large rotational inertia as buffer
- + self-synchronize through the grid
- + resilient voltage / frequency control
- slow actuation & control

## renewables & power electronics

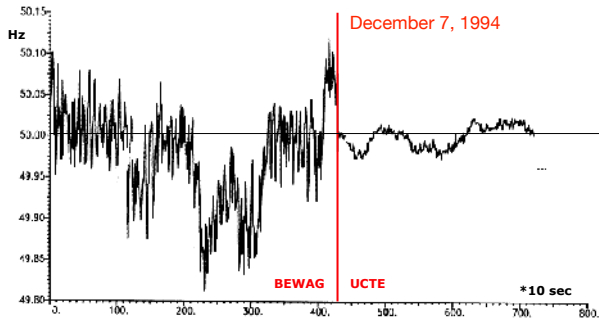
- + sustainable
- distributed & variable generation
- almost no energy storage
- no inherent self-synchronization
- fragile voltage / frequency control
- + fast / flexible / modular control

What do we see here ?



# West Berlin re-connecting to Europe

Source: *Energie-Museum Berlin*



**before** re-connection: islanded operation based on **batteries** & boiler

**afterwards** connected to European grid & **synchronous generation**

# The concerns are not hypothetical

issues broadly recognized by system operators, device manufacturers, & academia

theguardian

South Australia

South Australia blackout: entire state left without power after storms

## key events

- ▶ storm damages two lines
- ▶ control not resilient loss of 500 MW wind power
- ▶ between lines: conventional grid would have survived

## obstacle to sustainability

- ▶ integrating power electronics
- ▶ robust & resilient control

**INDEPENDENT**  
News > World > Australasia  
**Tesla's new mega-battery in Australia reacts to outages in 'record' time**  
One of Australia's biggest power plants suffered a drop in output - the new battery kicked in just 0.14 seconds later

**AEMO**  
Final Report – Queensland and South Australia system separation on 25 August 2018

**SIEMENS**

**Biblis A generator stabilizes the grid as a synchronous condenser**

However, as these sources are fully controllable, a regulation can be added to the inverter to provide "synthetic inertia". This can also be seen as a short term frequency support. On the other hand, these sources might be quite restricted with respect to the available

The relevance of inertia in power systems

Journal homepage: [www.elsevier.com/locate/epes](http://www.elsevier.com/locate/epes)

Review  
Committee  
ntso  
on  
Zone  
Group

# Critically re-visit modeling/analysis/control

## Foundations and Challenges of Low-Inertia Systems

(Invited Paper)

Federico Milano  
University College Dublin, Ireland  
email: federico.milano@ucd.ie

Florian Dörfler and Gabriela Hug  
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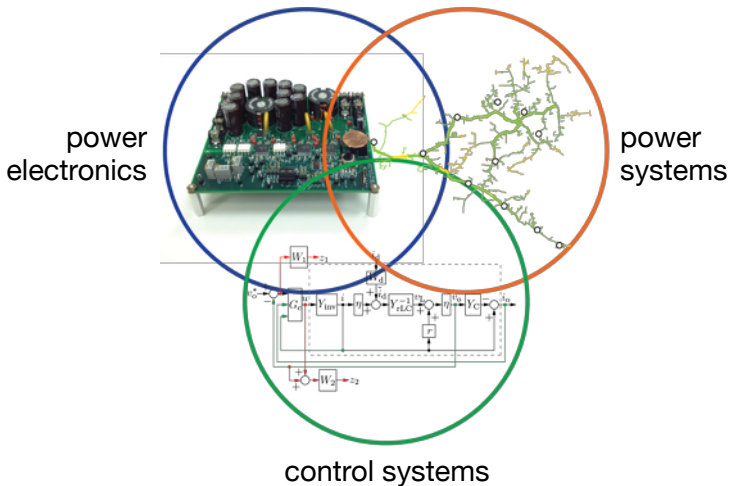
David J. Hill\* and Gregor Verbič  
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gregor.verbic@sydney.edu.au

The later sections contain many suggestions for further work, which can be summarized as follows:

- **New models** are needed which balance the need to include key features without burdening the model (whether for analytical or computational work) with uneven and excessive detail;
- **New stability theory** which properly reflects the new devices and time-scales associated with CIG, new loads and use of storage;
- Further **computational work** to achieve sensitivity guidelines including data-based approaches;
- **New control methodologies**, e.g. new controller to mitigate the high rate of change of frequency in low inertia systems;
- A power converter is a fully actuated, modular, and very fast control system, which are nearly antipodal characteristics to those of a synchronous machine. Thus, **one should critically reflect the control** of a converter as a virtual synchronous machine; and
- The lack of inertia in a power system does not need to (and **cannot**) be fixed by simply "adding inertia back" in the systems.

**a key unresolved challenge:** control of power converters in low-inertia grids  
→ industry & power community willing to explore **green-field approach** (see MIGRATE) with **advanced control** methods & **theoretical certificates**

# Exciting research domain bridging communities

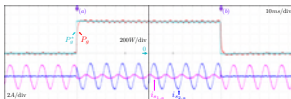
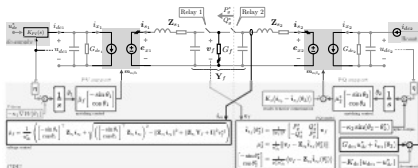




# Our research agenda

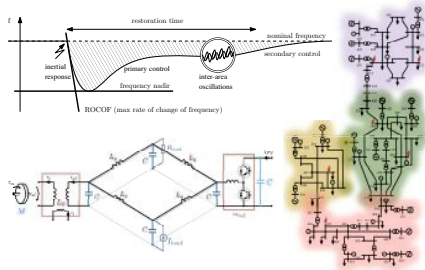
## device-level (power electronics)

- decentralized nonlinear **power converter control** strategies
- experimental **implementation, validation, & comparison**



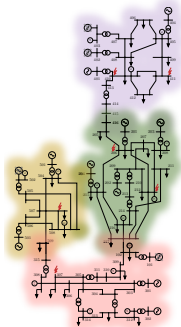
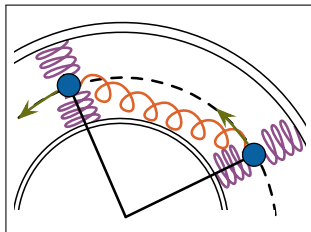
## system-level (low-inertia grid)

- low-inertia power system **models, stability, & performance** metrics
- **optimal allocation** of virtual inertia & fast-frequency response services



trying to **bridge the gap** from **device-level** to **system level** & from fundamental control **theory** to practical **experiments**

# Focus of today's tutorial



## modeling, control specifications, & game changers

- focus: fast time scales & old vs. new
- power system control specifications & limitations

## decentralized control of power converters

- grid-forming vs. grid-following: architectures & trade-offs
- grid-forming controls: VSM, droop, matching, & VOC

## effect of local controls in large-scale systems

- ancillary service perspective & optimal allocation

# All references & many more details in ...

## Foundations and Challenges of Low-Inertia Systems

(Invited Paper)

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## Distributed Control and Optimization for Autonomous Power Grids

Florian Dörfler   Saverio Bolognani   John W. Simpson-Porco   Sergio Grammatico

**Abstract**—The electric power system is currently undergoing a period of unprecedented changes. Environmental and sustainability concerns lead to replacement of a significant share of conventional fossil fuel-based power plants with renewable energy sources. As a result of this energy transition, centralized bulk generation based on fossil fuel and interfaced with synchronous machines is substituted by distributed

participation in general provide huge challenges as well as unprecedented opportunities to integrate an end-to-end automated and sustainable socio-technical system.

Parallel to these technological advances, the control, optimization, communication, computer science, and signal processing communities have developed novel methodological

<b>2020</b>		
<b>International Graduate School on Control</b>		
<a href="http://www.eeci-igsc.eu">www.eeci-igsc.eu</a>		
Independent Graduate Modules, one 21 hours module per week (3 ECTS)		
		
M11 – STOCKHOLM 13/04/2020-17/04/2020	Control and Optimization of Autonomous Power Systems	Florian Dörfler & Saverio Bolognani, Swiss Federal Institute of Technology (ETHZ), Switzerland

modeling,  
control specifications,  
& game changers

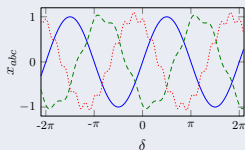
# Modeling: signal space in 3-phase AC

## three-phase AC

$$\begin{bmatrix} x_a(t) \\ x_b(t) \\ x_c(t) \end{bmatrix} = \begin{bmatrix} x_a(t+T) \\ x_b(t+T) \\ x_c(t+T) \end{bmatrix}$$

periodic with 0 average

$$\frac{1}{T} \int_0^T x_i(t) dt = 0$$

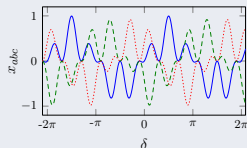


## balanced (nearly true)

$$= A(t) \begin{bmatrix} \sin(\delta(t)) \\ \sin(\delta(t) - \frac{2\pi}{3}) \\ \sin(\delta(t) + \frac{2\pi}{3}) \end{bmatrix}$$

so that

$$x_a(t) + x_b(t) + x_c(t) = 0$$

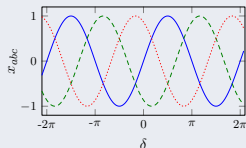


## synchronous (desired)

$$= A \begin{bmatrix} \sin(\delta_0 + \omega_0 t) \\ \sin(\delta_0 + \omega_0 t - \frac{2\pi}{3}) \\ \sin(\delta_0 + \omega_0 t + \frac{2\pi}{3}) \end{bmatrix}$$

const. freq & amp

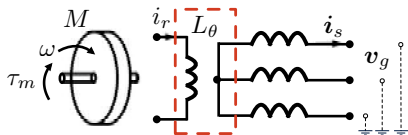
$\Rightarrow$  const. in rot. frame



**assumption** : balanced  $\Rightarrow$  2d-coordinates  $x(t) = [x_\alpha(t) \ x_\beta(t)]$  or  $x(t) = A(t)e^{i\delta(t)}$

**from currents/voltages to powers** : active  $p = v^\top i$  and reactive  $q = v^\top \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} i$

# Modeling: synchronous generator



$$\frac{d\theta}{dt} = \omega$$

$$M \frac{d\omega}{dt} = -D\omega + \tau_m + L_m i_r \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix}^\top \mathbf{i}_s$$

$$L_s \frac{d\mathbf{i}_s}{dt} = -R_s \mathbf{i}_s + \mathbf{v}_g - L_m i_r \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix} \omega$$

1. **primary energy supply**  $\tau_m$  from turbine converting thermal to mechanical energy (neglected)
2. mechanical  $(\theta, \omega)$  **swing dynamics** of rotor (flywheel) with inertia  $M$
3. **electro-mechanical energy conversion** through rotating magnetic field with inductance matrix

$$L_\theta = \begin{bmatrix} L_s & 0 & L_m \cos \theta \\ 0 & L_s & L_m \sin \theta \\ L_m \cos \theta & L_m \sin \theta & L_r \end{bmatrix}$$

(neglected  $i_r$  rotor current dynamics)

4.  $\mathbf{i}_s$  **stator flux dynamics** (sometimes including additional damper windings)
5. connection to grid with voltage  $\mathbf{v}_g$

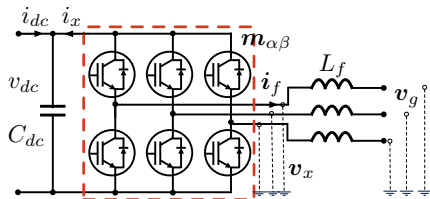
# Modeling: voltage source converter

1. **primary energy supply**  $i_{dc}$  from upstream DC boost converter or storage (neglected)
2.  $v_{dc}$  **DC charge dynamics** with capacitance  $C_{dc}$
3. **power electronics modulation**

$$i_x = -\mathbf{m}^\top \mathbf{i}_f \quad \text{and} \quad \mathbf{v}_x = \mathbf{m} v_{dc},$$

with averaged & normalized duty cycle ratios  $\mathbf{m} \in [-\frac{1}{2}, \frac{1}{2}] \times [-\frac{1}{2}, \frac{1}{2}]$

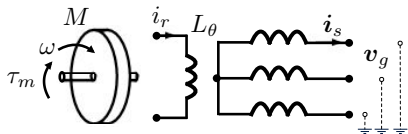
4.  $i_f$  **AC filter dynamics**  
(sometimes also LC or LCL filter)
5. connection to grid with voltage  $v_g$



$$C_{dc} \frac{dv_{dc}}{dt} = -G_{dc} v_{dc} + i_{dc} + \mathbf{m}^\top \mathbf{i}_f$$

$$L_f \frac{d\mathbf{i}_f}{dt} = -R_f \mathbf{i}_f + \mathbf{v}_g - \mathbf{m} v_{dc}$$

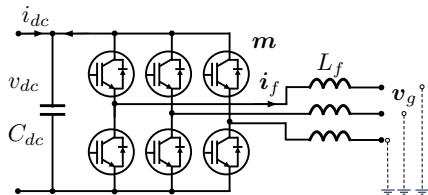
# Comparison: conversion mechanisms



$$\frac{d\theta}{dt} = \omega$$

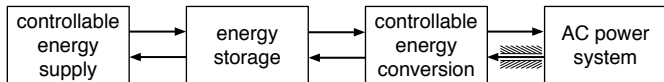
$$M \frac{d\omega}{dt} = -D\omega + \tau_m + L_m i_r \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix}^\top \mathbf{i}_s$$

$$L_s \frac{d\mathbf{i}_s}{dt} = -R_s \mathbf{i}_s + \mathbf{v}_g - L_m i_r \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix} \omega$$



$$C_{dc} \frac{dv_{dc}}{dt} = -G_{dc} v_{dc} + i_{dc} + \mathbf{m}^\top \mathbf{i}_f$$

$$L_f \frac{d\mathbf{i}_f}{dt} = -R_f \mathbf{i}_f + \mathbf{v}_g - \mathbf{m} v_{dc}$$



$\tau_m$  (slow)

vs.

$i_{dc}$  (fast)

$M$  (large)

vs.

$C_{dc}$  (small)

$L_\theta$  (physical)

vs.

$m$  (control)

resilient

vs.

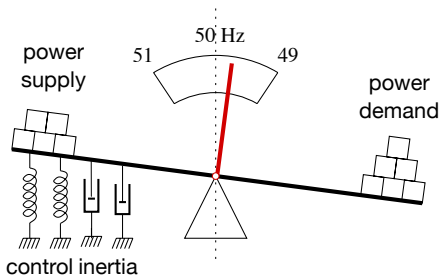
fragile

(over-currents)

physical & robust  
vs.  
controlled & agile  
**signal/energy  
transformer**



# Deceiving similarities & control limitations



**power balances** (neglecting small storage elements & losses):

$$\underbrace{\frac{d}{dt} \frac{1}{2} \omega^T M \omega}_{\text{internal energy}} = \underbrace{\omega^T \tau_m}_{\text{supply}} - \underbrace{i_s^T v_g}_{\text{demand}} + \underbrace{0}_{\text{conversion}}$$

$$\frac{d}{dt} \frac{1}{2} v_{dc}^T C_{dc} v_{dc} = i_{dc}^T v_{dc} - i_s^T v_g + 0$$

## Antipodal control characteristics

- large  $M$  vs. negligible  $C_{dc}$  **energy storage** for disturbance rejection
- slow  $\tau_m$  vs. fast  $i_{dc}$  **actuation of the energy supply** (though  $i_{dc}$  constrained)
- limited vs. full **actuation of the energy conversion** via  $L_\theta$  & modulation  $m$

- **state constraints:** tolerance to large vs. no over-currents

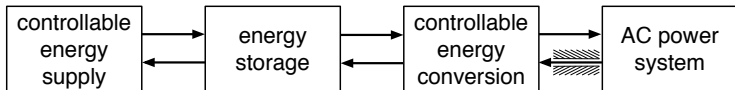
robust vs. agile  
resilient vs. fragile  
slow vs. fast actuation  
physical vs. control system

# Preview: pitfalls of naive inertia emulation

(naive) **baseline solution** :  
 inverter + storage + control  
 → emulate **virtual inertia**

... can & has been done **but**  
 recall **antipodal characteristics**

<p><b>Improvement of Transient Response in Microgrids Using Virtual Inertia</b>                  Nimaish Sen, Student Member, IEEE, Suryanarayana Doutha, Member, IEEE, and Mahesh C. Chandrasekar, Member, IEEE</p> <p><b>Virtual synchronous generator: A survey and new perspectives</b>                  Hassan Bevrani<sup>a,b,c</sup>, Toshifumi Ise<sup>b</sup>, Yushi Miura<sup>b</sup></p> <p><small><sup>a</sup>Dept. of Electrical and Computer Eng., University of Waterloo, 28 Wilfrid Laurier Ave. N., Waterloo, Ont. N2L 3G1, Canada; <sup>b</sup>Dept. of Electrical, Electronic and Information Eng., Osaka University, Suita, Japan; <sup>c</sup>Department of Electrical Engineering, University of Alameda, Alameda, CA, USA</small></p>	<p><b>Implementing Virtual Inertia in DFIG-Based Wind Power Generation</b>                  Amrindra Fakhri Mughaddam Azari, Student Member, IEEE, and Elhab-F. El-Saadany, Senior Member, IEEE</p> <p><b>Dynamic Frequency Control Support: A Virtual Inertia Provided by Distributed Energy Storage to Isolated Power Systems</b>                  Anthony Delille, Member, IEEE, Bruno François, Senior Member, IEEE, and Gilles Malarange</p>
<p><b>Inertia Emulation Control Strategy for VSC-HVDC Transmission Systems</b>                  Jibei Zhu, Campbell D. Booth, Grae P. Adam, Andrew J. Roscoe, and Chris G. Bright</p>	<p><b>Grid Tied Converter with Virtual Kinetic Storage</b>                  M.P.N van Wassenbeck<sup>1</sup>, S.W.H. de Haan<sup>1</sup>, Senior member, IEEE, P. Vantho<sup>1</sup> and K. Vincke<sup>2</sup></p>



slow vs. fast

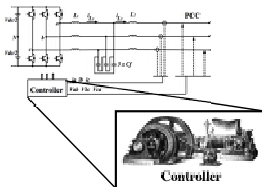
large vs. small

physics vs. control

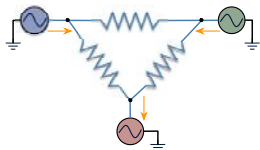
resilient vs. fragile

## telecom analogy (E. Mallada)

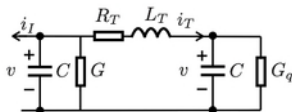
- works (under business-as-usual operation)
- there are better solutions (espec. for contingencies)



# Modeling: the network



interconnecting lines via  $\Pi$ -models & **ODEs**



- ▶ **conventional assumption:** quasi-steady state **algebraic model**

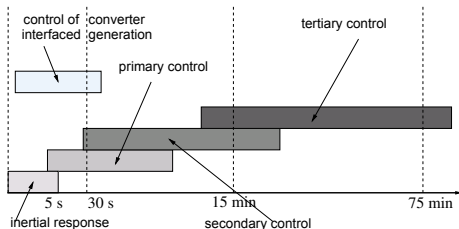
$$\underbrace{\begin{bmatrix} i_1 \\ \vdots \\ i_n \end{bmatrix}}_{\text{nodal injections}} = \underbrace{\begin{bmatrix} \vdots & \ddots & \vdots & \ddots & \vdots \\ -y_{k1} & \cdots & \sum_{j=1}^n y_{kj} & \cdots & -y_{kn} \\ \vdots & \ddots & \vdots & \ddots & \vdots \end{bmatrix}}_{\text{Laplacian matrix with } y_{k,j} = 1 / \text{complex impedance}} \underbrace{\begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}}_{\text{nodal potentials}}$$

- ▶ salient feature: **local** measurement reveals **synchronizing** coupling

$$\underbrace{i_k}_{\text{local variable}} = \underbrace{\sum_j y_{kj} (v_k - v_j)}_{\text{global sync}}$$

- ▶ but quasi-steady-state **assumption is flawed** in low-inertia systems

# Control specifications



- **nominal synchronous operation:**

- constant DC states:  $\dot{\omega} = \dot{v}_{dc} = 0$

- synchronous AC states at  $\omega_{ref}$ :

$$\dot{\theta} = \omega_{ref}, \quad \frac{d}{dt} \mathbf{i}_s = \begin{bmatrix} 0 & \omega_{ref} \\ -\omega_{ref} & 0 \end{bmatrix} \mathbf{i}_s, \dots$$

- set-points:  $\|\mathbf{v}_g\| = \mathbf{v}_{ref}$ ,

$$P \triangleq \mathbf{i}_s^T \mathbf{v}_g = P_{ref},$$

$$Q \triangleq \mathbf{i}_s^T \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \mathbf{v}_g = Q_{ref}$$

- **transient disturbance rejection & stabilization:**

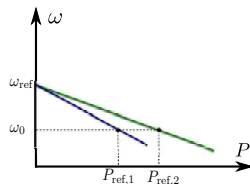
passively via physics (inertia) & actively via control

- **perturbed synchronous operation** at  $\omega \neq \omega_{ref}$  & power: deviations with specified sensitivities  $\partial P / \partial \omega$  (similar for  $v$ )

→ decentralized **droop/primary control**  $P - P_{ref} \propto \omega - \omega_{ref}$

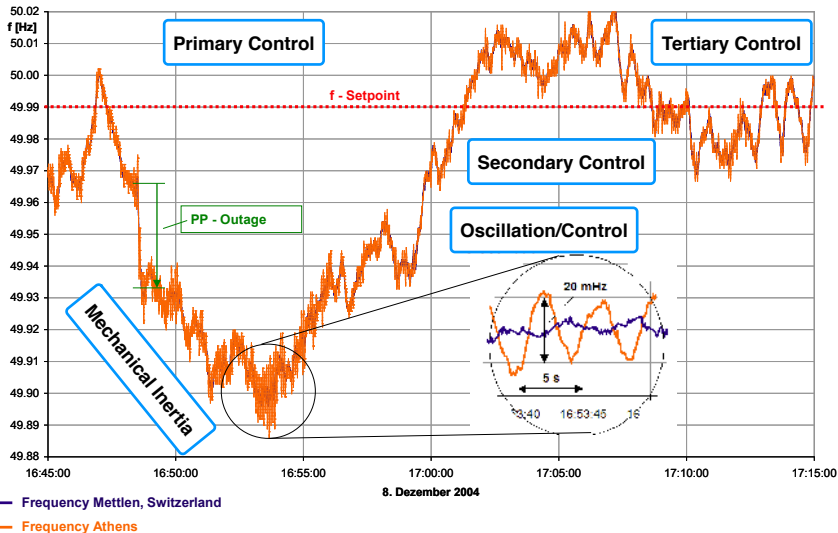
- **secondary control:** regulation of  $\omega \rightarrow \omega_{ref}$  (similar for  $v$ )

- **tertiary control:** (re)scheduling of set-points



} covered in other tutorials

# Controllers in action



Source: W. Sattinger, Swissgrid

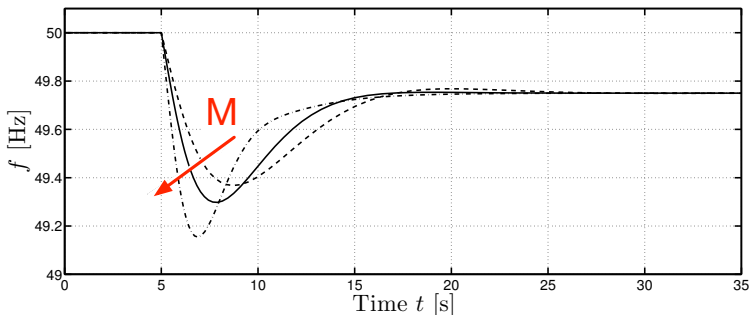
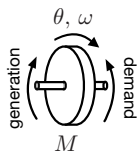
thought experiment:  
extrapolation to  
low-inertia systems

# Insight: loss of inertia & frequency stability

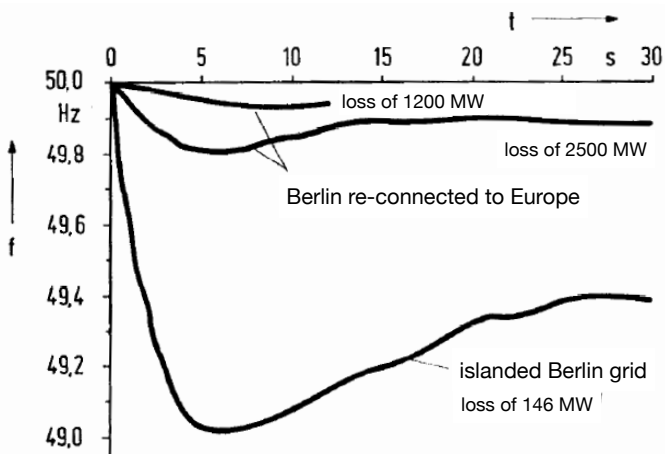
We lose our giant electromechanical low-pass filter:

$$M \frac{d}{dt} \omega(t) = P_{\text{generation}}(t) - P_{\text{demand}}(t)$$

change of kinetic energy = instantaneous power balance



# Berlin post-fault curves: before and after

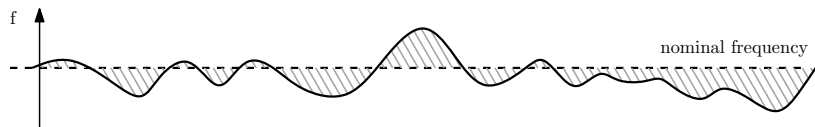


Source: *Energie-Museum Berlin*

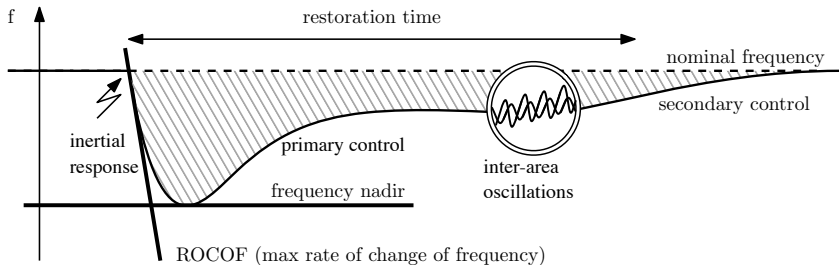


# This may be true up to first order ... **but**

- the **physics** of a low-inertia system are not any longer dominated by mechanical swing dynamics of synchronous machines
  - not just losing inertia but also tight **control** of frequency & voltage
  - distributed generation will lead to different **contingencies**
  - no more **separation** of  $(P, \omega)$  and  $(Q, \|v\|)$  dynamics / control
  - many **new phenomena**: line dynamics, subsynchronous oscillations, ...
- certainly more **brittle** behavior (faster time scales)
- for really low inertia levels **anything** can happen



In the long run: free yourself from thinking about power system stability / control as in the conventional text book picture



decentralized control  
of power converters

# Grid-forming & following converter control

	grid-following	grid-forming
<b>converter-type</b> (loose but very common definition)	current-controlled & frequency-following	voltage-controlled & frequency-forming
<b>measurement</b>	$(\omega, \ \mathbf{v}\ )$	$(P, Q)$
<b>set-point</b>	$(P, Q)$	$(\omega, \ \mathbf{v}\ )$
<b>dynamic reachability</b>	needs a stiff grid to track frequency	can operate in islanded mode & black-start grid

... **feedforward-controlled** (constant) power and voltage sources are forming & following → for many reasons **feedback control** is preferable

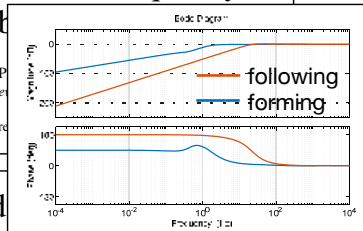
# Remark: definitions are debated

- put 20 experts in a room ... → no universal definition & many hybrid concepts
- many services can be provided both in grid-forming / -following mode
- previous definitions are **compromise** found in MIGRATE project, but we also came up with frequency-domain characterizations “sensitivity to grid frequency”

## Characterization of the Grid-forming function of a power source based on its external frequency

smoothing capacitor

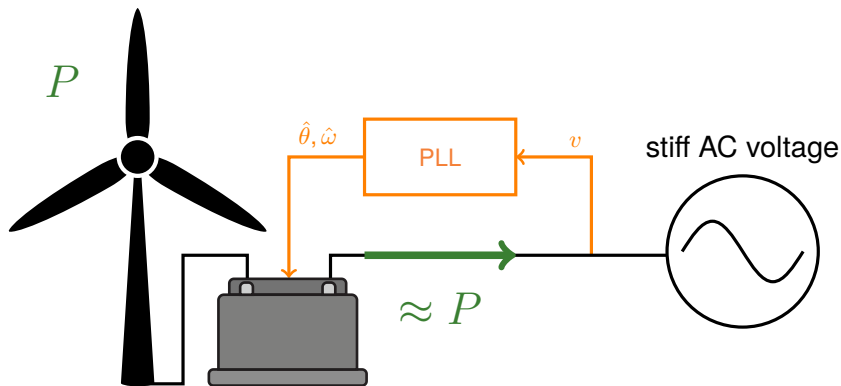
Debry Marie-Sophie, Denis Guillaume, P  
Réseau de Transport d'Electricité (Research and De  
La Défense  
marie-sophie.debry / guillaume.denis / thibault.pr



## $\mathcal{H}_\infty$ -Control of Grid-Connected Objectives and Decentralized Stability Certificates

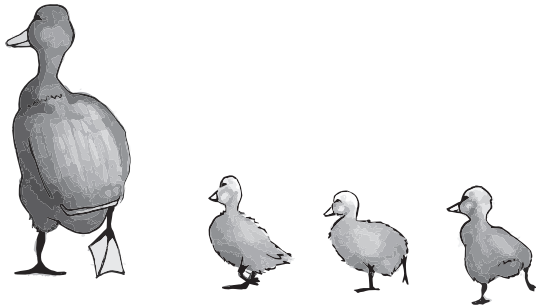
Linbin Huang, Huanhai Xin, and Florian Dörfler

# Limitations of grid-following control

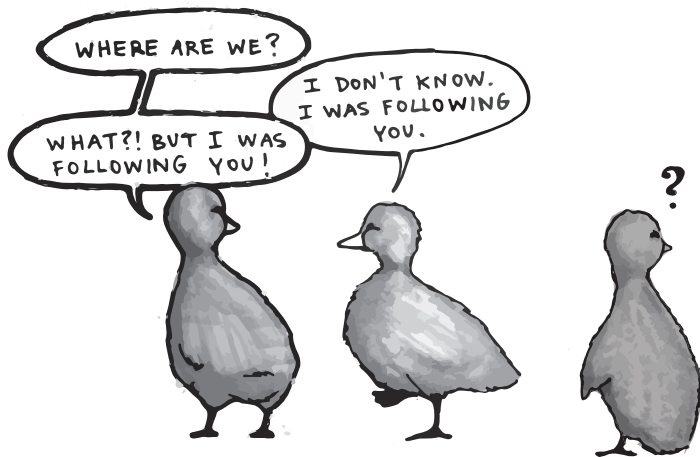


- **is good for** transferring power to a strong grid (main underlying assumption)
- **is not good for** providing a voltage reference, stabilization, or black start
- prevalent today, but **tomorrow's grid** needs (many) grid-forming sources

# A stiff grid with grid-following sources ...

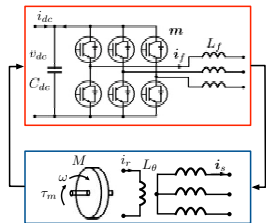


If everyone follows...

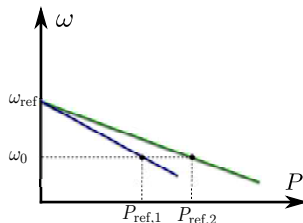




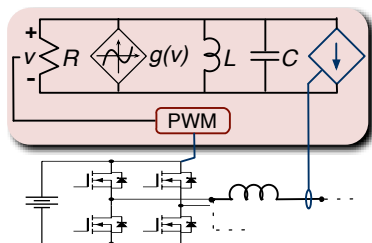
# Overview of grid-forming control strategies



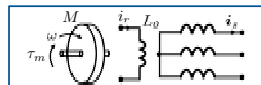
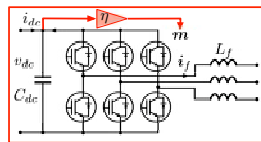
virtual synchronous machine



droop control

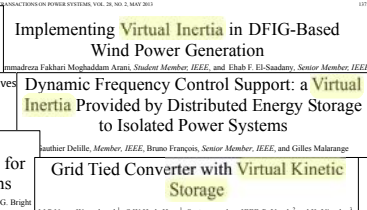
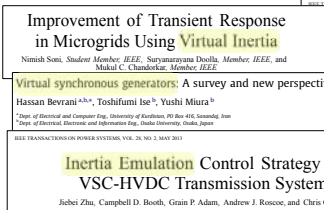


virtual oscillator control (VOC)



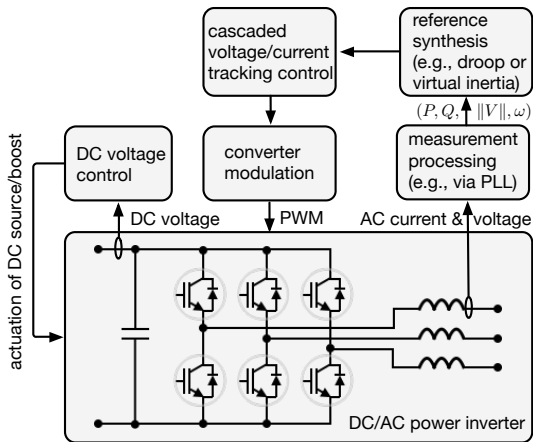
$v_{dc} \sim \omega$  matching control

# Naive baseline: virtual inertia emulation



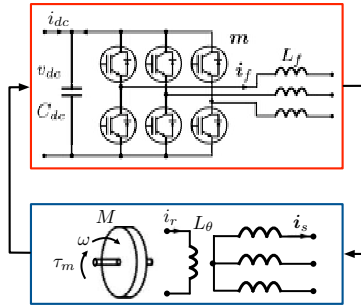
- ▶ **PD control** on  $\omega(t)$ :  $M \frac{d}{dt} \omega(t) + D(\omega(t) - \omega_0) = P_{\text{generation}}(t) - P_{\text{demand}}(t)$
- ▶ there are **smarter implementations** at the cost of algorithmic complexity

# Standard approach to converter control



1. acquiring & processing of **AC measurements**
2. synthesis of **references** (voltage/current/power)  
*"how would a synchronous generator respond now?"*
3. cascaded PI controllers to **track** references
4. **actuation** via modulation
5. **hidden assumption:** DC-side supply can **instantaneously** provide **unlimited power**

# Virtual synchronous machine emulation



- **reference** : detailed model of synchronous generator + controls
- **implementation** : low-pass filters for dissipation, virtual impedances for saturation, limiters,... tricks

→ most commonly **accepted solution** in **industry** (backward compatibility)

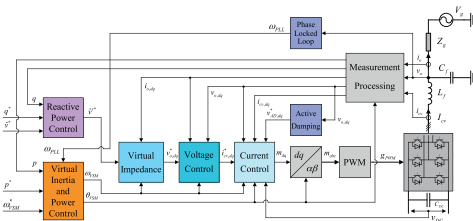
→ **over-parametrized** & ignores **DC source dynamics** and **limits**

→ **poor fit** for converter:

- converter: **fast** actuation & **no** significant **energy storage**
- machine: **slow** actuation & significant **energy storage**

→ **performs poorly** post-fault

→ **stability analysis is hopeless**



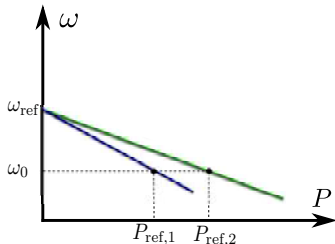
# Droop as simplest reference model

- ▶ **frequency control** by mimicking  $P - \omega$  droop property of synchronous machine:

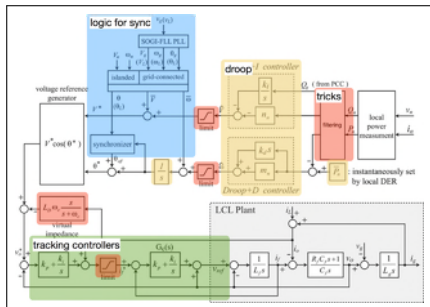
$$D(\omega - \omega_{\text{ref}}) = P - P_{\text{ref}}$$

- ▶ **voltage control** via  $Q - \|v\|$  droop:

$$\frac{d}{dt} \|v\| = -c_1(\|v\| - v_{\text{ref}}) - c_2(Q - Q_{\text{ref}})$$



- **reference** are generator controls
- direct control of  $(P, \omega)$  and  $(Q, \|v\|)$  **assuming they are independent** (approx. true only near steady state)
- ignores *DC* **source dynamics**
- requires **tricks in implementation** similar to virtual synchronous machine

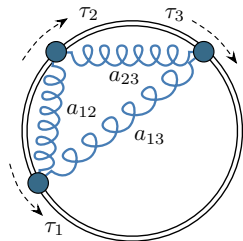


# Droop control is *de facto* baseline solution

- after (lots of) deliberate tuning, it **works well locally** near steady state
- admits both **grid-forming & grid-following** implementations
- simplified droop models are amenable to theoretic **analysis & certificates**

(simplified) **frequency droop** = coupled oscillators

$$\underbrace{\frac{d}{dt}\theta_i}_{\substack{\frac{d}{dt} \text{ terminal voltage}}} = \underbrace{\tau_i}_{\substack{\text{active power set-point}}} - \underbrace{\sum_j a_{ij} \sin(\theta_i - \theta_j)}_{\substack{\text{active power flows from grid}}}$$

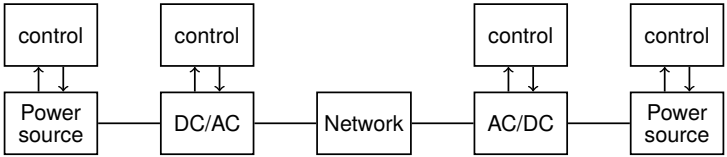


⇒ part of many **grid codes & ancillary service** markets

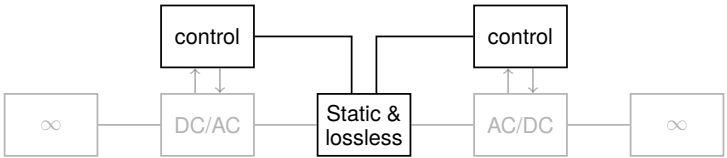
- Ø **poor post-fault performance** due to delays, wind-up, decoupling, SISO, ...
- Ø **no stability certificates** for detailed, nonlinear, & interconnected systems
- Ø unclear if droop control is the **long-term solution (?)** for low-inertia systems

matching control

# Power sources & signal transformers



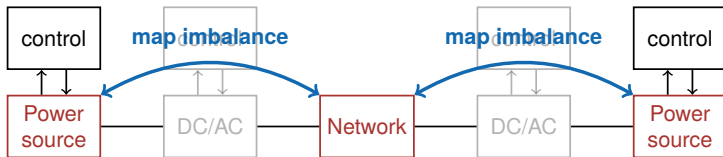
complete system model



abstraction & objectives of previous controllers



# Power sources & signal transformers cont'd



## power source

- ▶ governs **system-level behavior**
- ▶ response **time, power, ...**

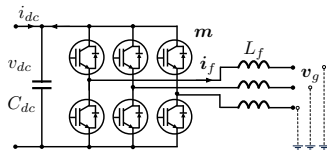
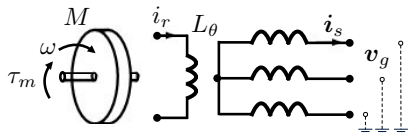
## DC/AC converter

- ▶ acts as signal **transformer**
- ▶ **negligible** controlled dynamics

## Key insights

- DC side & power source cannot be neglected
- focus on main **energy storage** elements & **energy source**
- synchronous machine **maps** power **imbalance** to **turbine** & **governor**
- converter maps imbalance power to ... ?

# Seeking more natural control strategies



$$\frac{d\theta}{dt} = \omega$$

$$M \frac{d\omega}{dt} = -D\omega + \tau_m + L_m i_r \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix}^\top \mathbf{i}_s$$

$$L_s \frac{d\mathbf{i}_s}{dt} = -R_s \mathbf{i}_s + \mathbf{v}_g - L_m i_r \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix} \omega$$

$$\frac{d\delta}{dt} = \eta \cdot v_{dc}$$

$$C_{dc} \frac{dv_{dc}}{dt} = -G_{dc} v_{dc} + i_{dc} + m_{\text{ampl}} \begin{bmatrix} -\sin \delta \\ \cos \delta \end{bmatrix}^\top \mathbf{i}_f$$

$$L_f \frac{d\mathbf{i}_f}{dt} = -R_f \mathbf{i}_f + \mathbf{v}_g - m_{\text{ampl}} \begin{bmatrix} -\sin \delta \\ \cos \delta \end{bmatrix} v_{dc}$$

1. modulation in polar coordinates:

$$\mathbf{m} = m_{\text{ampl}} \begin{bmatrix} -\sin \delta \\ \cos \delta \end{bmatrix} \quad \& \quad \dot{\delta} = m_{\text{freq}}$$

2. matching:  $m_{\text{freq}} = \eta v_{dc}$  with  $\eta = \frac{\omega_{\text{ref}}}{v_{dc, \text{ref}}}$

→ duality:  $C_{dc} \sim M$  is equivalent inertia

**structural similarities** (duality):

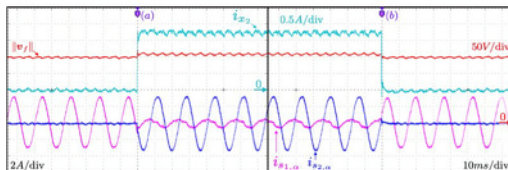
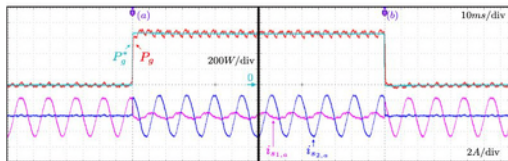
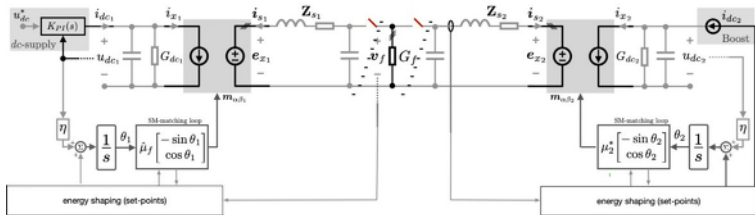
- states:  $\theta = \delta$ ,  $\omega = \eta v_{dc}$ ,  $\mathbf{i}_s = \mathbf{i}_f$
- control:  $u_{\text{ampl}} = L_m i_r$ ,  $i_{dc}/\eta = \tau_m$

→ equivalent inertia:  $M \equiv C_{dc}/\eta^2$  & energy imbalance signal  $\omega \equiv v_{dc}$

# Properties of matching control

- simple & robust **implementation**:  $v_{dc} \longrightarrow \boxed{\frac{\eta}{s}} \xrightarrow{\delta} \boxed{m_{\text{ampl}} \begin{bmatrix} -\sin \delta \\ \cos \delta \end{bmatrix}} \longrightarrow \mathbf{m}$
- exploits **structural similarities** & DC/AC **energy imbalance**
  - clarifies impact of DC side dynamics & limitations
  - similar results for higher-order machine models
  - saturating DC current  $\rightarrow$  saturating AC current (no need for virtual impedance etc.)
- can also be derived from **principled nonlinear control**
  - virtual angle + matching = internal model + passive interconnection (IDA-PBC)
- **energy shaping** via  $i_{dc}$  &  $u_{\text{mag}}$  to achieve further control objectives
  - damping/droop assignment (stability)
  - tracking set-points (PQ or PV)
  - add virtual capacitance (inertia)
- closed-loop **certificates**: incremental stability & passivity  $\approx$  energy decreasing
- closed loop is **only structurally equivalent** to synchronous machine
  - $\rightarrow$  time constants need to be tuned differently (later:  $\mathcal{H}_2$ -optimal tuning)

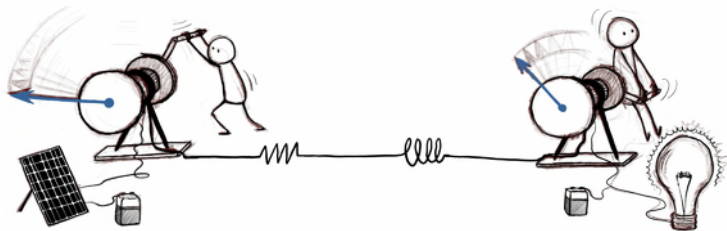
# Rapid prototyping experimental validation



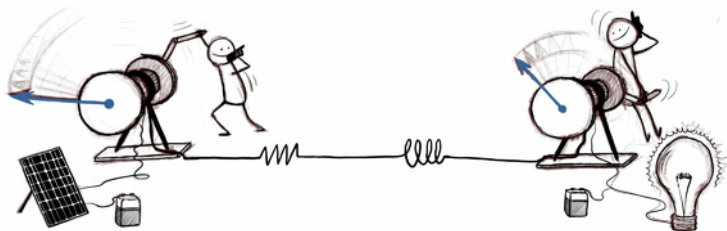
virtual oscillator control (VOC)

# Cartoon summary of VOC approach

**Conceptually**, inverters are oscillators that have to synchronize

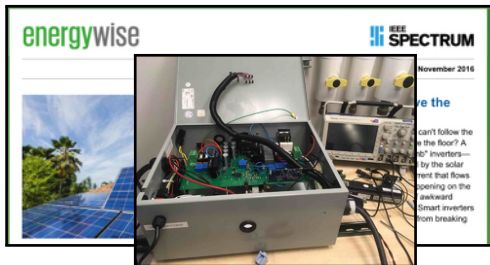
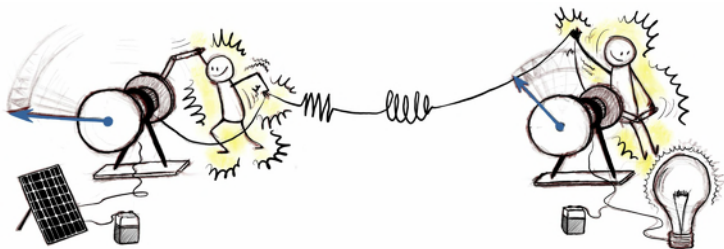


**Hypothetically**, they could sync by communication (not feasible)



# Cartoon summary of VOC approach

**Colorful idea:** inverters sync through physics & clever local control



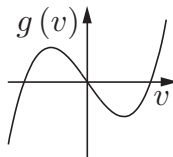
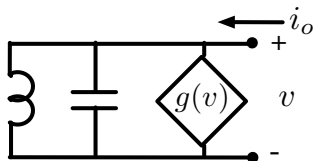
**theory:** sync of coupled oscillators & nonlinear decentralized control

power systems/electronics  
**experiments** @NREL  
outstanding performance  
(superior to droop control)

# Original Virtual Oscillator Control (VOC)

**nonlinear & open limit cycle oscillator** as reference model for terminal voltage (1-phase):

$$\ddot{v} + \omega_0^2 v + g(v) = i_o$$

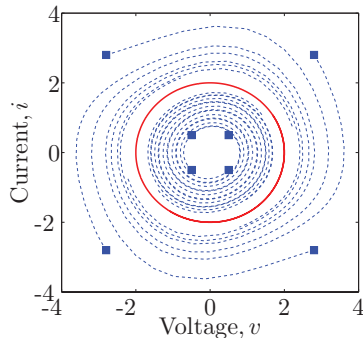


- simplified model amenable to theoretic analysis

→ **almost global synchronization & local droop**

- in practice proven to be **robust mechanism** with performance superior to droop & others

→ **problem**: cannot be controlled(?) to meet specifications on amplitude & power injections



[J. Aracil & F. Gordillo, '02], [Torres, Hespanha, Moehlis, '11],

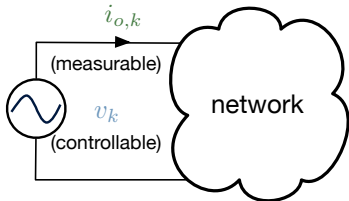
[Johnson, Dhople, Krein, '13], [Dhople, Johnson, Dörfler, '14]



improvement of original ad hoc  
virtual oscillator control (VOC)

# Model & control objectives

(assumptions easy to generalize)



## Simplified multi-converter system model

- ▶ converter = **terminal voltage**  $v_k \in \mathbb{R}^2$
- ▶ **line dynamics** = steady-state  $\Pi$ -model with line admittance  $\|Y_{jk}\| = 1/\sqrt{r_{kj}^2 + \omega_0^2 \ell_{kj}^2}$
- ▶ **homogeneous lines** with  $\kappa = \frac{\ell_{jk}}{r_{jk}}$  constant

## Desired steady-state behavior

- ▶ nominal **synchronous frequency**

$$\frac{d}{dt} v_k = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} v_k$$

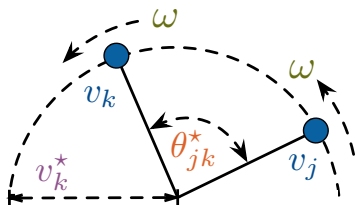
- ▶ voltage **amplitude** (uniform for this talk)

$$\|v_k\| = v^*$$

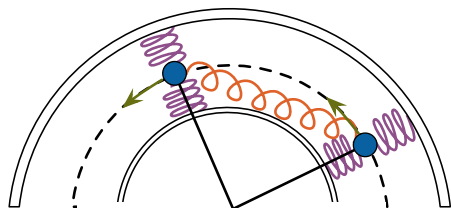
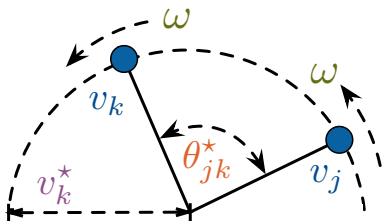
- ▶ active & reactive **power injection**

$$v_k^\top i_{o,k} = p_k^* \quad , \quad v_k^\top \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} i_{o,k} = q_k^*$$

$$\Leftrightarrow \text{relative angles: } v_k = \begin{bmatrix} \cos(\theta_{jk}^*) & -\sin(\theta_{jk}^*) \\ \sin(\theta_{jk}^*) & \cos(\theta_{jk}^*) \end{bmatrix} v_j$$



# Colorful idea: closed-loop target dynamics



$$\begin{aligned}
 \frac{d}{dt} \mathbf{v}_k &= \underbrace{\begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} \mathbf{v}_k}_{\text{rotation at } \omega} + c_1 \cdot \underbrace{\left( \|\mathbf{v}_k\|^{*2} - \|\mathbf{v}_k\|^2 \right) \mathbf{v}_k}_{\text{amplitude regulation to } \mathbf{v}_k^*} \\
 &+ c_2 \cdot \underbrace{\sum_{j=1}^n w_{jk} \left( \mathbf{v}_j - \begin{bmatrix} \cos(\theta_{jk}^*) & -\sin(\theta_{jk}^*) \\ \sin(\theta_{jk}^*) & \cos(\theta_{jk}^*) \end{bmatrix} \mathbf{v}_k \right)}_{\text{synchronization to desired relative angles } \theta_{jk}^*}
 \end{aligned}$$

# Decentralized implementation of dynamics

$$\underbrace{\sum_j w_{jk}(v_j - R(\theta_{jk}^*)v_k)}_{\text{need to know } w_{jk}, v_j, v_k \text{ and } \theta_{jk}^*} = \underbrace{\sum_j w_{jk}(v_j - v_k)}_{\text{"Laplacian" feedback}} + \underbrace{\sum_j w_{jk}(I - R(\theta_{jk}^*))v_k}_{\text{local feedback: } \mathcal{K}_k(\theta^*)v_k}$$

**insight I:** non-local measurements from **communication via physics**

$$\underbrace{\dot{i}_{o,k}}_{\text{local feedback}} = \underbrace{\sum_j y_{jk}(v_j - v_k)}_{\text{distributed feedback with } w_{jk} = y_{jk} = \|y_{kj}\| R(\kappa)^{-1}}$$

local feedback

distributed feedback with  $w_{jk} = y_{jk} = \|y_{kj}\| R(\kappa)^{-1}$

**insight II:** angle set-points & line-parameters from **power flow equations**

$$\left. \begin{aligned} p_k^* &= v^{*2} \sum_j \frac{r_{jk}(1 - \cos(\theta_{jk}^*)) - \omega_0 \ell_{jk} \sin(\theta_{jk}^*)}{r_{jk}^2 + \omega_0^2 \ell_{jk}^2} \\ q_k^* &= -v^{*2} \sum_j \frac{\omega_0 \ell_{jk}(1 - \cos(\theta_{jk}^*)) + r_{jk} \sin(\theta_{jk}^*)}{r_{jk}^2 + \omega_0^2 \ell_{jk}^2} \end{aligned} \right\} \Rightarrow \underbrace{\mathcal{K}_k(\theta^*)}_{\text{global parameters}} = \underbrace{\frac{1}{v^{*2}} R(\kappa)}_{\text{local parameters}} \begin{bmatrix} q_k^* & p_k^* \\ -p_k^* & q_k^* \end{bmatrix}$$

# Properties of virtual oscillator control

1. desired target dynamics can be realized via **fully decentralized control**

$$\frac{d}{dt} v_k = \underbrace{\begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} v_k}_{\text{rotation at } \omega_0} + c_1 \cdot R(\kappa) \underbrace{\left( \frac{1}{v^{\star 2}} \begin{bmatrix} q_k^* & p_k^* \\ -p_k^* & q_k^* \end{bmatrix} v_k - i_{o,k} \right)}_{\text{synchronization through physics}} + c_2 \cdot \underbrace{(v^{\star 2} - \|v_k\|^2)}_{\text{local amplitude regulation}} v_k$$

2. connection to **droop control** revealed in polar coordinates (for inductive grid)

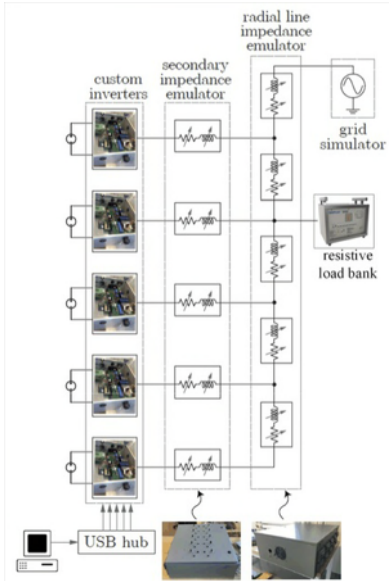
$$\frac{d}{dt} \theta_k = \omega_0 + c_1 \left( \frac{p_k^*}{v^{\star 2}} - \frac{p_k}{\|v_k\|^2} \right) \underset{\|v_k\| \approx 1}{\approx} \omega_0 + c_1 (p_k^* - p_k) \quad (p - \omega \text{ droop})$$

$$\frac{d}{dt} \|v_k\| \underset{\|v_k\| \approx 1}{\approx} c_1 (q_k^* - q_k) + c_2 (v^* - \|v_k\|) \quad (q - \|v\| \text{ droop})$$

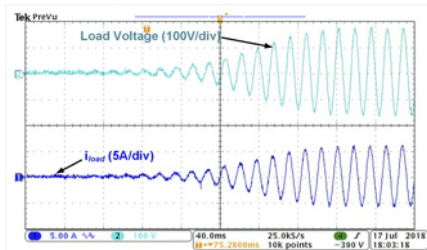
3. **almost global asymptotic stability** if

- **power transfer** “small enough” compared to **network connectivity**
- **amplitude control** slower than **synchronization control**

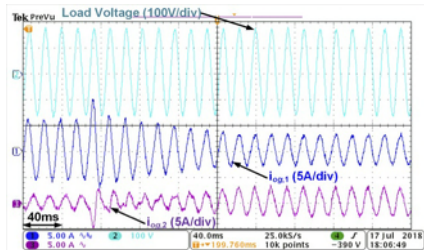
# Experimental setup @ NREL



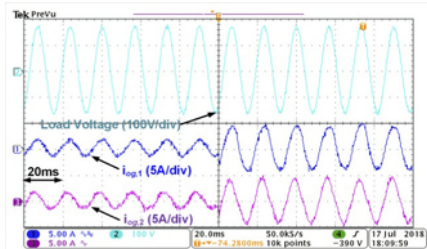
# Experimental results



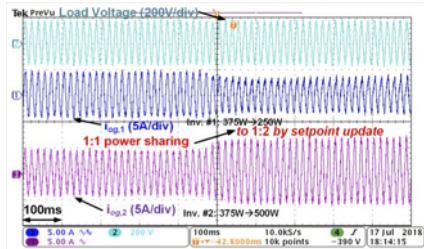
black start of inverter #1 under 500 W load  
 (making use of almost global stability)



connecting inverter #2 while inverter #1 is  
 regulating the grid under 500 W load



250 W to 750 W load transient with two  
 inverters active

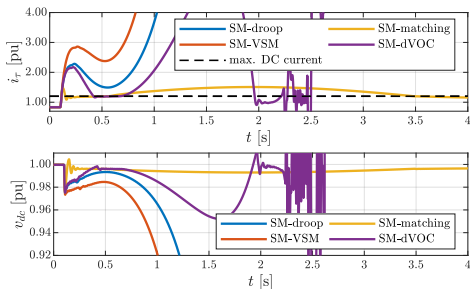


change of setpoint:  $p^*$  of inverter #2  
 updated from 250 W to 500 W

relative comparison



# Comparison of control strategies @AIT



- **all perform well** nominally & under minor disturbances

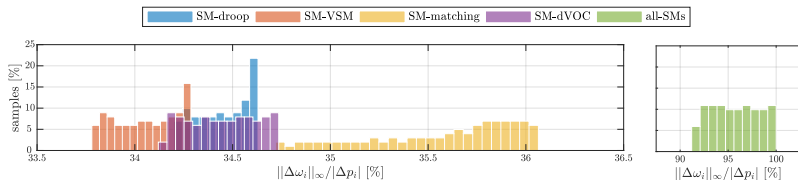
- **relative resilience** :  
matching > VOC > droop > virtual synchronous machine

→ it is a very poor strategy for a converter to emulate a flywheel

- promising **hybrid control** directions: VOC + matching

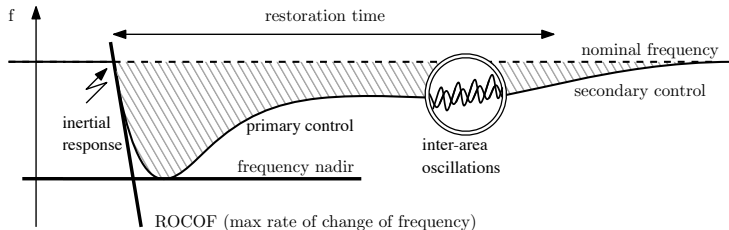
## Interactions of Grid-Forming Power Converters and Synchronous Machines – A Comparative Study

Ali Tayyebi, Dominic Groß, Adolfo Anta, Friederich Kupzog and Florian Dörfler



system-level optimization

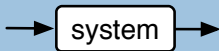
# Performance metrics for power systems



## System norm quantifying signal amplifications

### disturbances:

impulse (fault), step (loss of unit), white noise (renewables)



### performance outputs:

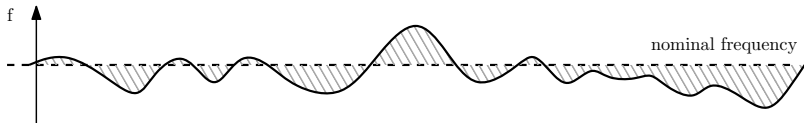
integral, peak, ROCOF, restoration time, ...

**Trade-offs:** inout/output combination & worst-case vs. average performance

# Integral-quadratic performance metric

recall: the post-fault response in a low-inertia system may look very different

$$\int_0^{\infty} x(t)^T Q x(t) dt$$



$\mathcal{H}_2$  **system norm** interpretation:  $\eta \rightarrow$  system  $\rightarrow y$

1. **performance output:**  $y = Q^{1/2} x$
2. **impulsive  $\eta$**  (faults)  $\rightarrow$  output energy  $\int_0^{\infty} y(t)^T y(t) dt$
3. **white noise  $\eta$**  (renewables)  $\rightarrow$  output variance  $\lim_{t \rightarrow \infty} \mathbb{E} (y(t)^T y(t))$

# System model & virtual inertia realization

## nonlinear DAE dynamics

$$\dot{x}_s = f_s(x_s, z_s)$$

$$0 = g_s(x_s, z_s, i)$$

$x_s$  : system & control states

$z_s$  : network signals

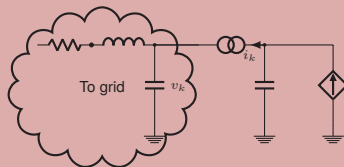
$i$  : current injections/faults

## virtual inertia & damping

= fast frequency response  
implemented by means of

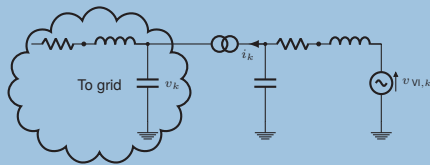
- converters & storage
- 2nd-order droop control

follow the grid: lock to  $\omega$



grid-following

form the grid: generate  $\omega$

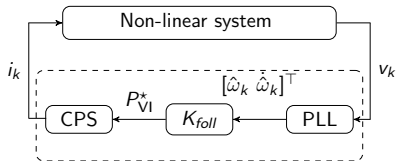


grid-forming

# Comparison: virtual inertia implementations

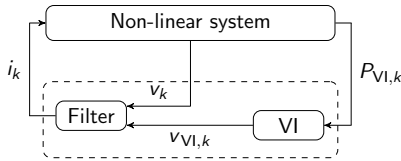
## grid-following

$$\omega \rightarrow P_{VI}$$



## grid-forming

$$P_{VI} \rightarrow \omega$$



### droop via phased-locked loop

$$\hat{\theta}_k = \hat{\omega}_k$$

$$\tau_k \dot{\hat{\omega}}_k = -\hat{\omega}_k - K_p v_{q,k} - K_i \int v_{q,k} d\tau$$

$$P_{VI,k}^* = K_{foll,k} [\hat{\omega}_k \ \dot{\hat{\omega}}_k]^T$$

### grid-forming 2nd-order droop

$$\dot{\theta}_{VI,k} = \omega_{VI,k}$$

$$\dot{\omega}_{VI,k} = -\tilde{d}_k \tilde{m}_k^{-1} \omega_{VI,k} - \tilde{m}_k^{-1} P$$

$$\dot{\omega}_{VI,k} = K_{form,k} [\omega_{VI,k} \ P]^T$$

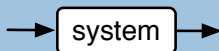
**Note:** control coefficients interpretable as “virtual inertia & damping” or “P & D”

# Closed-loop system & linearization

here: grid-following implementation; analogous for grid-forming implementation

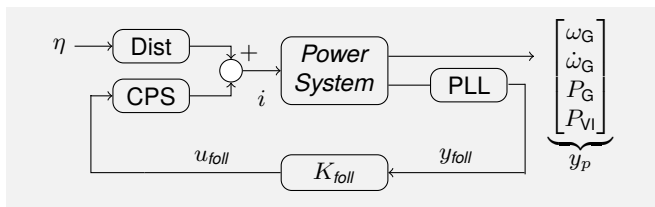
## disturbances:

impulse (fault), step (loss of unit), white noise (renewables)



## performance

outputs: integral, peak, ROCOF, restoration time, ...



$$\Delta \dot{x}_{cl} = (A + BK C) \Delta x_{cl} + B_g (\Pi^{1/2}) \eta, \quad \Delta y_p = C_p \Delta x_{cl}$$

**linearized closed loop** for optimal control design

# Optimal control formulation

$$\begin{array}{ll} \underset{P, K}{\text{minimize}} & \|G\|_{\mathcal{H}_2}^2 = \text{trace}(B_g^\top P B_g) \quad \text{performance metric} \\ \text{subject to} & K(m, d) \in S \quad \text{constraint set} \\ & P(A + BKC) + (A + BKC)^\top P + C_p^\top C_p = \mathbb{O} \quad \text{Lyapunov equation} \end{array}$$

## Key Insights:

1. **non-convex** problem in  $P, K$
2. **conservative** convex upper/lower bounds can be obtained
3. assume knowledge of **disturbance** profile  $\Pi$  or go for min-max
4. **gradient-based** approach: scalable, can handle constraints, suboptimal
5. **complexity** of gradient computation  $\mathcal{O}(n^3)$  via controllability Gramian

$$L(A + BKC)^\top + (A + BKC)L + B_g B_g^\top = \mathbb{O}$$



# Case-study: South-East Australian Grid

The Sydney Morning Herald

NATIONAL  
State in the dark: South Australia's  
major power outage



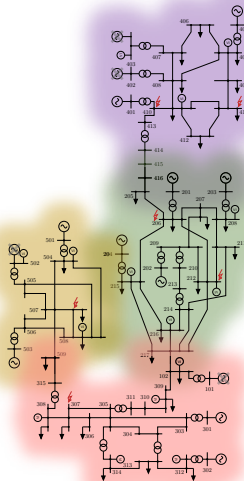
The New York Times



**Australia Powers Up the  
World's Biggest Battery  
— Courtesy of Elon Musk**

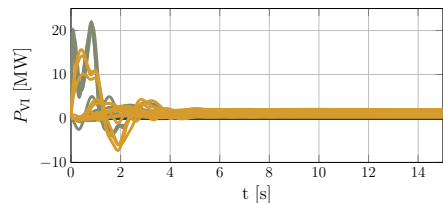
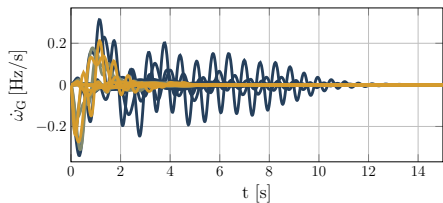
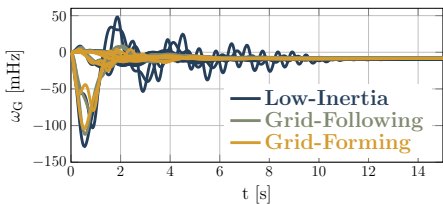


grid topology



simulink model

# Closed-loop simulations



## Non-linear model

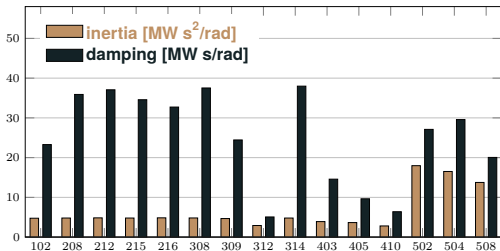
- 54 buses, 14 gens, & **15 converters**
- control: governors, AVRs, & PSSs
- **replace** generators with controllable power sources + virtual inertia
- numerically linearize model, choose performance inputs/outputs & gradient-based optimization routine
- non-linear closed-loop simulations: 200 MW disturbance at node 508

## Observations

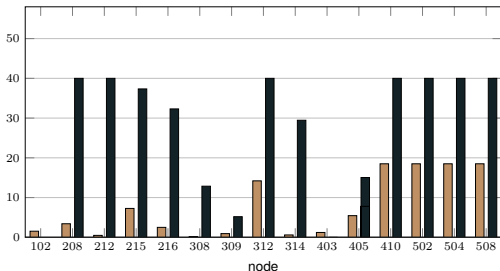
- system-level optimization **makes a difference** (even at same inertia)
- forming vs. following: **signal causality** has major impacts (e.g., peak power)

# Optimal allocation of virtual inertia/damping

(a) Grid-Forming



(b) Grid-Following

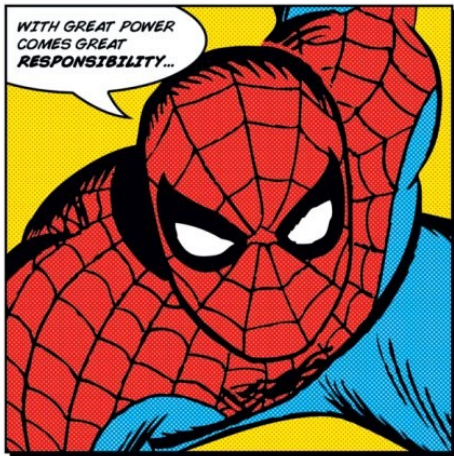


## Observations

- both control modes allocate virtual inertia in (blackout & battery) **area 5**
- **grid-following** : more reliance on damping (due to PLL-delay in  $\dot{\omega}$ )
- **grid-forming** : results in a more uniform (thus robust) allocations
  - **total inertia/damping** not crucial
  - **spatial allocation** matters a lot
  - implications for pricing & markets

# Conclusions

- low-inertia stability & converter control are **major bottlenecks** for sustainability
  - power system community & industry are open to **green-field** approaches
- **low-inertia systems — opportunity for collaboration: control & power**



**POWER IS NOTHING WITHOUT CONTROL**



# All references & many more details in ...

## Foundations and Challenges of Low-Inertia Systems

(Invited Paper)

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## Distributed Control and Optimization for Autonomous Power Grids

Florian Dörfler Saverio Bolognani John W. Simpson-Porco Sergio Grammatico

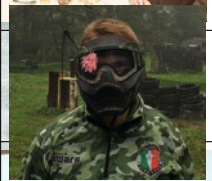
**Abstract**—The electric power system is currently undergoing a period of unprecedented changes. Environmental and sustainability concerns lead to replacement of a significant share of conventional fossil fuel-based power plants with renewable energy sources. As a result of this energy transition, centralized bulk generation based on fossil fuel and interfaced with synchronous machines is substituted by distributed

participation in general provide huge challenges as well as unprecedented opportunities to integrate an end-to-end automated and sustainable socio-technical system.

Parallel to these technological advances, the control, optimization, communication, computer science, and signal processing communities have developed novel methodological

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M11 – STOCKHOLM 13/04/2020-17/04/2020	Control and Optimization of Autonomous Power Systems	Florian Dörfler & Saverio Bolognani, Swiss Federal Institute of Technology (ETHZ), Switzerland

# my collaborators



Your Goal:

WHAT YOU CARE ABOUT

WHAT YOU'RE GOOD AT

JOY

HAPPY BUT POOR

RICH BUT BORED

SWEET CREAM

WHAT PAYS WELL

"Choose a job you love, and you will never have to work a day in your life."

us

60/60