Power system economics

Market-based operation: formulations, basic principles, problems and benefits
Spatial dimension of energy trading and power balancing
Ancillary services and real-time control

Andrej Jokić
Control Systems group
Faculty of Mechanical Engineering and Naval Architecture
University of Zagreb

smart grids ?

hidden technology

invisible hand of market

important (for the “smart“ part): get the fundamentals right and well

Outline

Deregulation




Unifying approach: optimization

In general terms, problems of a power system on global level can be summarized as follows

i) Economical efficiency subject to: Global *energy* balance + Transmission system security constraints
ii) Economical efficiency subject to: Accumulation of sufficient amount of ancillary service + Transmission system security constraints
iii) Economical and dynamical efficiency, subject to: Global *power* balance + Robust stability

ECONOMY versus **RELIABILITY**

- Formulation of **PROBLEMS**: structured, time-varying optimization problems
- **SOLUTIONS:**
  - not only algorithms that give solution (as desired output), but also:
  - efficient, robust (optimally account for trade-offs), scalable and flexible control and operational architecture (who does what and when? relations?)
  - long term benefits of markets due to different solution architecture compared to regulated system

Positioning in time scale

Market commodities

- Energy markets: commodity is energy [MWh]
- Ancillary services markets (power balancing): commodity is energy (options) and sometimes capacity (placed on disposal over some time) [MWh]

Observation: Commodities are defined over time intervals (necessary to quantify energy)

Program time unit (PTU)

Program time unit (PTU): a market trading period (5min to 1h) for forward and real-time markets.
Some markets trade with over longer intervals (days, months,...)

Positioning in time scale

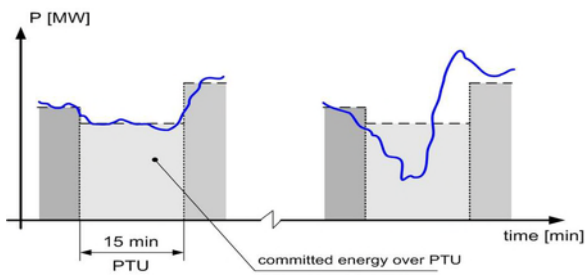
Market commodities

- Energy markets: commodity is energy [MWh]
- Ancillary services markets (power balancing): commodity is energy (options) and sometimes capacity (placed on disposal over some time) [MWh]

Positioning in time scale

Power versus energy

- Ancillary services: provision of power (real-time), trading in energy/capacity
- Congestion: constraints on power flows (real-time), trading in energy



Positioning in time scale

Power versus energy

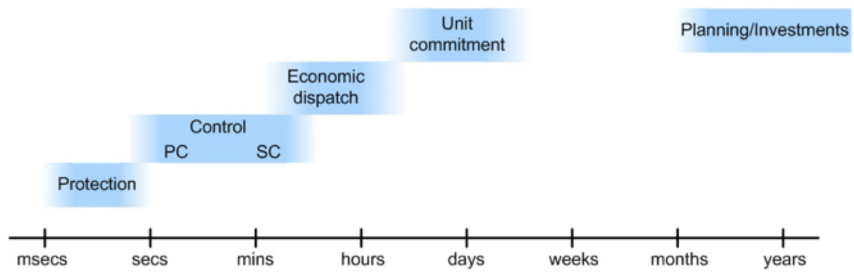
- Ancillary services: provision of power (real-time), trading in energy/capacity
- Congestion: constraints on power flows (real-time), trading in energy

Economy(energy), Control(power)

- Interplay between power and energy → coupling economy and physics/engineering (control)
- Increased uncertainties (renewables, decentralization) both in power and energy → tighter coupling economy, physics/control → requires design for efficiency and robustness

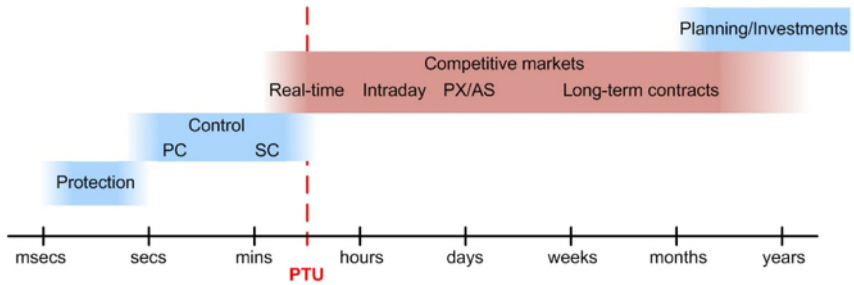
Out of scope in this talk: investments, legislation, details of regulation, political aspects

Positioning in time scale



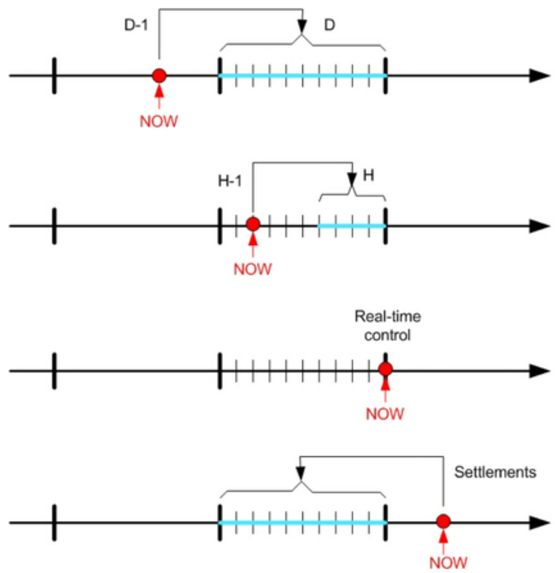
Traditional power system

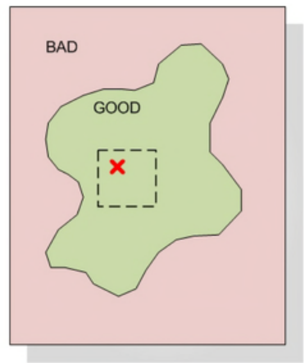
Positioning in time scale



Market based power system

Actions in time





Conditions for deregulation

Natural monopoly

- Economy of scale: Efficiency(100 MW plant) > Efficiency(10 MW plant) > Efficiency(1 MW plant)
- Large generating companies: one owner of many plants → cheaper production due to hiring of specialists, sharing parts and repair crews...

Conditions for successful deregulation

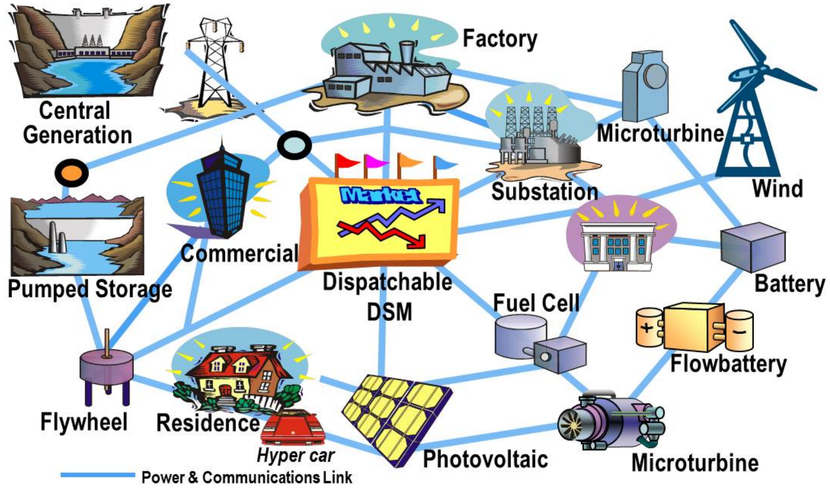
Lack of natural monopoly, or the conditions of natural monopoly should hold only weakly.

... if monopolist can produce power at significantly lower cost than the best competitive market, then regulation makes little sense.

Emerging playground for competition

More efficient low power plants (cheap gas turbines); renewable generation; smaller size distributed generation distributed on all levels in the system; price elastic demand,...

Conditions for deregulation



Maximizing social welfare

Energy market

- Production cost function: $C_i(p_i)$
- Consumption benefit function: $B_j(d_j)$

Social welfare maximization (isolated system)

$$\min_{p_1,\ldots,p_n,d_1,\ldots,d_m} \sum_{i=1}^{n} C_i(p_i) - \sum_{j=1}^{m} B_j(d_j) \qquad (= \max \text{ social welfare})$$

subject to

$$p_i \in \mathcal{P}_i, \quad i = 1,\ldots,n \qquad \text{(local production constraints)}$$

$$d_j \in \mathcal{D}_j, \quad j = 1,\ldots,m \qquad \text{(local consumption constraints)}$$

$$\sum_{i=1}^{n} p_i = \sum_{j=1}^{m} d_j \qquad \text{(balance supply and demand)}$$

example local constraints: $\mathcal{P}_i := \{p \mid \underline{p}_i \le p \le \overline{p}_i\}, \quad \mathcal{D}_j := \{d \mid \underline{d}_j \le d \le \overline{d}_j\}$

Intermezzo: Lagrange duality

Optimization problem

$$\min_x \{ f(x) \mid g(x) \le 0, \; h(x) = 0 \}$$

where $h : \mathbb{R}^n \to \mathbb{R}^m \quad g : \mathbb{R}^n \to \mathbb{R}^p$

Lower bounds

Let x be feasible point ($g(x) \le 0$, $h(x) = 0$). For arbitrary $\lambda \in \mathbb{R}^m$ and $\mu \in \mathbb{R}^p$ with $\mu \ge 0$ we have

$$L(x,\lambda,\mu) := f(x) + \lambda^\top h(x) + \mu^\top g(x) \le f(x).$$

After infimization we have

$$\ell(\lambda,\mu) := \inf_x L(x,\lambda,\mu) \le \inf_{\{x \mid g(x)\le 0,\, h(x)=0\}} f(x)$$

Since λ and $\mu \ge 0$ were arbitrary we conclude

$$\sup_{\{\lambda,\mu \mid \mu \ge 0\}} \ell(\lambda,\mu) \quad \le \quad \inf_{\{x \mid g(x)\le 0,\, h(x)=0\}} f(x)$$

Intermezzo: Lagrange duality

Terminology and observations

- Lagrange function: $L(x,\lambda,\mu) := f(x) + \lambda^\top h(x) + \mu^\top g(x)$
- Lagrange dual cost: $\ell(\lambda,\mu) := \inf_x L(x,\lambda,\mu)$
- Lagrange dual problem: $d_{opt} = \sup_{\{\lambda,\mu \,|\, \mu\geq 0\}} \ell(\lambda,\mu)$
- Primal problem: $p_{opt} = \inf_{\{x \,|\, g(x)\leq 0,\, h(x)=0\}} f(x)$

Dual problem is **concave maximization** problem. Constraints are often simpler than in primal problem.

Weak duality (lower bounds)

Dual optimal value (d_{opt}) $\leq$ Primal optimal value (p_{opt})

Weak duality is always true.

Intermezzo: Lagrange duality

Lagrange Duality Theorem

Weak duality always holds: $d_{opt} \leq p_{opt}$

Let primal problem be **convex** with satisfied **Slater's constraint qualification**. Then strong duality holds: $d_{opt} = p_{opt}$.

Strong duality in compact form

$$\max_{\{\lambda,\mu \,|\, \mu\geq 0\}} \left(\inf_x f(x) + \lambda^\top h(x) + \mu^\top g(x) \right) = \inf_{\{x \,|\, g(x)\leq 0,\, h(x)=0\}} f(x)$$

Slater's constraint qualification

Define sets $\mathcal{I}_n, \mathcal{I}_a$: $i \in \mathcal{I}_n$ if $g_i(\cdot)$ is nonlinear; $i \in \mathcal{I}_a$ if $g_i(\cdot)$ is affine.
Slater CQ: the set

$$\{x \mid h(x) = 0,\ g_i(x) < 0 \text{ for } i \in \mathcal{I}_n,\ g_i(x) \leq 0 \text{ for } i \in \mathcal{I}_a, \}$$

is nonempty.

Maximizing social welfare via dual problem

Energy market

Primal

$$\min_{p_i\in\mathcal{P}_i, d_j\in\mathcal{D}_j} \quad \sum_{i=1}^n C_i(p_i) - \sum_{j=1}^m B_j(d_j)$$

$$\text{subject to} \quad \sum_{i=1}^n p_i = \sum_{j=1}^m d_j$$

Dual

$$\max_{\lambda\in\mathbb{R}} \ell(\lambda)$$

where

$$\ell(\lambda) = \min_{p_i\in\mathcal{P}_i, d_j\in\mathcal{D}_j} \quad \sum_{i=1}^n C_i(p_i) - \sum_{j=1}^m B_j(d_j) + \lambda\Big(\sum_{j=1}^m d_j - \sum_{i=1}^n p_i\Big)$$

Assumption: convexity. $C_i(\cdot)$ convex functions, $B_j(\cdot)$ concave fun., $\mathcal{P}_i, \mathcal{D}_j$ convex sets.

Maximizing social welfare via dual problem

Energy market

Dual

$$\max_{\lambda\in\mathbb{R}} \ell(\lambda)$$

where

$$\ell(\lambda) = \min_{p_i\in\mathcal{P}_i, d_j\in\mathcal{D}_j} \quad \sum_{i=1}^n C_i(p_i) - \sum_{j=1}^m B_j(d_j) + \lambda\Big(\sum_{j=1}^m d_j - \sum_{i=1}^n p_i\Big)$$

Observation 1: Lagrange dual cost function $\ell(\lambda)$ is decomposable (for a fixed λ, can be decomposed into $n+m$ separate minimization problems)

Observation 2: $\max_{\lambda\in\mathbb{R}} \ell(\lambda)$ is attained when $\sum_{j=1}^m d_j = \sum_{i=1}^n p_i$ ((sub)gradient of $\ell(\lambda)$ is zero).

Maximizing social welfare via dual problem

Energy market

$$\max_{\lambda \in \mathbb{R}} \ell(\lambda)$$

Supplier's *local* minimizations

$$\min_{\mathcal{P}_1} \; C_1(p_1) - \lambda p_1$$
$$\min_{\mathcal{P}_2} \; C_2(p_2) - \lambda p_2$$
$$\vdots$$
$$\min_{\mathcal{P}_n} \; C_n(p_n) - \lambda p_n$$

Demand's *local* minimizations

$$\min_{\mathcal{D}_1} \; \lambda d_1 - B_1(d_1)$$
$$\min_{\mathcal{D}_2} \; \lambda d_2 - B_1(d_2)$$
$$\vdots$$
$$\min_{\mathcal{D}_m} \; \lambda d_m - B_1(d_m)$$

Maximizing social welfare via dual problem

Energy market

Market operator

$$\max_{\lambda \in \mathbb{R}} \ell(\lambda) \quad \Leftrightarrow \quad \text{determine } \lambda : \sum_{j=1}^{m} d_j^\star = \sum_{i=1}^{n} p_i^\star$$

Rational behaviour of market players (max its own benefits)

Supplier's *local* minimizations

$$p_1^\star = \text{argmin}_{p_1 \in \mathcal{P}_1} \; C_1(p_1) - \lambda p_1$$
$$p_2^\star = \text{argmin}_{p_2 \in \mathcal{P}_2} \; C_2(p_2) - \lambda p_2$$
$$\vdots$$
$$p_n^\star = \text{argmin}_{p_n \in \mathcal{P}_n} \; C_n(p_n) - \lambda p_n$$

Demand's *local* minimizations

$$d_1^\star = \text{argmin}_{d_1 \in \mathcal{D}_1} \; \lambda d_1 - B_1(d_1)$$
$$d_2^\star = \text{argmin}_{d_2 \in \mathcal{D}_2} \; \lambda d_2 - B_1(d_2)$$
$$\vdots$$
$$d_m^\star = \text{argmin}_{d_m \in \mathcal{D}_m} \; \lambda d_m - B_1(d_m)$$

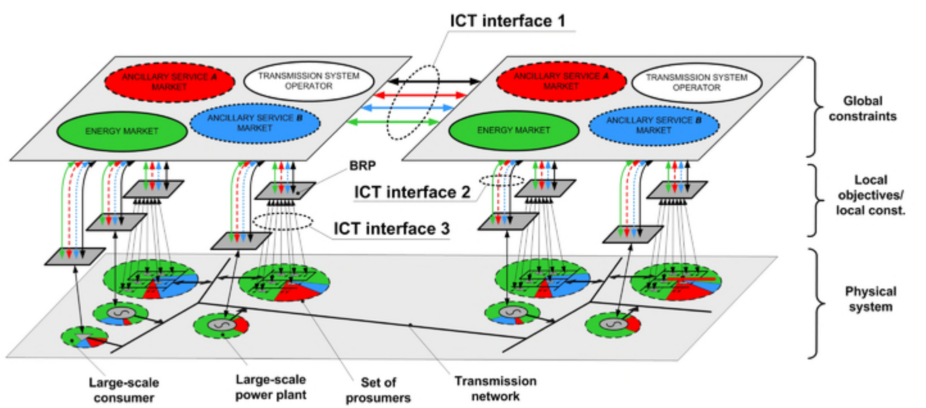
λ^* which solves the above problem is the (market clearing) price

Market based operation

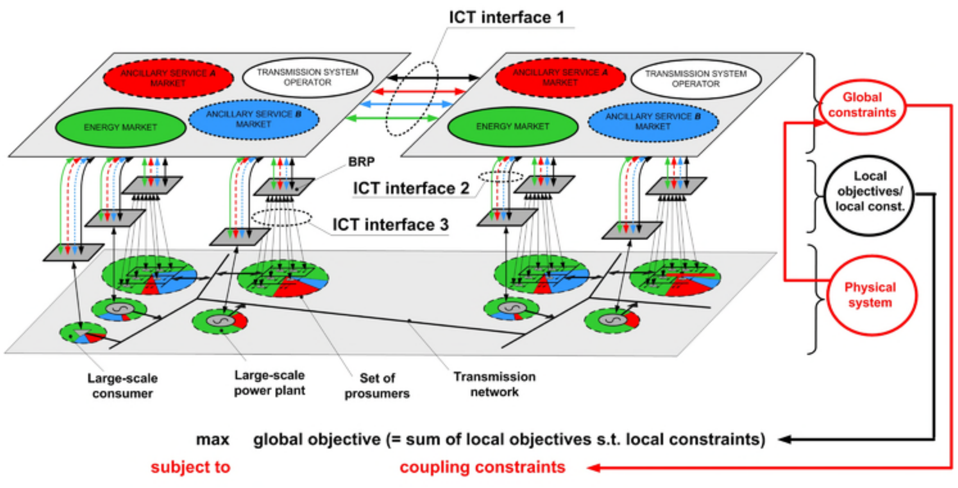
Some observations/remarks

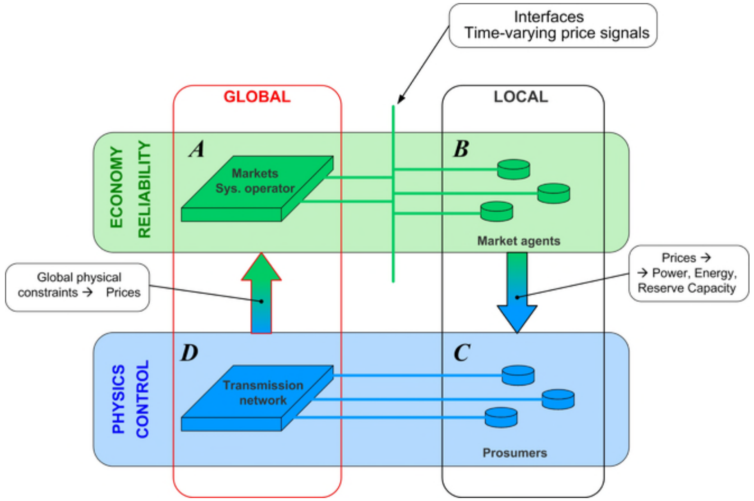
- change from regulated and single utility owned and operated system to the market based system can be seen as shift from explicitly solving **primal** problem to explicitly solving **dual** problem
- Lagrange dual (and "complementarity problems"): suitable as manipulates with both **physical** (**primal**) variables and **economy** related variables - prices (**dual**)
- generic approach: assign prices to **global** constraints (i.e. power balance) and use them to coordinate **local** behaviours to meet the **global** constraints
- By shifting to solving dual problem we have introduced different **solution architecture**: *i)* new players: market operators, competing market agents; *ii)* we have defined who does what; *iii)* we have introduced prices and bids as protocols for coordination among players.
- Large-scale complex systems: rely on **protocols**, **modularity** and **architecture** (Internet: TCP/IP; power system: 50 Hz is a "protocol"; money / bid format;... a bit wider view: passivity in control as a protocol...)

Market based operation



Market based operation





Time varying price signals as

- Protocols and defining ingredients of uniform interfaces in communication between producers, consumers, market and system operators
- Signals for coordination and time synchronization of local behaviours to achieve global goals

Market based operation

Supplier: $p_i^\star = \operatorname{argmin}_{p_i \in \mathcal{P}_i} \quad C_i(p_i) - \lambda p_i$
Consumer: $d_j^\star = \operatorname{argmin}_{d_j \in \mathcal{D}_j} \quad \lambda d_1 - B_j(d_j)$

Suppose λ is given such that $p_i^\star \in$ interior of $\mathcal{P}_i$, $d_j^\star \in$ interior of $\mathcal{D}_j$, then we have

$$\frac{\mathrm{d}C_i(p_i^\star)}{\mathrm{d}p_i} = \lambda$$

$$\frac{\mathrm{d}B_j(d_j^\star)}{\mathrm{d}d_j} = \lambda$$

i.e., social welfare is maximized when all *prosumers* (producers/consumers) adjust their prosumption levels so that marginal cost/benefit functions are equal to the price.

Market clearing problem

Bids from marginal costs/benefits

$$\frac{\mathrm{d}C_i(p_i)}{\mathrm{d}p_i} = \lambda \quad \Leftrightarrow \quad p_i = \gamma_i^p(\lambda) \quad \Leftrightarrow \quad \lambda = \beta_i^p(p_i)$$

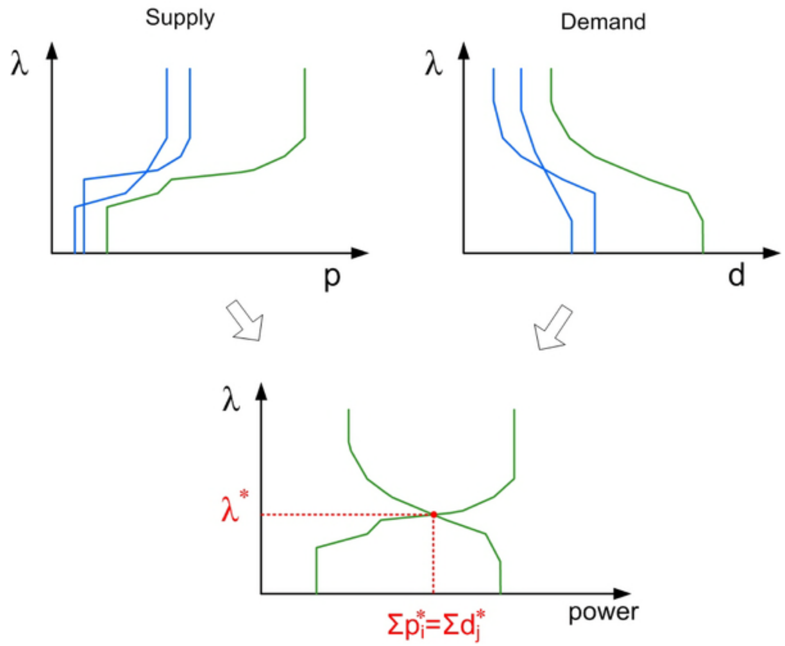
$$\frac{\mathrm{d}B_j(d_j)}{\mathrm{d}d_j} = \lambda \quad \Leftrightarrow \quad d_j = \gamma_j^d(\lambda) \quad \Leftrightarrow \quad \lambda = \beta_j^d(d_i)$$

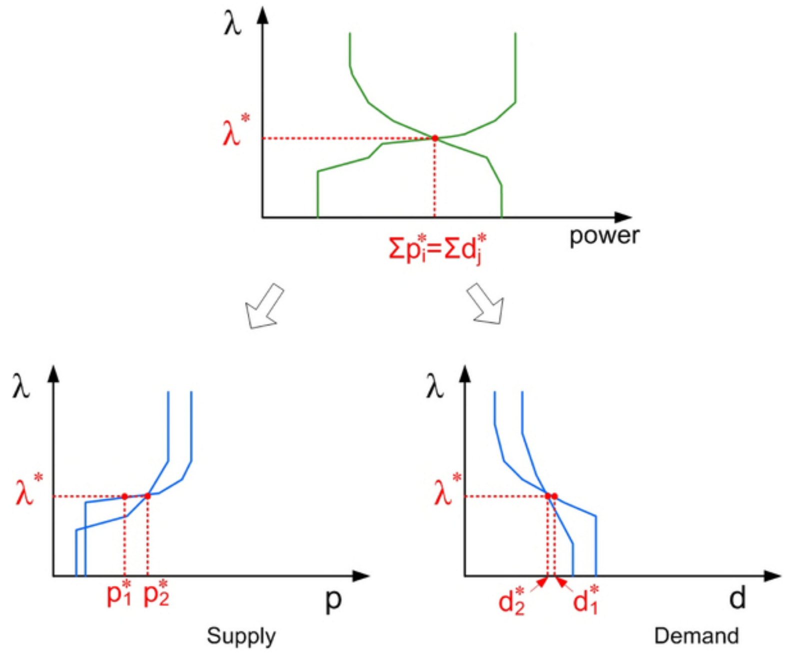
Market clearing problem in practice

Find the **market clearing price** $\lambda^\star$ at intersection of the aggregated supply bid curve $\tilde{\gamma}^p(\lambda) := \sum_i \gamma_i^p(\lambda)$ with the aggregated demand bid curve $\tilde{\gamma}^d(\lambda) := \sum_j \gamma_j^d(\lambda)$:

$$\sum_{i=1}^{n} p_i^\star = \sum_{i=1}^{n} \gamma_i^p(\lambda^\star) = \tilde{\gamma}^p(\lambda^\star) = \tilde{\gamma}^d(\lambda^\star) = \sum_{j=1}^{m} \gamma_j^d(\lambda^\star) = \sum_{i=1}^{m} d_i^\star$$

Remark: extension to cases when assumptions $p_i^\star \in$ interior of $\mathcal{P}_i$, $d_j^\star \in$ interior of $\mathcal{D}_j$ are not valid are straightforward. Easy to include constraints in the bids.

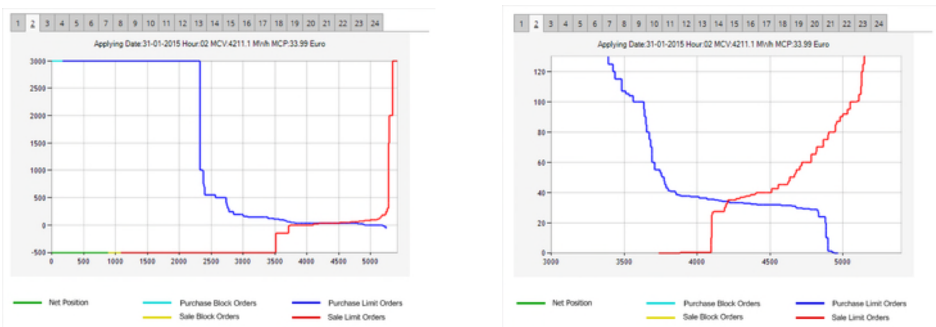




Market clearing: example

APX, aggregated bids

30. January 2015, 2 a.m.

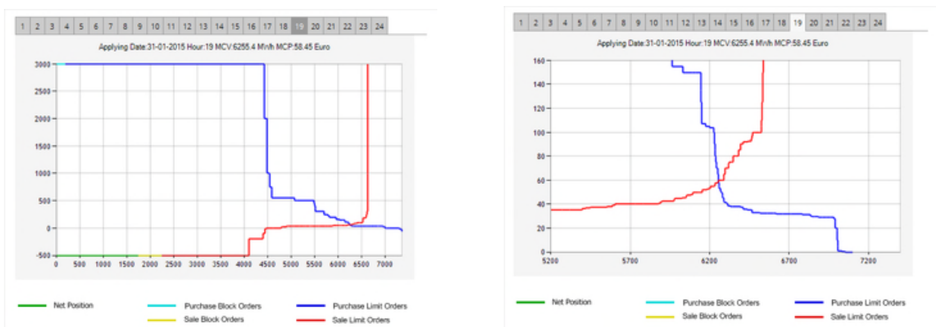


In some markets (e.g., APX) block bids are possible (bids for more trading periods; convenient to account for start-up costs. Origin of nonconvexity.)

Market clearing: example

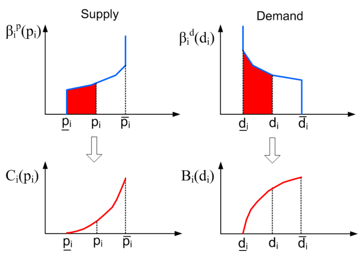
APX, aggregated bids

30. January 2015, 7 p.m.



In some markets (e.g., APX) block bids are possible (bids for more trading periods; convenient to account for start-up costs. Origin of nonconvexity.)

Market clearing problem



Terminology: "all supply bids smaller than some price are accepted

Exercise 1. Prove the following:

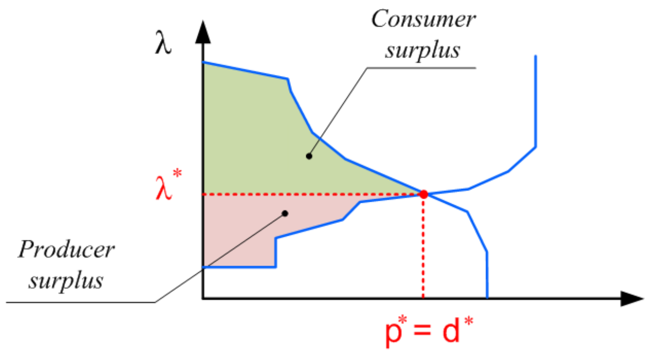
Non-decreasing $\beta_i^p(\cdot)$ $\Rightarrow$ $C_i(\cdot)$ is convex
Non-increasing $\beta_i^d(\cdot)$ $\Rightarrow$ $B_i(\cdot)$ is concave

$$C_i(p_i) = \int_{\underline{p}_i}^{p_i} \beta_i^p(\xi)\mathrm{d}\xi, \quad B_i(d_i) = \int_{\underline{d}_i}^{d_i} \beta_i^d(\xi)\mathrm{d}\xi$$

Market operators require bids to be non-decreasing/non-increasing (irrespective of true marginal costs/benefits).

Exercise 2.
Let the bids be piecewise constant (non-decreasing for supply, non-increasing for demand). Formulate market clearing problem as an optimization problem (primal).

Maximizing social welfare via dual problem



Consumer surplus

$CS(d_i) := B_i(d_i) - \lambda d_i$

Producer surplus

$PS(p_i) := \lambda d_i - C_i(p_i)$

Remarks:
In fact graphical interpretation of solving dual problem.
Maximized areas (surpluses) = optimal value of Lagrange multiplier (price).

In practice it is often told that all the bids till Market clearing volume / Market clearing price are accepted.

Balance responsible party

Balance responsible party (BRP)

- BRP is a legal entity that is capable and allowed to trade on energy and ancillary service markets.
- BRP is defined by specification of its responsibilities (operational rules) and interfaces with other subsystems in the operational architecture of the overall system.

By defining the interfaces and responsibilities, we are in fact defining the BRPs as crucial building blocks (modules) of the system.

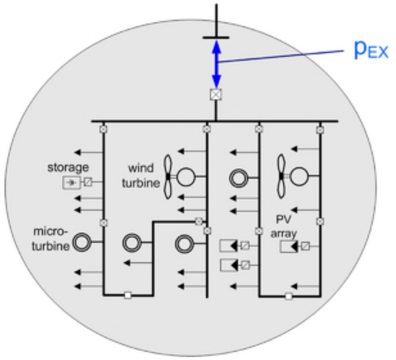
- Responsible for own production and load prediction;
- Responsible for behavior in markets (e.g. market power misuses);
- Responsible for behavior in power system (e.g. responsibility to react on real-time SC signal from TSO);
- Can pay bills;

Bidding

Basics of bidding

BRPs portfolio: • m generators $\{C_i(p_i), \underline{p}_i, \overline{p}_i\}_{i=1,\ldots,m}$; • n controllable loads $\{B_i(d_i), \underline{d}_i, \overline{d}_i\}$; • aggregated price inelastic power injection q

How could the BRP bid for its aggregated prosumption p_{EX}? $\beta_{BRP}(p_{EX}) = ?$



Bidding

Basics of bidding

Approach I

$$\min_{\{p_i\},\{d_j\},p_{EX}} \sum_{i=1}^{m} C_i(p_i) - \sum_{j=1}^{n} B_j(d_j) - \lambda p_{EX}$$

$$\text{subject to } \sum_{i=1}^{m} p_i - \sum_{j=1}^{n} d_j + q = p_{EX}$$

$$\underline{p}_i \leq p_i \leq \overline{p}_i, \quad i = 1, \ldots, m$$

$$\underline{d}_j \leq d_j \leq \overline{d}_j \quad j = 1, \ldots, n$$

λ as parameter, calculate p_{EX}

Approach II

$$\min_{\{p_i\},\{d_j\}} \sum_{i=1}^{m} C_i(p_i) - \sum_{j=1}^{n} B_j(d_j)$$

$$\text{subject to } \sum_{i=1}^{m} p_i - \sum_{j=1}^{n} d_j + q = p_{EX} \; \clubsuit$$

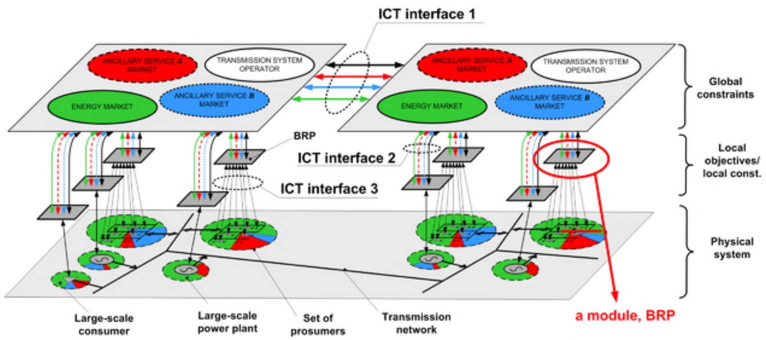
$$\underline{p}_i \leq p_i \leq \overline{p}_i \quad i = 1, \ldots, m$$

$$\underline{d}_j \leq d_j \leq \overline{d}_j \quad j = 1, \ldots, n$$

p_{EX} as parameter, Lagrange multiplier to ♣ as price

Exercise 3: Show equivalence between Approach I and Approach II.

Balance responsible party

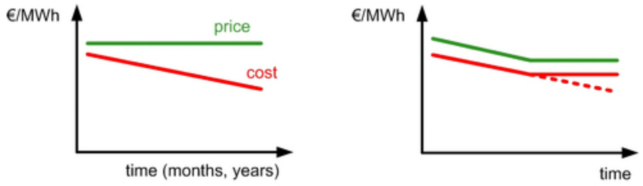


- All market participants interact with markets through a BRP, or are a BRP themselves.
- BRP as a **module** (building block)
- Heterogeneity, local "issues".... all "hidden" behind the interface ("Interface 2")
- Example: bids are requested to be increasing functions (CONVEXITY) - simple and "smart" way to deal with complexity
- Later on: BRP will have to internally "decouple" services to comply with protocols

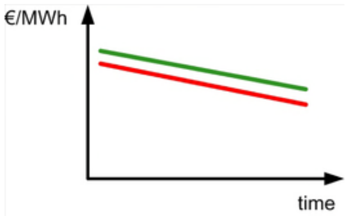
Outline

Benefits of market-based (price-based) operation

In mathematical terms we reached (via dual) the same solution (as primal).
Why deregulation?



a) price-cap regulation b) cost of service regulation



Benefits of market-based (price-based) operation

In mathematical terms we reached (via dual) the same solution (as primal).
Why deregulation?

Competitive markets simultaneously

- hold prices down to marginal cost
- minimize cost

Regulation can do one or the other, but not both.

Perfect competition

Adam Smith (*"Wealth of Nations"*):

- perfectly competitive market $\Longrightarrow$ economic efficiency
- "invisible hand of market" (*Solution architecture matters*)

Perfect competition (conditions)

- large number of generators (market agents)
- each agent act competitively (attempts to maximize its profits)
- price taking agents
- good information (market prices are publicly known)
- well-behaved costs

Well-behaved costs = convexity. Important for existence of equilibrium.
Difficulties: start up costs

Competitive equilibrium

A market condition in which supply equals demand and traders are price takers.

Particularities of markets in power systems

Problems with electrical energy as commodity

- **No buffering.** Cannot be efficiently stored in large quantities. Consumed as produced $\rightarrow$ fast changing production costs.
- **No free routing.** Other transportation systems have free choices among alternative paths between source and destination. Power transmission system: power flows governed by physical laws.

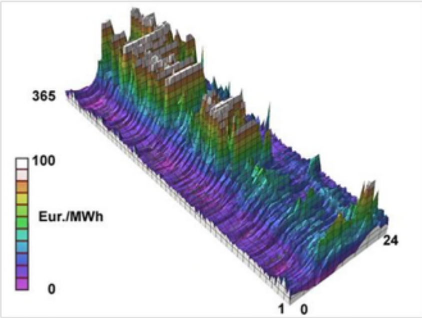
Demand-side flaws

- Lack of metering and real-time billing. Customers disconnected from market (do not respond to real-time fluctuations in price/cost of supply)
- Lack of real-time control of power flow to specific customers. Ability of load to take power from the grid without prior contract with a generator.

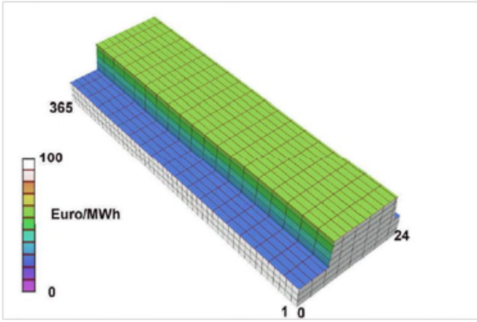
Consequences: necessity of an **independent system operator** as supplier in real-time, responsible for balancing;
necessity of well designed **market architecture**

Prices

Demand-side flaws



Yearly market prices (APX)



Prices for consumers

Benefits of market-based (price-based) operation

Some expected benefits:

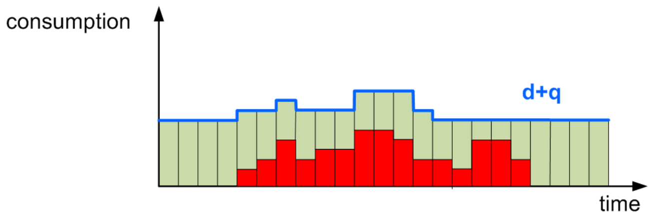
- large benefits expected to come from demand side (price-elastic consumers in "smart grids") when exposed to real-time prices (smart meters)
- $\rightarrow$ lower demand when generation is most costly
- $\rightarrow$ in long run: less generators to be built, reduced production costs

Load factor

$$\text{load factor} = \frac{\text{average demand}}{\text{peak demand}}$$

Real-time pricing reduces load factor (but in the most general case does not achieve load factor of 1).

Benefits of market-based (price-based) operation



$p(k)$=controllable power production at time k
$q(k)$=uncontrollable load or negated uncontrollable power
$d(k)$=controllable load
$C(p)$=cost function for producing at power level p
$B(d)$=benefit function of consuming at power level d
Energy constrained load: $\sum_{k=1}^{N} d(k) = E_N$
(with $B(d) = const.$, the goal of consumption profile $d(1), \dots, d(N)$ is to *shift the load* to minimize payments while satisfying energy production over the time horizon)

Benefits of market-based (price-based) operation

Example

Social welfare maximization ($\equiv$ market solution under perfect competition)

$$\min_{\{p(k),d(k)\}_{k=1,\dots,N}} \sum_{k=1}^{N} \Big(C(p(k)) - B(d(k)) \Big)$$
$$\text{subject to} \quad p(k) = d(k) + q(k), \quad k = 1, \dots, N$$
$$\sum_{k=1}^{N} d(k) = E_N$$

- With $C(\cdot), B(\cdot)$ strictly convex/concave and q is not constant in time, power factor is necessarily smaller than 1.
- With $B(\cdot) \equiv 0$, load shifting leads to power factor 1 even with $q \neq \mathbf{1}c$

Exercise 4: Prove the above statements.

Benefits of market-based (price-based) operation

Example

Social welfare maximization ($\equiv$ market solution under perfect competition)

$$\begin{aligned} \min_{\{p(k),d(k)\}_{k=1,\dots,N}} \quad & \sum_{k=1}^{N} \Big(C(p(k)) - B(d(k)) \Big) \\ \text{subject to} \quad & p(k) = d(k) + q(k), \quad k = 1, \dots, N \\ & \sum_{k=1}^{N} d(k) = E_N \end{aligned}$$

Constant power profiles

($q = 0$) Let $C_i(\cdot)$ be strictly convex function ($B_i(\cdot)$ strictly concave function). Then optimal power production (consumption) profile to produce (consume) certain amount of *energy* over some PTU is a *constant production (consumption) profile*.

...observation in favour of dealing with real-time power balancing and congestion.

Benefits of market-based (price-based) operation

Load shifting (load factor improvement) caused by pricing is in some cases self-limiting

still ...

(+) changing load factor from 60% to 80% gives 25% reduction in needed generation capacity.

but...

(-) with more loads as baseload, reduction of for peaking generators: fixed costs reduction of $\approx 12\%$ (peaking generators cost roughly half of an average generator costs per installed megawatt). Overall reduction in **cost** of supply relatively low (several percent). [Stoft "Power system economics"]

but ...

(+) price-elastic demand side reduces conditions for **market power**

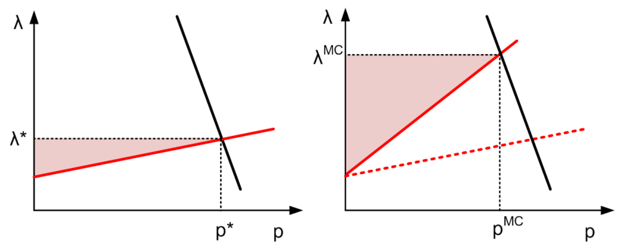
Outline

Market power

Market power

The ability to alter *profitably* prices away from competitive levels.

"profitably": important in definition. Some baseload plant (e.g. nuclear power plant) can influence the system when needed, even if it looses money by exercising this influence (e.g. by shutting down).



(λ^{MC}, p^{MC}) = monopolistic equilibrium
$(\lambda^{\star}, p^{\star})$ = competitive equilibrium

$$\max \ \lambda^{MC}(\beta(p))\, p^{MC}(\beta(p)) - C\big(p^{MC}(\beta(p))\big)$$

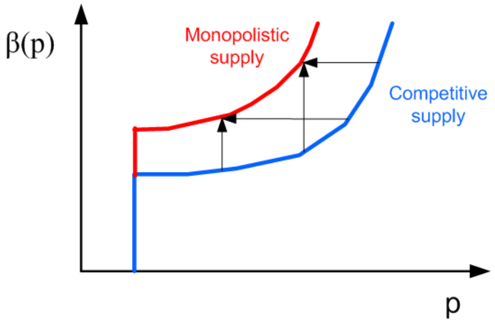
Market power

Market power

- on supply side: monopoly power. result: price higher than competitive
- on demand side: monopsony power. result: price lower than competitive

Exercising monopoly power

- quantity withholding (reducing output)
- financial withholding (raising the price for output)

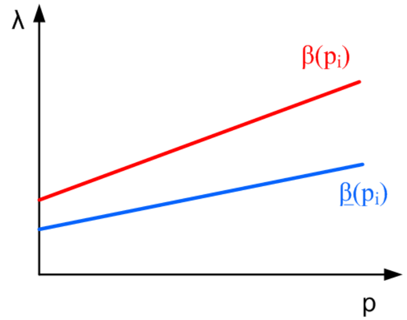


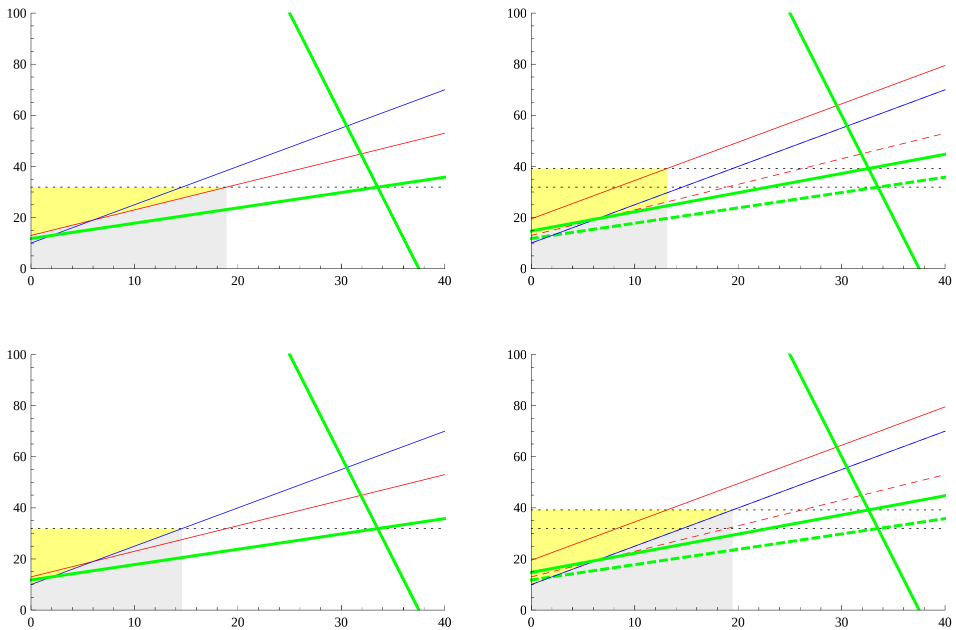
Market power

Example

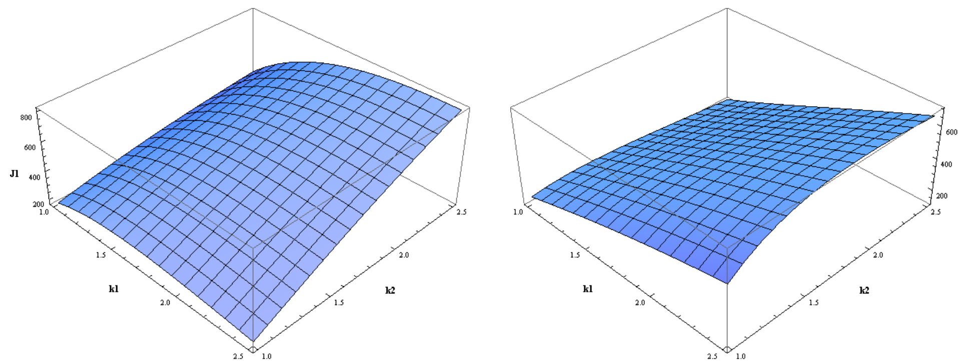
Incremental costs of a supplier: $a_i p_i + b_i$, with $a_i > 0$

Strategy: selecting $k_i \geq 0$ for the bid $\beta_i(p_i) = k_i \underline{\beta}(p_i) = k_i a_i p_i + k_i b_i$





Market power



Market power

Competitive equilibrium (Walrasian equilibrium)

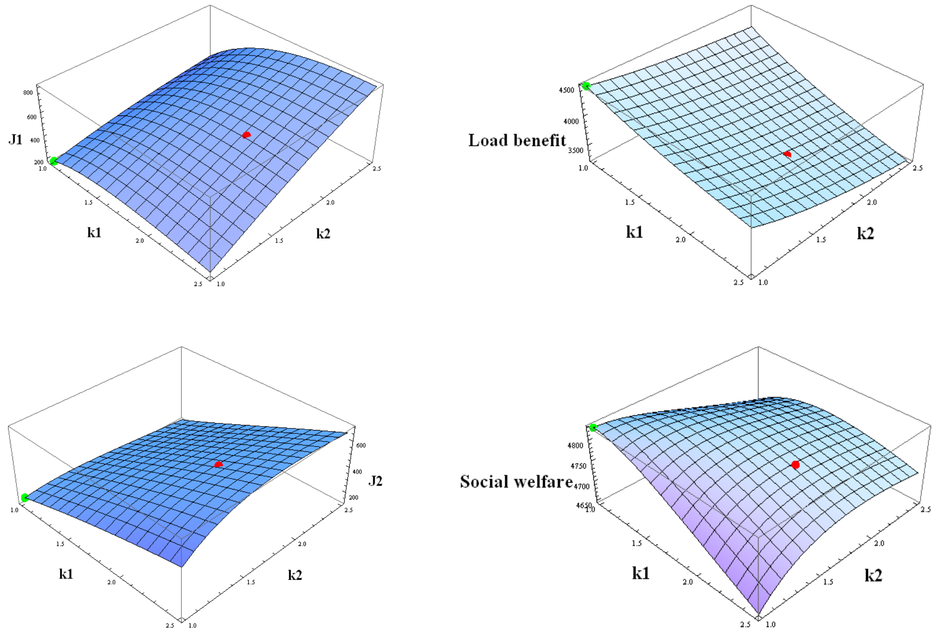
A market condition in which supply equals demand and traders are price takers.

Nash equilibrium

None of the players can increase its benefits by changing its own strategy, provided that other players continue with their strategies.

Strategy S_i of a player i (algorithm for playing in the market)
$J_i(s_1, \ldots, s_n)$: benefits of player i, as outcome of all strategies

$$\forall i, s_i \in S_i : J_i(s_1^*, \ldots, s_{i-1}^*, s_i^*, s_{i+1}^*, \ldots, s_n^*) \geq J_i(s_1^*, \ldots, s_{i-1}^*, s_i, s_{i+1}^*, \ldots, s_n^*)$$



green dot ← perfect competition; red dot ← Nash equilibrium

Market power

Elasticity of demand (e)

With aggregated demand $D := \sum_i d_i$ and price λ

$$e = -\frac{\Delta D}{D} \Big/ \frac{\Delta \lambda}{\lambda} \quad \rightarrow \quad e = -\frac{\mathrm{d}D}{\mathrm{d}\lambda}\frac{\lambda}{D}$$

Market share

$$s_i = \frac{p_i}{\sum_i p_i}$$

Lerner index for Cournot oligopoly (group of uncoordinated suppliers)

$$L_x = \frac{s}{e}$$

For monopoly: $s = 1, L_x = 1/e$.

Summary/illustration of problems

including time couplings

- Forward time BRP bidding over finite horizon of N PTUs.
- Similar formulation: internal BRP re-scheduling / real-time (MPC type) control over one or several PTUs

$\mathbf{p_i} := (p_i(1), \ldots, p_i(N)), \quad \mathbf{d_i} := (d_i(1), \ldots, d_i(N))$
$q(k)$ = (predicted) uncontrollable prosumption at k-th PTU for the considered BRP

BRP's problem with time couplings (example)

$$\min_{\{p_i\},\{d_j\}} \sum_{k=1}^{N} \Big(\sum_i C_i(p_i(k)) - \sum_j B_j(d_j(k)) \Big) - \lambda(k) p_{EX}(k)$$

$$\text{subject to} \quad \sum_i p_i(k) - \sum_j d_j(k) + q(k) = p_{EX}(k)$$

$$p_i(k) \in \mathcal{P}_i(\mathbf{p_i(k)}), \quad d_j(k) \in \mathcal{D}_j(\mathbf{d_j(k)}) \quad (\textit{dynamics}, \textit{constraints})$$

Summary/illustration of problems

including time couplings

$$\min_{\{p_i\},\{d_j\}} \sum_{k=1}^{N} \Big(\sum_i C_i(p_i(k)) - \sum_j B_j(d_j(k)) \Big) - \lambda(k) p_{EX}(k)$$

$$\text{subject to} \quad \sum_i p_i(k) - \sum_j d_j(k) + q(k) = p_{EX}(k)$$

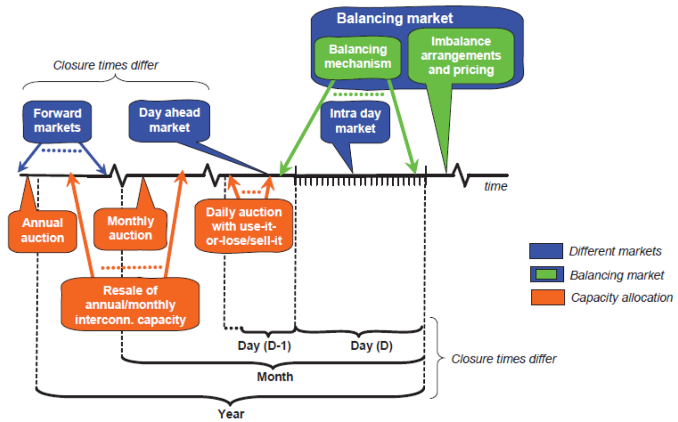
$$p_i(k) \in \mathcal{P}_i(\mathbf{p_i(k)}), \quad d_j(k) \in \mathcal{D}_j(\mathbf{d_j(k)}) \quad (dynamics, constraints)$$

General philosophy: keep market operator's job simple and transparent; let BRPs cope with their problems

- Market operator services for time couplings: block bids, intra-day market
- Similarity with hierarchical/distributed (dual decomposition based) MPC
- Iterations replaced with bids (functions relating primal-dual variables)
- Complexity: largely on the BRP's side, behind the "market interface", behind bid
- Market power, game theory: $\lambda(k, p_{EX}(k))$

Market architecture

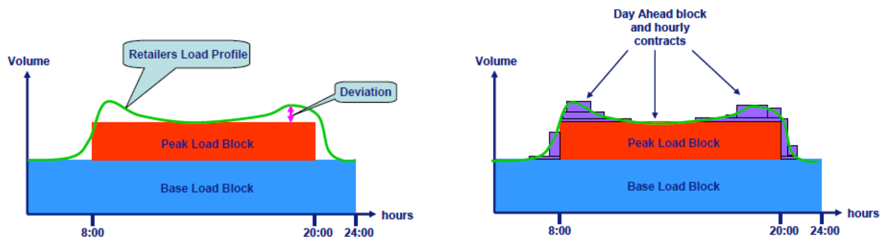
Architecture = functionality allocation: "who does what?", "how are the subsystems interrelated and connected?"



Forward time markets (Bilateral markets; "Over the counter (OTC) trade": reducing risks
Day ahead market: adapting to $D-1$ state/prediction. competition; liquidity
Intraday markets: adaptation to $H-1$ state/prediction (some similarity with MPC)
Balancing market: reflecting true physical transactions

Market architecture

"Submarkets"



(a) The base and peak load contracted on OTC (b) Realized power

The base and peak load on energy markets

Market architecture

Market types

Two basic ways to arrange trades between buyers and sellers

- bilateral (trade directly)
- mediated (over intermediary)

Arrangement	Type of Market				
Bilateral:	Search	Bulletin Board	Brokered		
Mediated:			Dealer	Exchange	Pool
	Less organized			More centralized	

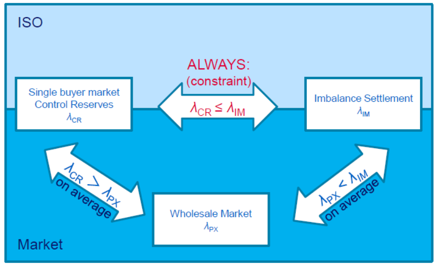
- Currently there is no consensus on the best list of submarkets from which to construct an entire power market.
- Design of market architecture must consider market structure in which it is embedded.
- Market structure = properties of the market closely tied to technology and ownership.

Market architecture

Linkages

- implicit (e.g., prices on forward markets (longer term) try to approximate expected spot prices (short term))
- explicit

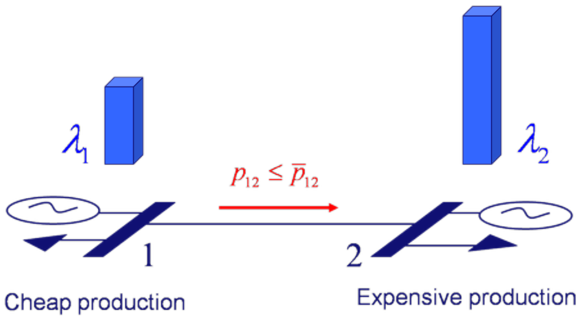
Implicit linkages are important part of market architecture (e.g., they create incentives for certain business opportunities.)



Relations between prices on different markets (TenneT NL)

Outline

Congestion management

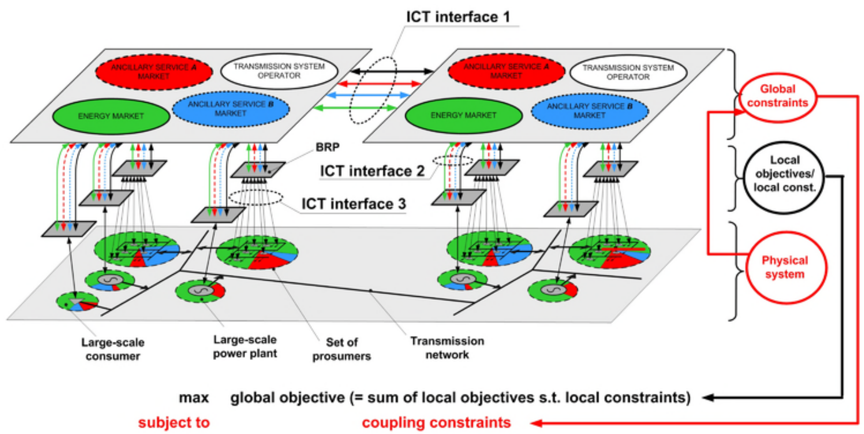


Line flow limits:

- physical: thermal limits, stability limits
- contingency limits (robustness): physical limits following contingency

Congestion is a problem on more time-scales (day-ahead, real-time).

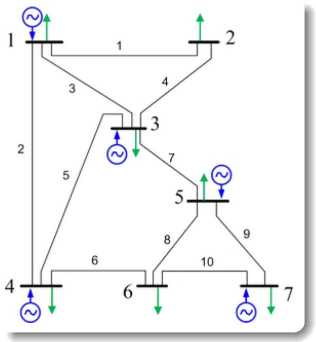
Congestion management



Traditional system: vertically integrated utility with full knowledge and control.
Market-based system. Responsible party: Transmission system operator (TSO).
Transmission system used in different way than planned. One of the toughest problems in market-based operation. Several solution architectures in practice

Recall: power flow equations (DC)

Transmission system: connected undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$



DC power flow model:

$$p_{ij} = b_{ij}(\theta_i - \theta_j) = -p_{ji}$$

b_{ij} = susceptance of line $\epsilon_{ij} \in \mathcal{E}$,
θ_i = voltage phase angle at node (bus) $v_i \in \mathcal{V}$.

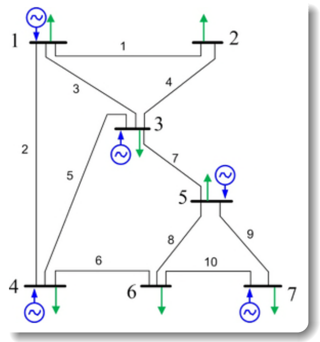
Node v_i with neighbouring nodes $\mathcal{N}_i$, power balance:
$p_i = \sum_{j \in \mathcal{N}_i} p_{ij}$

p_i = node aggregated controllable power injection

- $p_i < 0$ consumption
- $p_i > 0$ production

Recall: power flow equations (DC)

Transmission system: connected undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$



$$\begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{pmatrix} = \begin{pmatrix} b_{\mathcal{N}_1} & -b_{12} & \dots & -b_{1n} \\ -b_{12} & b_{\mathcal{N}_2} & \dots & -b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -b_{1n} & -b_{2n} & \dots & b_{\mathcal{N}_n} \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{pmatrix}$$

with $b_{\mathcal{N}_i} := \sum_{j \in \mathcal{N}_i} b_{ij}$

Power flow equations

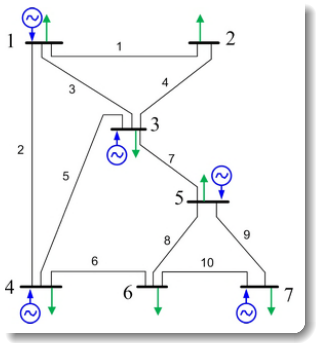
$$p = B\theta$$

Remark: $B^\top = B, \quad B\mathbf{1}_n = 0.$

Line flow limits

$$L\theta \leq \overline{e}_{\mathcal{E}}$$

Power Transfer Distribution Factors (PTDF)



Power Transfer Distribution Factors (PTDF)

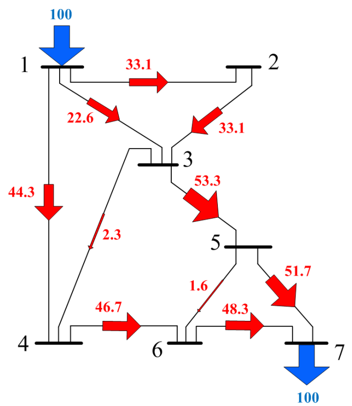
PTDF (*of a line with respect to a transaction*) is the coefficient of the linear relationship between the amount of transaction and the flow on the line.

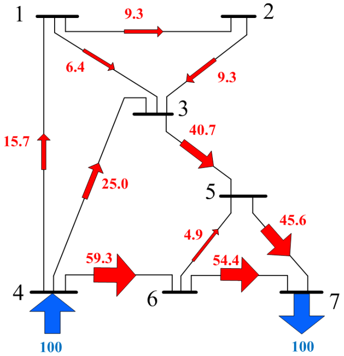
A transaction = specific amount of power injected at one (specified) node and removed at another (specified) node.

PTDF is the fraction of the amount of a transaction from one node to the other that flows over a given transmission line.

Power Transfer Distribution Factors (PTDF)

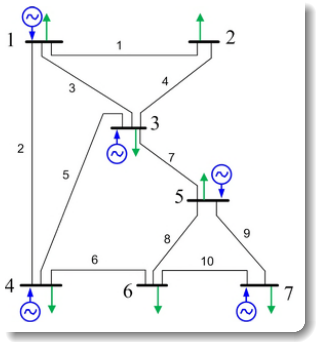
Example.





↓ No free routing.
(↑ Frequency as global variable.)

Power Transfer Distribution Factors (PTDF)



Set $\theta_1 = 0$. With abbreviations
$\tilde{p} := \begin{pmatrix} p_2 & \dots & p_n \end{pmatrix}^\top$, $\tilde{\theta} := \begin{pmatrix} \theta_2 & \dots & \theta_n \end{pmatrix}^\top$:

$$\underbrace{\begin{pmatrix} p_1 \\ \tilde{p} \end{pmatrix}}_{p} = \underbrace{\begin{pmatrix} \tilde{B}_{11} & \tilde{B}_{21}^\top \\ \tilde{B}_{21} & \tilde{B}_{22} \end{pmatrix}}_{B} \underbrace{\begin{pmatrix} 0 \\ \tilde{\theta} \end{pmatrix}}_{\theta}$$

$$\underbrace{\begin{pmatrix} \theta_1 \\ \tilde{\theta} \end{pmatrix}}_{\theta} = \underbrace{\begin{pmatrix} 0 & \mathbf{0}_{n-1}^\top \\ \mathbf{0}_n & \tilde{B}_{22}^{-1} \end{pmatrix}}_{F} \underbrace{\begin{pmatrix} p_1 \\ \tilde{p} \end{pmatrix}}_{p}$$

$\psi_{ij,mn}$ the fraction of transaction from node m to node n, which flows over line ij.

$$\psi_{ij,mn} = b_{ij}(F_{im} - F_{in} - F_{jm} + F_{jn})$$

Optimal power flow problem

p_i = node aggregated controllable power injection with assigned economic objective function $J_i(p_i)$:

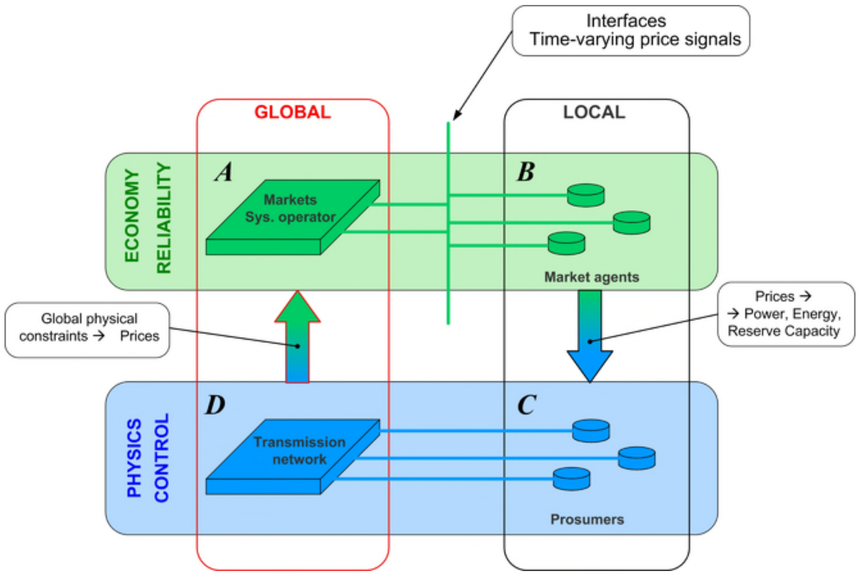
- $p_i < 0$, net consumption, $J_i(p_i) = -B_i(p_i)$
- $p_i > 0$, net production, $J_i(p_i) = C_i(p_i)$

q_i = uncontrollable, price inelastic, nodal power injection (net consumption: $q_i < 0$, net production : $q_i > 0$).

Optimal power flow problem (OPF)

$$\begin{aligned} \min_{p,\theta} \quad & \sum_{i=1}^{n} J_i(p_i) \\ \text{subject to} \quad & p + q - B\theta = 0 \\ & \underline{p} \le p \le \overline{p} \\ & L\theta \le \overline{e}_{\mathcal{E}} \end{aligned}$$

Market-based solution?



Outline

Congestion management approaches

Allocation methods

- Nodal pricing (Locational marginal pricing)
- Zonal pricing:
  - Market splitting
  - Flow-based coupling
- Explicit auctioning
- ...other.. (uniform pricing with congestion relief,...)

Alleviation methods

- Generation dispatching
- Buy-back countertrade

Congestion management approaches

- common: maintaining security; different: impact on market economy
- Why such diversity? previous market developments (history) and conservative engineering, national politics and economic developments, strategic approach to market players, specific topologies, generation portfolios, policy, young filed (?)...
- Congestion management is depended on the energy market architecture

Nodal pricing

Given: bids $\beta(p) := \big(\beta_1(p_1) \quad \dots \quad \beta_n(p_n)\big)^\top$. Deduced: prosumption limits $\{\underline{p}_i, \overline{p}_i\}$, $\underline{p} < \overline{p}$, cost functions $J_i(p_i) := \int_{\underline{p}_i}^{p_i} \beta_i(\xi)\mathrm{d}\xi$ for $p_i \geq 0$ and $J_i(p_i) := \int_{p_i}^{\overline{p}_i} \beta_i(\xi)\mathrm{d}\xi$ for $p_i < 0$

Optimal pricing problem

with $\lambda = \big(\lambda_1 \quad \dots \quad \lambda_n\big)^\top$

$$\min_{p,\theta,\lambda} \sum_{i=1}^{n} J_i(p_i) \quad \text{(max welfare)}$$

subject to

$$\beta(p) = \lambda$$
$$p - B\theta = 0$$
$$L\theta \leq \overline{e}_\mathcal{E}$$

OPF problem

$$\min_{p,\theta} \sum_{i=1}^{n} J_i(p_i)$$

subject to

$$p - B\theta = 0 \quad \clubsuit$$
$$\underline{p} \leq p \leq \overline{p}$$
$$L\theta \leq \overline{e}_\mathcal{E}$$

Proposition

Vector of optimal dual variables related to the constraint (♣) in the dual to OPF problem is the vector of optimal nodal prices.

Intermezzo: Lagrange duality, KKT conditions

$f : \mathbb{R}^n \to \mathbb{R}, \quad h : \mathbb{R}^n \to \mathbb{R}^m, \quad g : \mathbb{R}^n \to \mathbb{R}^p$

$$\min_x \; f(x)$$
$$\text{subject to} \quad h(x) = 0$$
$$g(x) \leq 0$$

Lagrange function

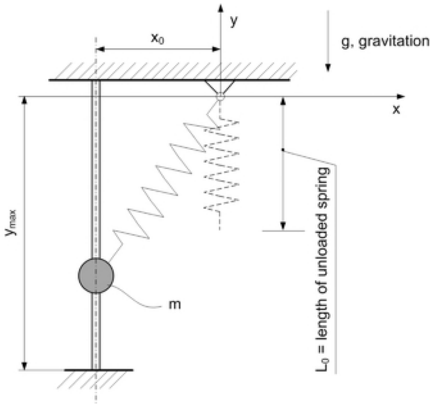
$$L(x, \lambda, \mu) := f(x) + \lambda^\top h(x) + \mu^\top g(x)$$

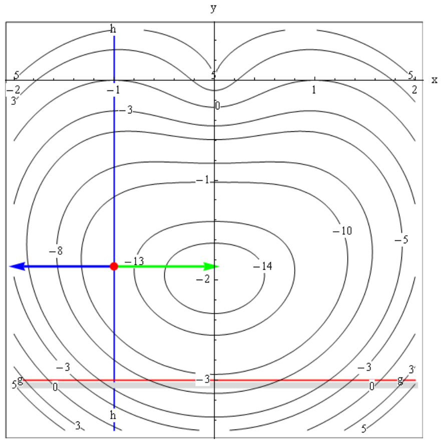
KKT optimality conditions

$$\nabla f(x) + \sum_{i=1}^{m} \lambda_i \nabla h_i(x) + \sum_{i=1}^{p} \mu_i \nabla g_i(g) = 0$$
$$h(x) = 0$$
$$0 \leq -g(x) \perp \mu \geq 0$$

Intermezzo: Lagrange duality, KKT conditions

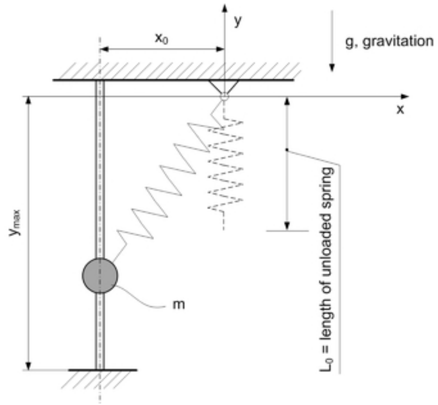
Illustrative example

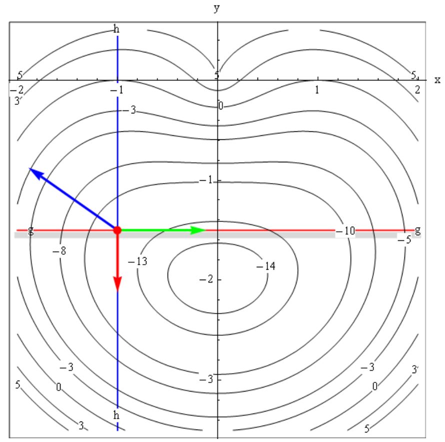




Intermezzo: Lagrange duality, KKT conditions

Illustrative example





Nodal pricing

KKT conditions (after "including back" the limits $\{\underline{p}_i, \overline{p}_i\}$ into the bids $\beta_i(p_i)$)

OPF problem

$$\min_{p,\theta} \sum_{i=1}^{n} J_i(p_i)$$
$$\text{subject to } p - B\theta = 0$$
$$\underline{p} \le p \le \overline{p}$$
$$L\theta \le \overline{e}_{\mathcal{E}}$$

KKT conditions

$$\beta(p^\star) - \lambda^\star = 0$$
$$p^\star - B\theta^\star = 0$$
$$B\lambda^\star + L^\top \mu^\star = 0$$
$$0 \le (-L\theta^\star + \overline{e}_{\mathcal{E}}) \quad \perp \quad \mu^\star \ge 0$$

Singe price in case of no congestion

$$-L\theta^\star + \overline{e}_{\mathcal{E}} < 0 \implies \mu^\star = 0 \implies B\lambda^\star = 0 \implies \lambda^\star = \mathbf{1}_n\hat{\lambda},\ \hat{\lambda} \in \mathbb{R}$$

In case of singe congested line, optimal nodal price in general have different value for each node. $(B\lambda^\star = -L^\top \mu^\star)$

Nodal pricing

Accounting for contingencies

OPF problem with contingencies

$$\min_{p,\theta} \sum_{i=1}^{n} J_i(p_i)$$
$$\text{subject to } p - B\theta = 0$$
$$p - B_c\theta_c = 0$$
$$\underline{p} \le p \le \overline{p}$$
$$L\theta \le \overline{e}_{\mathcal{E}}$$
$$L_c\theta_c \le \overline{e}_c$$

KKT conditions

$$\beta(p^\star) - \underbrace{(\lambda_n^\star + \lambda_c^\star)}_{\lambda^\star} = 0$$
$$p^\star - B\theta^\star = 0$$
$$p^\star - B\theta_c^\star = 0$$
$$B\lambda_n^\star + L^\top \mu_n^\star = 0$$
$$B_c\lambda_c^\star + L_c^\top \mu_c^\star = 0$$
$$0 \le (-L\theta^\star + \overline{e}_{\mathcal{E}}) \quad \perp \quad \mu^\star \ge 0$$
$$0 \le (-L_c\theta_c^\star + \overline{e}_c) \quad \perp \quad \mu_c^\star \ge 0$$

Accounting for overloads when a singe circuit is out: "N-1 criteria.

Usually post contingency flow limits are higher than nominal $(\overline{e}_{\mathcal{E}} \le \overline{e}_c)$

Nodal pricing

Congestion revenue (collected by the market operator): $-(p^{\star})^{\top}\lambda^{\star}$

Congestion revenue (merchandise surplus) is nonnegative

With losses neglected (DC), it always hold that

$$-(p^{\star})^{\top}\lambda^{\star} \geq 0.$$

In case of at least one line congested (line flow constraint active), we have

$$-(p^{\star})^{\top}\lambda^{\star} > 0.$$

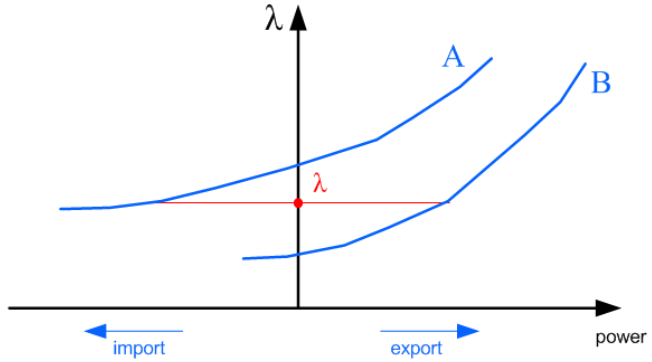
With $p = p_g + p_d$ where $p_g \geq 0$ are generator injections and $p_d \leq 0$ load, we have

$$-(p^{\star})^{\top}\lambda^{\star} \geq 0 \quad \Longrightarrow \quad (\lambda^{\star})^{\top}|p_d| - (\lambda^{\star})^{\top}|p_g| \geq 0 \quad \text{(market operator profits)}$$

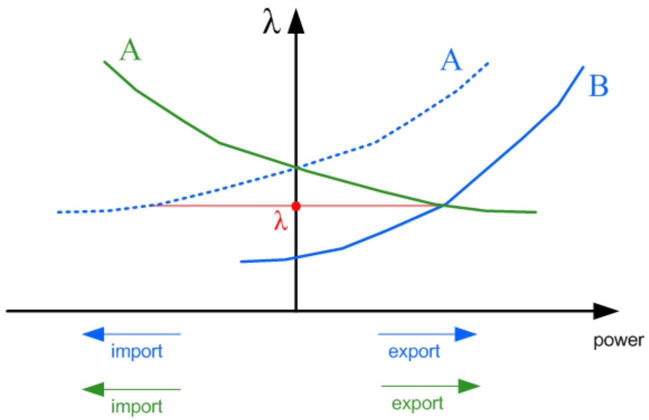
where $|\cdot|$ is elementwise applied absolute value on the vector.

Exercise 5: prove that congestion revenue is always nonnegative
(Hint: multiply optimality condition $B\lambda^{\star} + L^{\top}\mu^{\star} = 0$ from left with $(\theta^{\star})^{\top}$.)

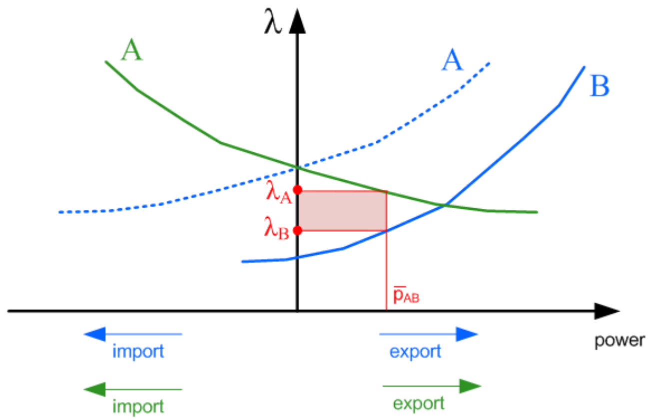
Nodal pricing



Nodal pricing



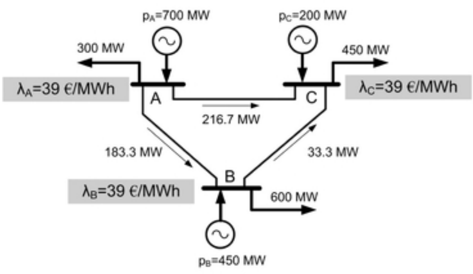
Nodal pricing

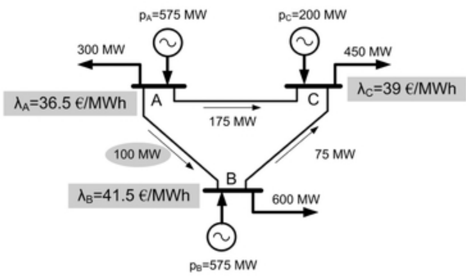


Nodal pricing

Example I

Exercise 6: Solve the nodal pricing problem from the figure.

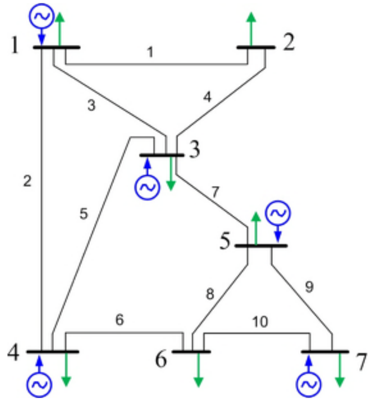


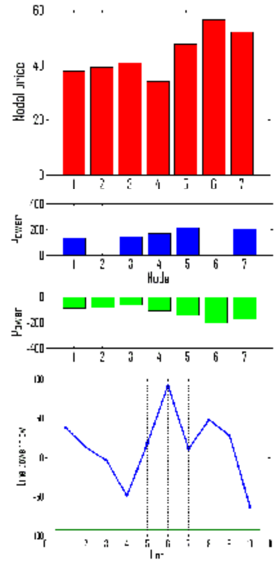


- The bids (incremental costs): $\beta_A(p_A) = 25 + 0.02p_A$, $\beta_B(p_B) = 30 + 0.02p_B$, $\beta_C(p_C) = 35 + 0.02p_C$
- Load is price inelastic.
- Line flow limits: only line $A - B$ has a limit on power flow, which is set to 100MW.
- All three lines are identical

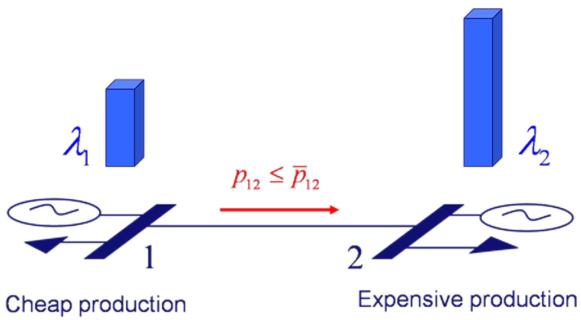
Nodal pricing

Example II





Congestion and market power



- Bid lower then incremental cost in one location to induce congestion and profit by exercising market power in other location.
- Positive side of market power due to congestion or number of generators: larger prices "invite" new players/investments.
- Market power due to exploration of holes in market rules or exploitation of conflict of interest: no useful economic signals

Transmission rights

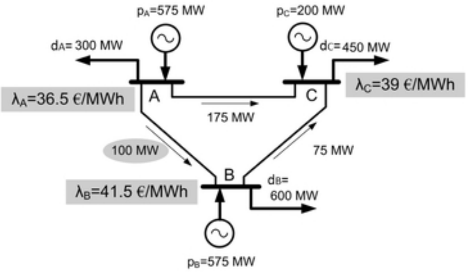
Transmission is scarce.

There is an extra money (congestion rent).

↓

Organize market for transmission rights. Use extra money to control financial risks of congestion induced price variations.

Transmission rights



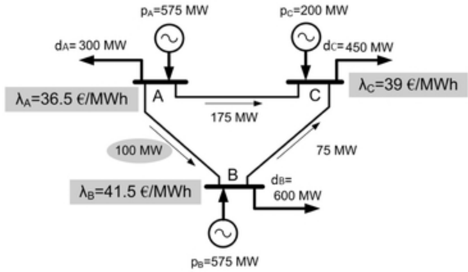
CR = congestion rent

$$CR = \lambda_A(d_A - p_A) + \lambda_B(d_B - p_B) + \lambda_C(d_C - p_C)$$
$$= p_{AB}(\lambda_B - \lambda_A) + p_{BC}(\lambda_B - \lambda_C) + p_{AC}(\lambda_C - \lambda_A)$$
$$= 750$$

Example a)

- d_B has contract for 150MW from p_A.
- Physically max transaction from A to B = 150MW (2/3 of transaction flows across line AB and 1/3 across path $AC - CB$).
- p_B buys 150MW of its power at locational price of node A: pays $d_B * \lambda_B$ but gets compensated (paid by generator in A) in amount $150 * (\lambda_B - \lambda_A) = 750$.
- Market operator compensates generator at A for $750 = CR$

Transmission rights



CR = congestion rent

$$CR = \lambda_A(d_A - p_A) + \lambda_B(d_B - p_B) + \lambda_C(d_C - p_C)$$
$$= p_{AB}(\lambda_B - \lambda_A) + p_{BC}(\lambda_B - \lambda_C) + p_{AC}(\lambda_C - \lambda_A)$$
$$= 750$$

Example b)

- d_C has contract for 300MW from p_A.
- Physically max transaction from A to C = 300MW (1/3 of transaction flows across path $AB - BC$ and 2/3 across line AC).
- p_C buys 300MW of its power at locational price of node A: pays $d_C * \lambda_C$ but gets compensated (paid by generator in A) in amount $300 * (\lambda_C - \lambda_A) = 750$.
- Market operator compensates generator at A for $750 = CR$

Transmission rights

Optimal nodal prices are competitive prices. → Well designed markets with perfect competition will find the same set of prices as calculated via Lagrange multipliers.

So, using optimization (duality) is a "shortcut". However...

- One might purchase a transmission right to protect itself against locational price swings due to congestion (congestion implies more local balancing → local conditions are more volatile than global (no aggregation) → volatility of locational prices)
- Owning a transmission right protects loads from market power exercise of local producers
- Market operator might have losses if contracted transmission rights are in excess of transmission capacity across a congested interface (sell according to worst case contingency)
- With limited amount of transmission rights, not all loads are protected from market power in case of congestion

Zonal pricing (market splitting)

Given: bids $\beta(p) := \begin{pmatrix}\beta_1(p_1) & \dots & \beta_n(p_n)\end{pmatrix}^\top$
Deduced: cost functions $J_i(p_i)$

Optimal pricing problem

with $\lambda = \begin{pmatrix}\mathbf{1_{n1}}^\top \lambda_{\mathcal{Z}_1} & \dots & \mathbf{1_{nK}}^\top \lambda_{\mathcal{Z}_K}\end{pmatrix}^\top$

$$\min_{p,\theta,\lambda} \sum_{i=1}^{n} J_i(p_i) \quad \text{(max welfare)}$$

subject to

$$\beta(p) = \lambda$$
$$p - B\theta = 0$$
$$L\theta \leq \bar{e}_{\mathcal{E}}$$

Different types of bids - different class of optimization problem:

i) QP for $\{\beta_i(p_i)\}_{i=1,\dots,n}$ affine with no saturation
ii) MILP for $\{\beta_i(p_i)\}_{i=1,\dots,n}$ piecewise constant (often in current practice)
iii) MIQP $\{\beta_i(p_i)\}_{i=1,\dots,n}$ affine with saturations

No simple characterization via duality, except for (i).

$\lambda_{\mathcal{Z}_i}$ zonal price for n_i nodes in zone i (zone $\mathcal{Z}_i$).
First n_1 nodes in zone $\mathcal{Z}_2$, then next n_2 nodes in zone $\mathcal{Z}_2$,...

Zonal pricing (market splitting)

Given: bids $\beta(p) := \big(\beta_1(p_1) \quad \dots \quad \beta_n(p_n)\big)^\top$
Deduced: cost functions $J_i(p_i)$

Optimal pricing problem

with $\lambda = \big(\mathbf{1_{n1}}^\top \lambda_{\mathcal{Z}_1} \quad \dots \quad \mathbf{1_{nK}}^\top \lambda_{\mathcal{Z}_K}\big)^\top$

$$\min_{p,\theta,\lambda} \sum_{i=1}^{n} J_i(p_i) \quad \text{(max welfare)}$$

subject to

$$\beta(p) = \lambda$$
$$p - B\theta = 0 \quad \clubsuit$$
$$L\theta - \bar{e}_{\mathcal{E}} \leq 0 \quad \spadesuit$$

Zonal prices for affine bids (case (i))

$\gamma_i(\cdot) = \beta_i^{-1}(\cdot)$

$\tilde{\mu}$ opt. Lagrange multiplier for $\spadesuit$
$\tilde{\lambda}$ opt. Lagrange multiplier for $\clubsuit$ ("auxiliary nodal prices", note that $B\tilde{\lambda} + L^\top\tilde{\mu} = 0$)

$$\sum_{j \in \mathcal{Z}_i} (\tilde{\lambda}_j - \lambda_{\mathcal{Z}_i})\gamma_j'(\lambda_{\mathcal{Z}_i}) = 0, \quad i = 1, \dots, K$$

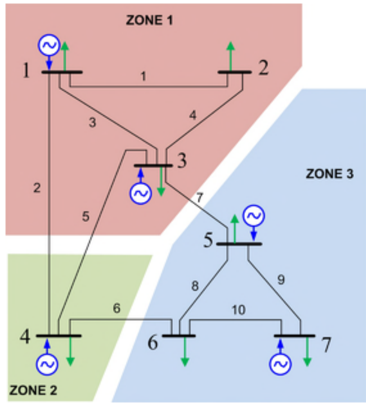
where $\gamma_j'(\cdot)$ is derivative of $\gamma_j(\cdot)$.

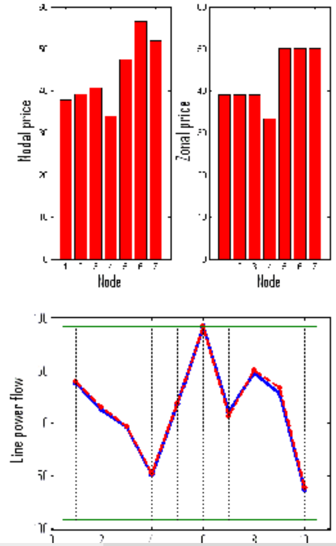
In case of affine bids, zonal prices can be calculated as averaged sum of auxiliary nodal prices, where the weights are derived from the bids.

Zonal pricing (market splitting)

Example

Exercise 7 (on next slide)





INTERMEZZO: Exercise 7

Exercise 7
For network with topology on previous slide calculate: nodal prices, zonal prices, PTDFs for transactions of choice, ...

line i-j	x_{ij}	flow limit
1-2	0.0576	100
1-4	0.092	100
1-3	0.17	100
2-3	0.0586	100
3-4	0.1008	100
4-6	0.072	100
3-5	0.0625	100
3-5	0.161	100
3-5	0.085	100
3-5	0.0856	100

node i	a_i	b_i	load
1	0.13	1.73	88
2	-	-	87
3	0.13	1.86	64
4	0.09	2.13	110
5	0.10	2.39	147
6	-	-	203
7	0.12	2.53	172

Cost function of generator at node i:
$C_i(p_i) = a_i p_i^2 + b_i p_i$

Zonal pricing (flow-based market coupling)

CWE FB market coupling

CWE = Central Western Europe
NWE = North-West Europe
The market coupling evolved from market splitting.
In EU, price zones already exist (national networks).
Goal: coupling of price zones (pan-EU market).

- Available Transfer Capacity (ATC) based market coupling: in 2010 for NWE
- Flow-based market coupling: parallel run and testing for CWE region
  - estimated increase in day-head market welfare: 95M Euro / year (report 9 May 2014)

Zonal pricing (flow-based market coupling)

CWE FB market coupling

Market coupling

- matching orders on several power exchanges (market operators)
- implicit (transfer) capacity allocation mechanism
- market prices and net positions of the connected markets simultaneously determined
- goal: efficient and safe usage of transmission system under coupled markets

Zonal pricing (flow-based market coupling)

CWE FB market coupling

$e_{\mathcal{C}} \in \mathbb{R}^T$ vector power flows in T congestion critical lines
$e_{\mathcal{C}}^{ref} \in \mathbb{R}^T$ vector of predicted (reference) line power flows in congestion critical lines
$p_{\mathcal{Z}_i} \in \mathbb{R}$ aggregated prosumption in zone i
$p_{\mathcal{Z}_i}^{ref} \in \mathbb{R}$ predicted aggregated prosumption in zone i
$\Psi \in \mathbb{R}^{T\times K}$ matrix of "zonal" Power Transfer Distribution Factors (PTDF)
$p_{\mathcal{Z}} := \left(p_{\mathcal{Z}_1}, \ldots, p_{\mathcal{Z}_K}\right)^\top$, $p_{\mathcal{Z}}^{ref} := \left(p_{\mathcal{Z}_1}^{ref}, \ldots, p_{\mathcal{Z}_K}^{ref}\right)^\top$

$$e_{\mathcal{C}} = e_{\mathcal{C}}^{ref} + \Psi(p_{\mathcal{Z}} - p_{\mathcal{Z}}^{ref})$$

Generation Shift Key (GSK)

$$\Psi = \tilde{\Psi}\underbrace{\mathrm{diag}(M_1, \ldots, M_K)}_{M}$$

$M_i \in \mathbb{R}^{R_i}$ = Generation Shift Key (GSK) = mapping from aggregated zone power variation (scalar value) into variations of R_i nodal "market active" power injections in that zone.
$\tilde{\Psi} \in \mathbb{R}^{T\times(R_1+\ldots+R_K)}$ = matrix of "standard" PTDF factors

From aggregated zonal bids $\beta_{\mathcal{Z}_i}(p_{\mathcal{Z}_i})$ deduce objective functions $J_i(p_{\mathcal{Z}_i})$.
$p_{\mathcal{Z}} := \left(p_{\mathcal{Z}_1}, \ldots, p_{\mathcal{Z}_K}\right)^\top$, $p_{\mathcal{Z}_i} \in \mathbb{R}$ (not sign restricted, possible net import and net export)
$\lambda_{\mathcal{Z}} := \left(\lambda_{\mathcal{Z}_1}, \ldots, \lambda_{\mathcal{Z}_K}\right)^\top$, $\lambda_{\mathcal{Z}_i} \in \mathbb{R}$, $s_{\mathcal{C}}$ is vector of reliability margins

Market coupling problem

$$\begin{aligned} \min_{p_{\mathcal{Z}},\lambda_{\mathcal{Z}}} \quad & \sum_{i=1}^{K} J_{\mathcal{Z}_i}(p_{\mathcal{Z}_i}) \\ \text{subject to} \quad & \beta_{\mathcal{Z}}(p_{\mathcal{Z}}) = \lambda_{\mathcal{Z}} \\ & \sum_{i=1}^{K} p_{\mathcal{Z}_i} = 0 \\ & \underbrace{e_{\mathcal{C}}^{ref} + \boxed{\tilde{\Psi}M(p_{\mathcal{Z}} - p_{\mathcal{Z}}^{ref})}}_{e_{\mathcal{C}}} + s_{\mathcal{C}} - \bar{e}_{\mathcal{C}} \le 0 \end{aligned}$$

Market coupling problem ♣

$$\begin{aligned} \min_{p_{\mathcal{Z}},\theta,\lambda_{\mathcal{Z}}} \quad & \sum_{i=1}^{K} J_{\mathcal{Z}_i}(p_{\mathcal{Z}_i}) \\ \text{subject to} \quad & \beta_{\mathcal{Z}}(p_{\mathcal{Z}}) = \lambda_{\mathcal{Z}} \\ & \boxed{Mp_{\mathcal{Z}}} - B\theta = 0 \\ & \underbrace{e_{\mathcal{C}}^{ref} + L\theta}_{e_{\mathcal{C}}} + s_{\mathcal{C}} - \bar{e}_{\mathcal{C}} \le 0 \end{aligned}$$

$\boxed{\text{boxed parts}}$ = relaxation of difficult part for zonal pricing (origin of nonconvexity).

citation:"*...due to convexity pre-requisite of the flow based domain, the GSK must be linear...*"
There is more structure in ♣ formulation (possible to exploit).

Zonal pricing (flow-based market coupling)

CWE FB market coupling

Remarks

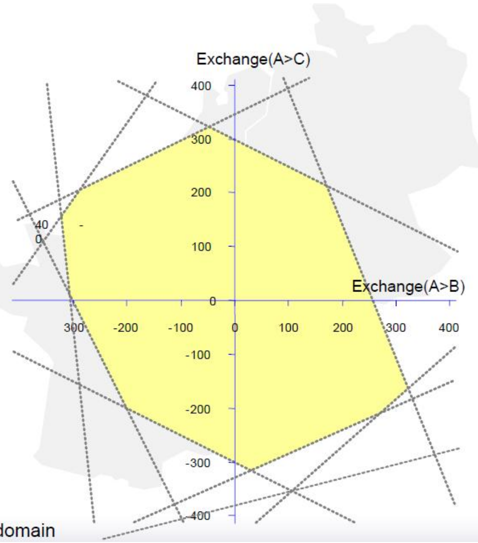
- "a critical branch is considered to be significantly impacted by CWE cross border trade, if its maximum CWE zone-to-zone PTDF is larger then 5%"
- regularly updated (D-2 days) detailed transmission system model and parameters estimation in detailed model used for PTDF calculation
- regular cooperation of all TSO's in gathering data
- reliability margins $s_{\mathcal{C}}$: to capture uncertainties, among others from GSK approximation

Zonal pricing (flow-based market coupling)

CWE FB market coupling

⚠ Numbers are for illustration only

Monitored Lines	Outage scenario	Margin left (MW)	Influence of exchange on lines (PTDF)		
			A→B	A→C	B→C
Line 1	No outage	150	1%	10%	3%
	Outage 1	120	5%	20%	1%
	Outage 2	100	6%	25%	1%
Line 2	No outage	150	-2%	0	5%
	Outage 3	100	-12%	0	10 %
Line 3	No outage				
	Outage 4				

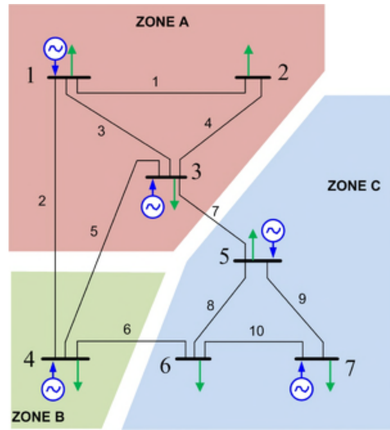


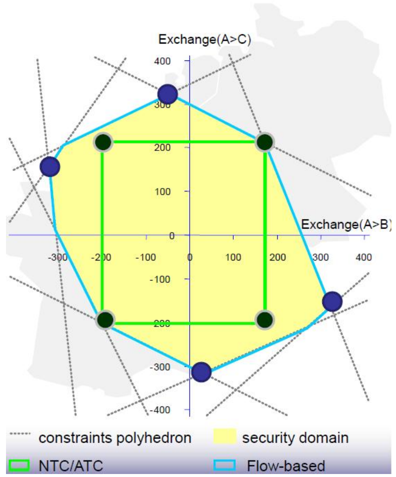
Constraints

Security of Supply domain

Zonal pricing (flow-based market coupling)

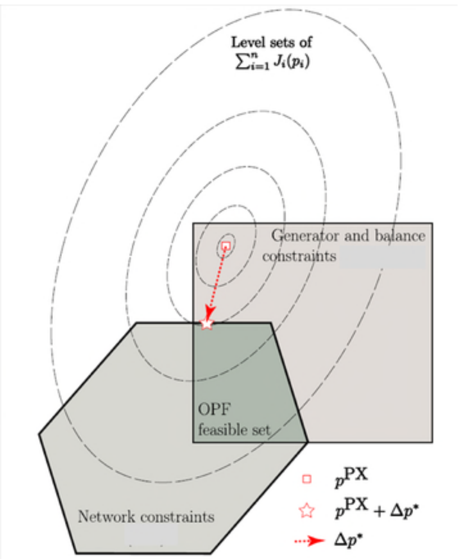
CWE FB market coupling





Alleviation methods

Illustration of optimal redispatch



1) Clear energy market ignoring (internal) line flow limits $\rightarrow (p^{PX}, \theta^{PX})$
2) Redispatch if a line flow limit violated

$$\min_{\Delta p, \Delta\theta} \sum_i J_i(\Delta p_i)$$
$$\text{subject to } \Delta p - B\Delta\theta = 0$$
$$L(\theta^{PX} + \Delta\theta) \leq \bar{e}_{\mathcal{E}}$$

3) Based on $\Delta p^{\star}$, the TSO pays $J_i(\Delta p_i)$ to i-th prosumer

Outline

Convexification of OPF

Bus injection model

$\mathbf{v_k}, \mathbf{i_k}, \mathbf{s_k}$ = voltage, current, power (all complex) at node k
$\mathbf{Y}$ admittance matrix
e_k column vector with 1 in the k-th entry, zero elsewhere

$\mathbf{s_k} = p_k + iq_k$

$\mathbf{s_k} = \mathbf{v_k}\mathbf{i_k}^* = (e_k^\top \mathbf{v})(e_k^\top \mathbf{Y}\mathbf{v})^* = \text{tr}\ (\mathbf{Y}^* e_k e_k^\top)\mathbf{v}\mathbf{v}^*$

with $\mathbf{Y_k} = e_k e_k^\top \mathbf{Y}$, $\quad \Phi_k := \frac{1}{2}(\mathbf{Y_k}^* + \mathbf{Y_k})$, $\quad \Psi_k := \frac{1}{2i}(\mathbf{Y_k}^* - \mathbf{Y_k})$, $\quad J_k := e_k e_k^\top$

$$p_k = \text{tr}\ \Phi_k \mathbf{v}\mathbf{v}^*$$
$$q_k = \text{tr}\ \Psi_k \mathbf{v}\mathbf{v}^*$$
$$|\mathbf{v_k}|^2 = \text{tr}\ J_k \mathbf{v}\mathbf{v}^*$$

Convexification of OPF

OPF problem (QCQP)

$$\min_{\mathbf{v}} \sum_k \text{tr}\ C_k \mathbf{v}\mathbf{v}^*$$

subjet to

$$\underline{p}_k \leq \text{tr}\ \Phi_k \mathbf{v}\mathbf{v}^* \leq \overline{p}_k$$
$$\underline{q}_k \leq \text{tr}\ \Psi_k \mathbf{v}\mathbf{v}^* \leq \overline{q}_k$$
$$\underline{\mathbf{v}}_\mathbf{k}^2 \leq \text{tr}\ J_k \mathbf{v}\mathbf{v}^* \leq \overline{\mathbf{v}}_\mathbf{k}^2$$

SDP formulation of the OPF problem

$$\min_{\mathbf{v}} \sum_k \text{tr}\ C_k W$$

subjet to

$$\underline{p}_k \leq \text{tr}\ \Phi_k W \leq \overline{p}_k$$
$$\underline{q}_k \leq \text{tr}\ \Psi_k W \leq \overline{q}_k$$
$$\underline{\mathbf{v}}_\mathbf{k}^2 \leq \text{tr}\ J_k W \leq \overline{\mathbf{v}}_\mathbf{k}^2$$
$$W \succeq 0$$
$$\text{rank}(W) = 1$$

SDP relaxation of the OPF problem

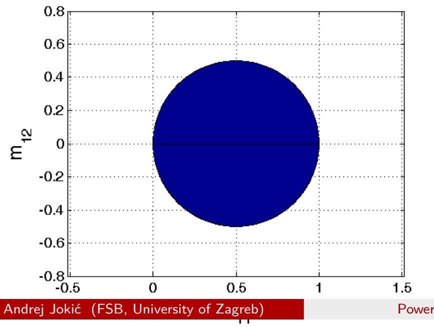
Omit the constraint $\text{rank}(W) = 1$

Convexification of OPF

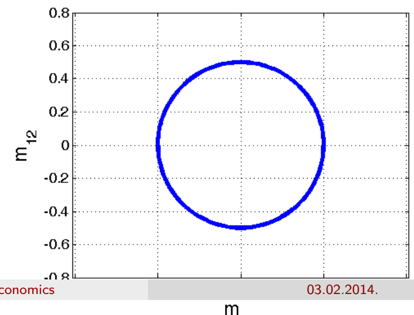
Example. Rank constraint as origin of nonconvexity.

$$M = \begin{pmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{pmatrix}$$

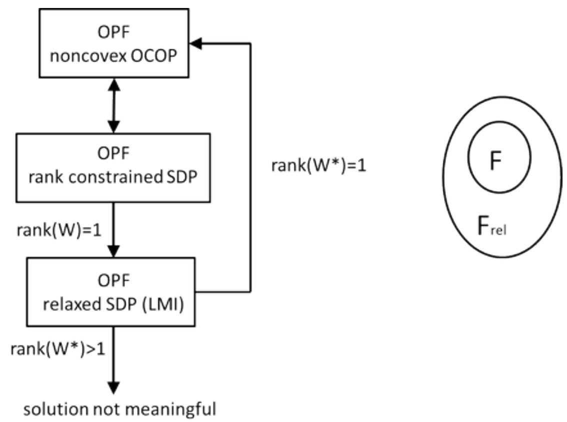
$$M \succeq 0$$
$$\text{trace}(M) = 1$$



$$M \succeq 0$$
$$\text{trace}(M) = 1$$
$$\text{rank}(M) = 1$$



Convex relaxation of OPF



- Radial networks: ∃ (mild) sufficient conditions for exactness of relaxation
- Branch flow model: radial net → exact
- Mesh networks: convexification via phase shifters
- When exact: strong duality

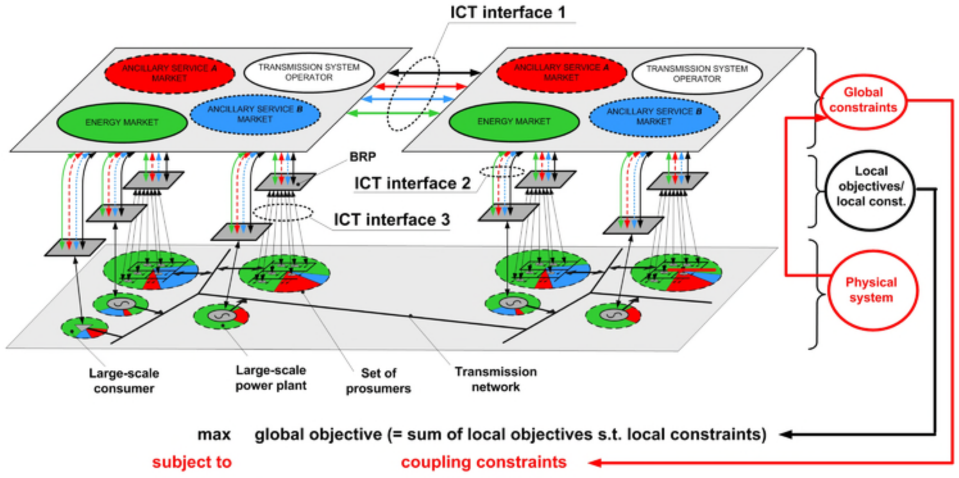
Convex relaxation of OPF

Mesh network

OPF Problem	SDP Relaxation of OPF
Minimize $\sum_{k\in\mathcal{G}} f_k(P_{G_k})$ over $\mathbf{P}_G$, $\mathbf{Q}_G$, $\mathbf{V}$	Minimize $\sum_{k\in\mathcal{G}} f_k(P_{G_k})$ over $\mathbf{P}_G$, $\mathbf{Q}_G$, $\mathbf{W}\in\mathbb{H}_+^n$
Subject to:	Subject to:
1- A capacity constraint for each line $(l,m)\in\mathcal{L}$	1- A convexified capacity constraint for each line
2- The following constraints for each bus $k\in\mathcal{N}$:	2- The following constraints for each bus $k\in\mathcal{N}$:
$P_{G_k}-P_{D_k}=\sum_{l\in\mathcal{N}(k)}\mathrm{Re}\{V_k(V_k^*-V_l^*)y_{kl}^*\}$ (1a)	$P_{G_k}-P_{D_k}=\sum_{l\in\mathcal{N}(k)}\mathrm{Re}\{(W_{kk}-W_{kl})y_{kl}^*\}$ (2a)
$Q_{G_k}-Q_{D_k}=\sum_{l\in\mathcal{N}(k)}\mathrm{Im}\{V_k(V_k^*-V_l^*)y_{kl}^*\}$ (1b)	$Q_{G_k}-Q_{D_k}=\sum_{l\in\mathcal{N}(k)}\mathrm{Im}\{(W_{kk}-W_{kl})y_{kl}^*\}$ (2b)
$P_k^{\min}\le P_{G_k}\le P_k^{\max}$ (1c)	$P_k^{\min}\le P_{G_k}\le P_k^{\max}$ (2c)
$Q_k^{\min}\le Q_{G_k}\le Q_k^{\max}$ (1d)	$Q_k^{\min}\le Q_{G_k}\le Q_k^{\max}$ (2d)
$V_k^{\min}\le \lvert V_k\rvert\le V_k^{\max}$ (1e)	$(V_k^{\min})^2\le W_{kk}\le (V_k^{\max})^2$ (2e)

Capacity constraint for line $(l,m)\in\mathcal{L}$	Convexified capacity constraint for line $(l,m)\in\mathcal{L}$
$\lvert\theta_{lm}\rvert=\lvert\measuredangle V_l-\measuredangle V_m\rvert\le\theta_{lm}^{\max}$ (3a)	$\mathrm{Im}\{W_{lm}\}\le\mathrm{Re}\{W_{lm}\}\tan(\theta_{lm}^{\max})$ (4a)
$\lvert P_{lm}\rvert=\lvert\mathrm{Re}\{V_l(V_l^*-V_m^*)y_{lm}^*\}\rvert\le P_{lm}^{\max}$ (3b)	$\mathrm{Re}\{(W_{ll}-W_{lm})y_{lm}^*\}\le P_{lm}^{\max}$ (4b)
$\lvert S_{lm}\rvert=\lvert V_l(V_l^*-V_m^*)y_{lm}^*\rvert\le S_{lm}^{\max}$ (3c)	$\lvert(W_{ll}-W_{lm})y_{lm}^*\rvert\le S_{lm}^{\max}$ (4c)
$\lvert V_l-V_m\rvert\le\Delta V_{lm}^{\max}$ (3d)	$W_{ll}+W_{mm}-W_{lm}-W_{ml}\le(\Delta V_{lm}^{\max})^2$ (4d)

Solution architecture: Some challenges and potentials



Solution architecture: Some challenges and potentials

- do not use PTDF - easier to decompose on Interface 1
- Keeping voltage phase angles preserves the structure
- Interface 1 in reality replaced with higher hierachical level, not reflecting toplogy of the system
- Both interface 1 and 2 require parts of variables of the power flow
- Interface 3 currently hardly exists - large potentials
- Full AC with uncertainties - robust solutions, conservatism? Stohastic settings...

Outline

Ancillary services (AS)

Regulated system: AS bundled with energy
Deregulated system: unbundling of AS, creation of competitive markets for AS

Ancillary services

- Real power balancing
- Voltage support (voltage stability)
- Network congestion relief (transmission security)
- Economic dispatch
- Financial trade enforcement
- Black start

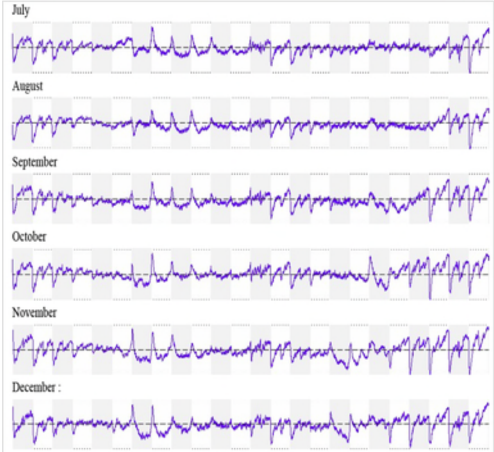


Figure 6.1. Monthly frequency profiles for 2007
Scaling is +/- 50 mHz (vertical) and 24 hours (horizontal)

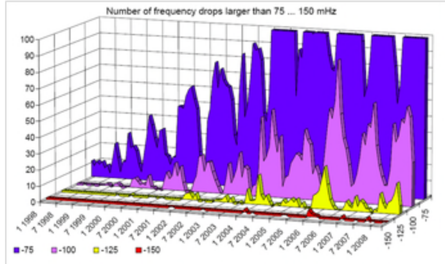
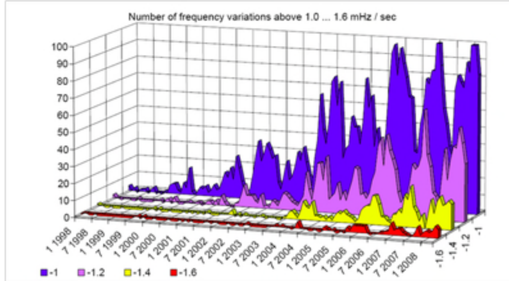
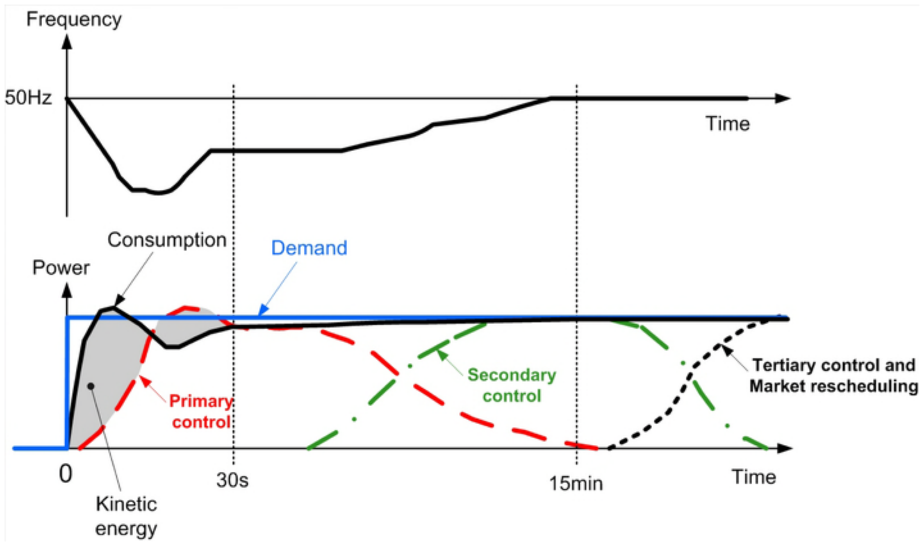


Figure 3.4: Evolution of number of 75, 100, 125, 150 mHz frequency drops



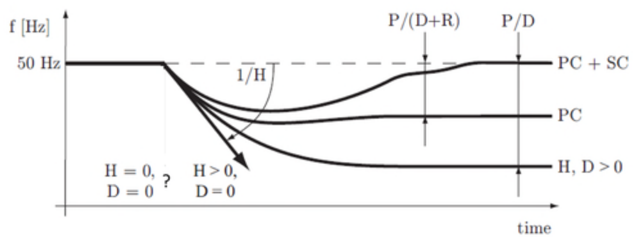
Power balancing ancillary services



Commodities

Related AS commodities

- Inertia: not a commodity.
- Primary control (PC) commodities: capacity (usually mapped into control gain (droop). (Control gain as market commodity!)
- Secondary control (SC) commodities: activated energy; allocated capacity (various arrangements)
- Tertiary control commodities: capacity and energy

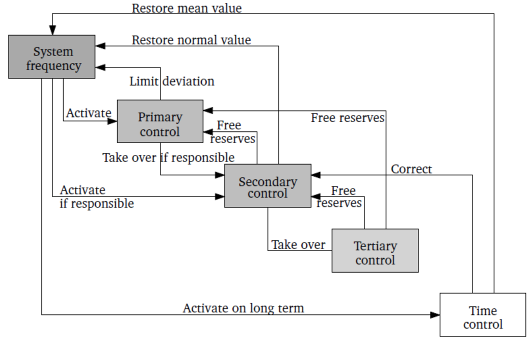


Some questions:
Can one benefit from investing in flywheel?
What about inertia in future?

Category.	Function	Reserves
FCR	contain frequency deviations	primary reserves, FCR
FRR	restore nominal frequency	secondary reserves LFC, AR, FADR tertiary reserves
RR	replace used FCR and FRR	tertiary reserves, FADR

ENTSO

FCR = Frequency containment reserves (local, automatic, activation time 30s)
FRR = Frequency restoration reserves (central, automatic or manual, 30s to 15 min)
RR = Replacement reserves (several min to 1 h)



Continental Europe synchronous system

- primary reserve
- secondary reserve
- tertiary reserve

Category.	Function	Reserves
FCR	contain frequency deviations	primary reserves, FCR
FRR	restore nominal frequency	secondary reserves LFC, AR, FADR tertiary reserves
RR	replace used FCR and FRR	tertiary reserves, FADR

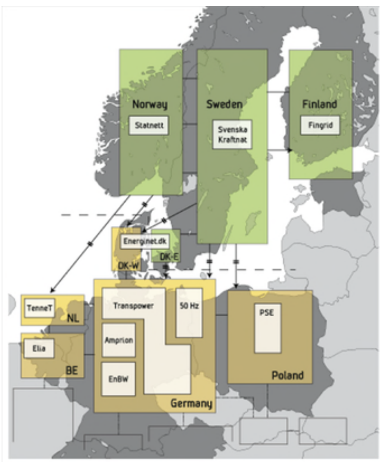
ENTSO

FCR = Frequency containment reserves (local, automatic, activation time 30s)
FRR = Frequency restoration reserves (central, automatic or manual, 30s to 15 min)
RR = Replacement reserves (several min to 1 h)

Nordic synchronous system

FCNR = Frequency controlled normal reserve (automatic, instantaneous; with rapid change to 49.9/50.1 Hz, up/down regulation within 2-3 min)
FCDR = Frequency controlled disturbance reserve (automatic, instantaneous; with rapid change to 49.5 Hz, up regulation within 2-3 min)
AR = Automatic reserves
FADR = Fast active disturbance reserve (manual, 15 min)

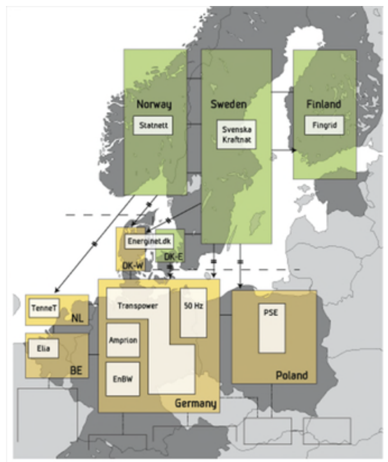
Service objectives and commodities



		DE	NL	BE	DK-W
Primary	capacity	weekly pay-as-bid	mandatory -	4-yearly bilateral	daily marginal
	energy	unpaid -	unpaid -	unpaid -	unpaid -
Secondary	capacity	weekly pay-as-bid	annually bilateral	2-yearly pay-as-bid	monthly pay-as-bid
	energy	weekly average	daily marginal	daily pay-as-bid	daily spot-based
Tertiary	capacity	daily pay-as-bid	unpaid -	4-yearly bilateral	daily marginal
	energy	daily average	daily marginal	daily mixed	daily marginal

Balancing services in continental Europe synchronous system (yellow TSOs in the Fig.) [source: S. Jaehnert, PhD thesis]
Remark: from 2014 in TenneT PC capacity is commodity.

Service objectives and commodities



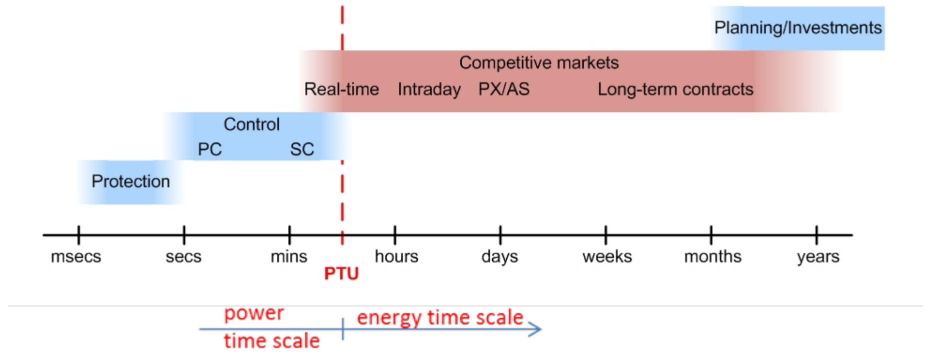
		NO	SE	FI	DK-E
FCR	capacity	yearly / daily marginal	weekly / hourly pay-as-bid	yearly / daily pay-as-bid	daily pay-as-bid
	energy	unpaid -	unpaid -	unpaid -	unpaid -
AR	capacity energy	to be decided			
FADR	capacity	yearly / weekly marginal	yearly bilateral	yearly pay-as-bid	daily pay-as-bid
	energy	hourly marginal			

Balancing services in Nordic synchronous system (green TSOs in the Fig.)

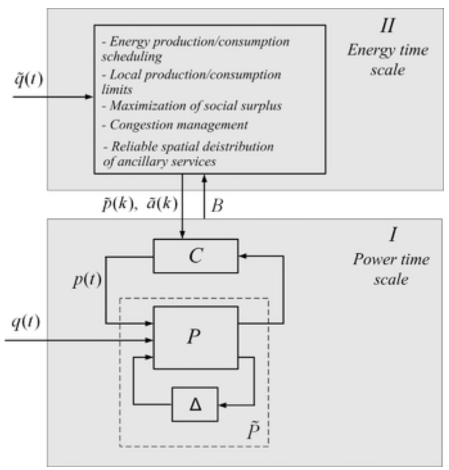
Sync. Area	Process	Product	Activation	Local/Central	Dynamic/ Static	Full Activation Time
BALTIC	Frequency Containment	Primary Reserve	Auto	Local	D	30 s
Cyprus	Frequency Containment	Primary Reserve	Auto	Local	D	20 s
Iceland	Frequency Containment	Primary Control Reserve	Auto	Local	D	variable
Ireland	Frequency Containment	Primary operating reserve	Auto	Local	D/S	5 s
Ireland	Frequency Containment	Secondary operating reserve	Auto	Local	D/S	15 s
NORDIC	Frequency Containment	FNR (FCR N)	Auto	Local	D	120 s -180 s
NORDIC	Frequency Containment	FDR (FCR D)	Auto	Local	D	30 s
RG CE	Frequency Containment	Primary Control Reserve	Auto	Local	D	30 s
UK	Frequency Containment	Frequency response dynamic	Auto	Local	D	Primary 10 s / Secondary 30 s
UK	Frequency Containment	Frequency response static	Auto	Local	S	variable
BALTIC	Frequency Restoration	Secondary emergency reserve	Manual	Central	S	15 Min
Cyprus	Frequency Restoration	Secondary Control Reserve	Auto/Manual	Local/Central	D/S	5 Min
Iceland	Frequency Restoration	Regulating power	Manual	Central	S	10 Min
Ireland	Frequency Restoration	Tertiary operational reserve 1	Auto/Manual	Local/Central	D/S	90 s
Ireland	Frequency Restoration	Tertiary operational reserve 2	Manual	Central	S	5 Min
Ireland	Frequency Restoration	Replacement reserves	Manual	Central	S	20 Min
NORDIC	Frequency Restoration	Regulating power	Manual	Central	S	15 Min
RG CE	Frequency Restoration	Secondary Control Reserve	Auto	Central	D	≤ 15 Min
RG CE	Frequency Restoration	Direct activated Tertiary Control Reserve	Manual	Central	S	≤ 15 Min
UK	Frequency Restoration	Various Products	Manual	D/S	N/A	variable
BALTIC	Replacement	Tertiary (cold) reserve	Manual	Central	S	12 h
Cyprus	Replacement	Replacement reserves	Manual	Central	S	20 min
Iceland	Replacement	Regulating power	Manual	Central	S	10 Min
Ireland	Replacement	Replacement reserves	Manual	Central	S	20 Min
NORDIC	Replacement	Regulating power	Manual	Central	S	15 Min
RG CE	Replacement	Schedule activated Tertiary Control Reserve	Manual	Central	S	individual
RG CE	Replacement	Direct activated Tertiary Control Reserve	Manual	Central	S	individual
UK	Replacement	Various Products but the main one is Short Term Operating Reserve (STOR)	Manual	D/S	N/A	from 20 min to 4 h

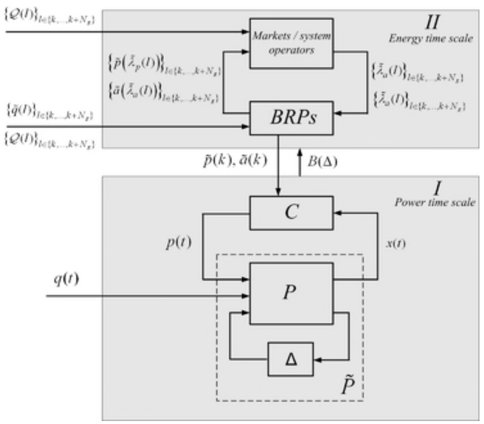
entsoe

Power balancing ancillary services in time scale



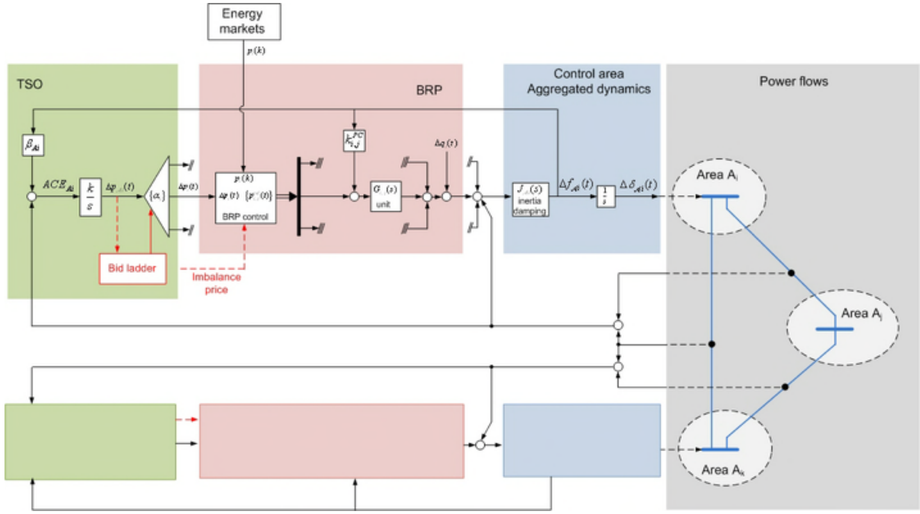
TSO is responsible for balancing within the PTU
BRP is responsible for their balance over whole PTU





Outline

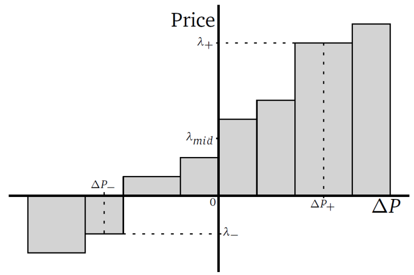
AS provision



AS provision

Primary control

- Sold capacity (market commodity) mapped into PC control gain (local droop)

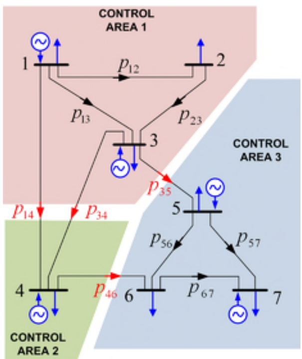


Secondary control

- ACE is matched with bidding ladder every 4 seconds
- Bid ladder changes every PTU (changing parameters in SC loop)

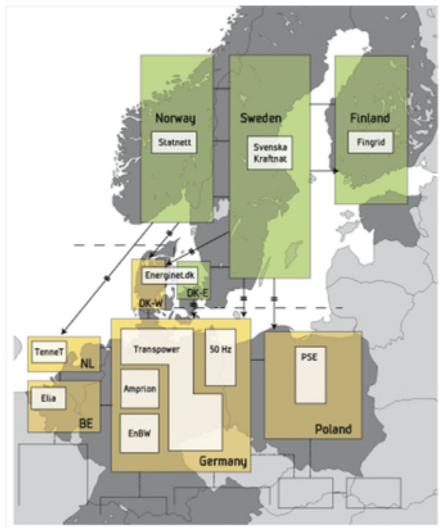
AS provision

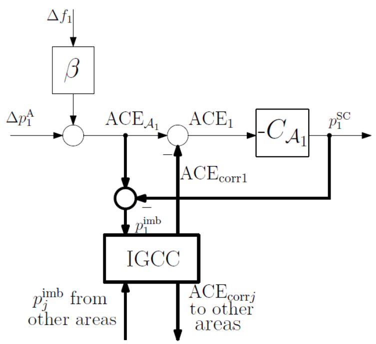
Exercise 8: show that $ACE_i = 0, \forall i \rightarrow \Delta f = 0$ total power exchanges among control areas as at scheduled values. Hint: write down the equations for a simple example (e.g. in the figure), and generalize.



$$\text{ACE}_1 = \beta_1 \Delta f_1 + \Delta p_1^{ex}, \quad p_1^{ex} = \Delta p_{14} + \Delta p_{34} + \Delta p_{35}$$

Inter Control Area Cooperation (IGCC)

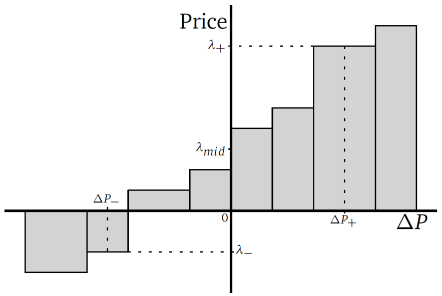




Outline

Imbalance settlement

Example of TenneT NL



BSP (Balance Service Provider) = BRP asked for active contribution

other BRPs: contribute on their own (passive contribution)

λ_p = penalty/incentive price

			BSP			BRP		
			Short	0	Long	Short	0	Long
Situation	-1	(long)	$-(\lambda_-)$	0	n.a.	$-(\lambda_- + \lambda_p)$	0	$\lambda_- - \lambda_p$
	0		n.a.	0	n.a.	$-(\lambda_{mid} + \lambda_p)$	0	$\lambda_{mid} - \lambda_p$
	1	(short)	n.a.	0	λ_+	$-(\lambda_+ + \lambda_p)$	0	$\lambda_+ - \lambda_p$
	2	(both)	$-(\lambda_-)$	0	λ_+	$-(\lambda_+ + \lambda_p)$	0	$\lambda_- - \lambda_p$

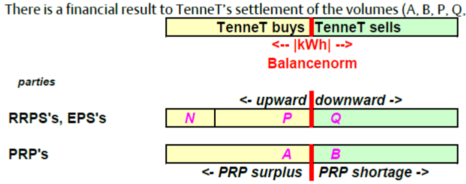
Imbalance settlement

Example of TenneT NL

state	meaning	occurrence
1	no imbalance in whole PTU	0.14%
-1	the system is long (surplus), requested only negative options	51.77%
0	the system is short (deficit), requested only positive options	38.25%
0	the system has been both long and short within PTU	9.85%

			BSP			BRP		
			Short	0	Long	Short	0	Long
Situation	-1	(long)	$-(\lambda_-)$	0	n.a.	$-(\lambda_- + \lambda_p)$	0	$\lambda_- - \lambda_p$
	0		n.a.	0	n.a.	$-(\lambda_{mid} + \lambda_p)$	0	$\lambda_{mid} - \lambda_p$
	1	(short)	n.a.	0	λ_+	$-(\lambda_+ + \lambda_p)$	0	$\lambda_+ - \lambda_p$
	2	(both)	$-(\lambda_-)$	0	λ_+	$-(\lambda_+ + \lambda_p)$	0	$\lambda_- - \lambda_p$

There is a financial result to TenneT's settlement of the volumes (A, B, P, Q, N) at the designated prices.



The basic formula that applies to the financial result is:

(Q * Pdo + B * Pshort) - (N * Pem + P * Pup + A * Psurp)

Or:

B * Pshort – A * Psurp + Q * Pdown – P * Pup – N * Pem

Elaborated per regulation state, this becomes:

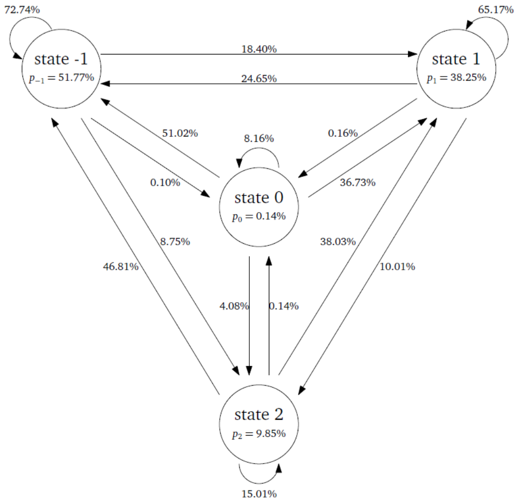
reg. state: 0	B * (Pmid + ic)	- A * (Pmid - ic)		
-1	B * (Pdo + ic)	- A * (Pdo - ic)	+ Q * Pdo - P * Pup	
+1	B * (Pup + ic)	- A * (Pup - ic)	+ Q * Pdo - P * Pup	
2	B * (Pup + ic)	- A * (Pdo - ic)	+ Q * Pdo - P * Pup	
-1, em	B * (Pdo + ic)	- A * (Pdo - ic)	+ Q * Pdo - P * Pup	- N * Pem
+1, em	B * (max(Pem, Pup) + ic)	- A * (max(Pem, Pup) - ic)	+ Q * Pdo - P * Pup	- N * Pem
2, em	B * (max(Pem, Pup) + ic)	- A * (Pdo - ic)	+ Q * Pdo - P * Pup	- N * Pem

Where Pem > Pup, and after a bit of reshuffling this becomes:

reg. state: 0	(B - A) * Pmid			+ (A + B) * ic
-1	(B - A + Q) * Pdo	- P * Pup		+ (A + B) * ic
+1	(B - A - P) * Pup	+ Q * Pdo		+ (A + B) * ic
2	((A + B) - (P + Q)) * (Pup - Pdo)/2			+ (A + B) * ic
-1, em	(B - A + Q) * Pdo	- (P + N) * Pem	+ P * (Pem-Pup)	+ (A + B) * ic
+1, em	(B - A - P - N) * Pem	+ Q * Pdo	+ P * (Pem-Pup)	+ (A + B) * ic
2, em	((A + B) - (P + N + Q)) * (Pem - Pdo)/2		+ P * (Pem-Pup)	+ (A + B) * ic

Risk of bidding less or equal than the risk of not bidding

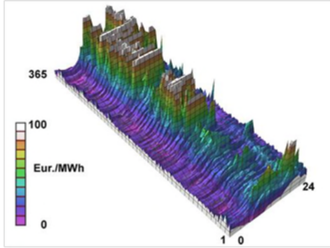
Risk of requested action less or equal than risk of unrequested actions



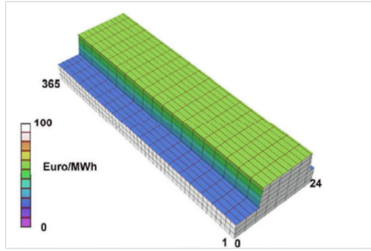
The last info I have:

"Afraid" to announce current situation in real time (delay of one PTU), and close the loop

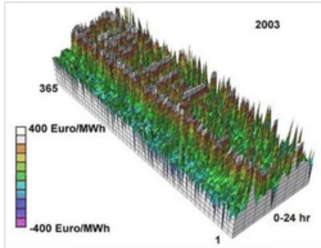
Prices



Day ahead market prices (APX)



Prices for consumers



Balancing prices (TenneT)

Bidding

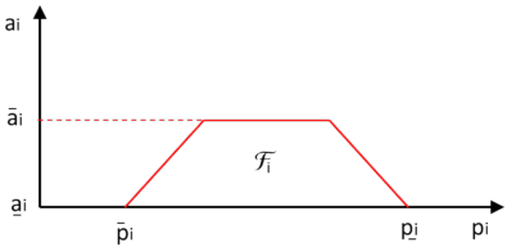
$$\min_{\{p_i\},\{a_i\}} \sum_i C_i(p_i)$$

subject to

$$(p_i, a_i) \in \mathcal{F}_i$$

$$\sum_i p_i - d_{int} = P_{ex} \quad (\lambda_P)$$

$$\sum_i a_i - a_{int} = A_{ex} \quad (\lambda_A)$$



a_i AS allocated capacity at unit i

p_i power production from unit i

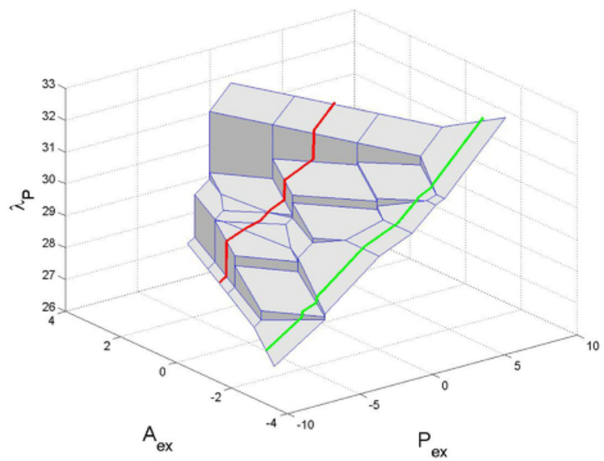
d_{int} internal BRP demand

a_{int} internal BRP's request for local AS capacity

Most often: sequential clearing of markets

Bidding

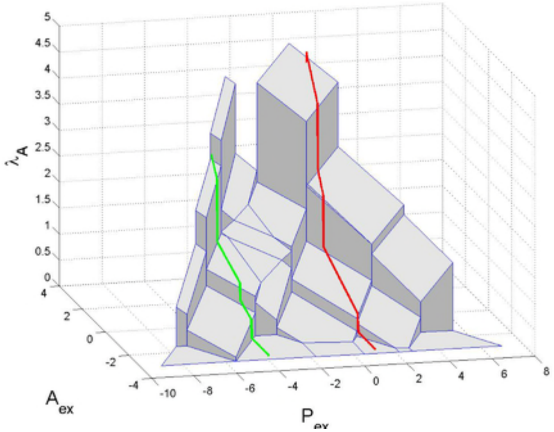
"Behind the interface"; inside BRP



$$\beta(P_{ex}, A_{ex}) \quad \rightarrow \quad \tilde{\beta}(P_{ex})$$

Bidding

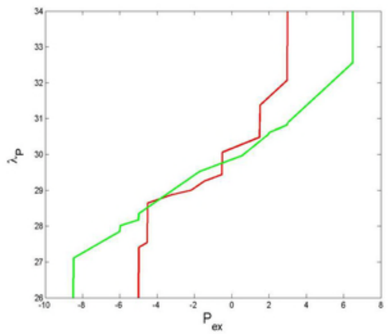
"Behind the interface"; inside BRP

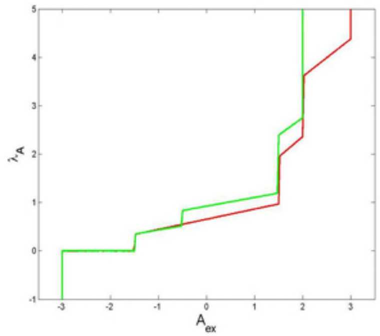


$$\beta(P_{ex}, A_{ex}) \quad \rightarrow \quad \tilde{\beta}(A_{ex})$$

Bidding

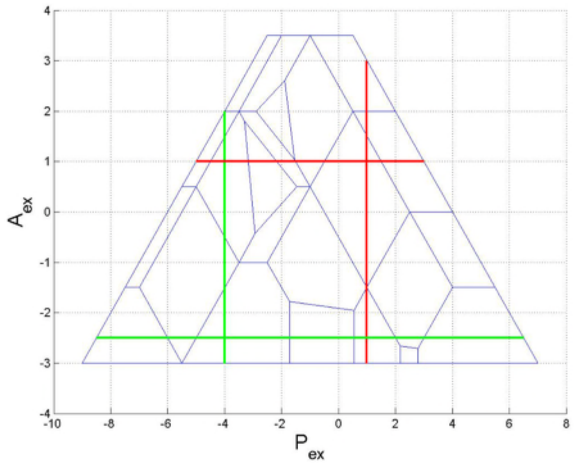
"for the outside world"





$\tilde{\beta}(P_{ex})$ $\tilde{\beta}(A_{ex})$

Bidding



Bidding

$$\min \ \ell + \frac{1}{1-\beta}\left(\sum_{s=1}^{L}\pi_s^{AS+}[f_s-\ell]^+ + \sum_{s=L+1}^{2L}\pi_{s-L}^{AS-}[f_i-\ell]^+\right) \tag{6.4a}$$

$$\text{s.t.} \ f_s = \sum_{j=1}^{n}\frac{C_j u_{sj}}{M_j\left(a_{2j}\left(\frac{u_{sj}}{TP_{max,j}}\right)^2 + a_{1,j}\frac{u_{sj}}{TP_{max,j}} + a_{0,j}\right)} + [\lambda_{imb,s}x_{imb,s}]^- - \lambda_p^{PX}x_p^{PX} - \lambda_s^{AS+}x_{up,s}^{AS}, \ s=1,\dots,L, \tag{6.4b}$$

$$f_s = \sum_{j=1}^{n}\frac{C_j u_{sj}}{M_j\left(a_{2j}\left(\frac{u_{sj}}{TP_{max,j}}\right)^2 + a_{1,j}\frac{u_{sj}}{TP_{max,j}} + a_{0,j}\right)} + |\lambda_{imb,s}x_{imb,s}| - \lambda_p^{PX}x_p^{PX} + \lambda_{s-L}^{AS-}x_{do,s-L}^{AS}, \ s=L+1,\dots,2L, \tag{6.4c}$$

$$\underline{u_j} \le u_{sj} \le \overline{u_j}, \ j=1,\dots,n, \ s=1,\dots,2L, \tag{6.4d}$$

$$\sum_{j=1}^{n} u_{sj} - x_p^{PX} - x_{up,s}^{AS} = x_{imb,s}, \ s=1,\dots,L, \tag{6.4e}$$

$$\sum_{j=1}^{n} u_{sj} - x_p^{PX} + x_{do,s-L}^{AS} = x_{imb,s}, \ s=L+1,\dots,2L, \tag{6.4f}$$

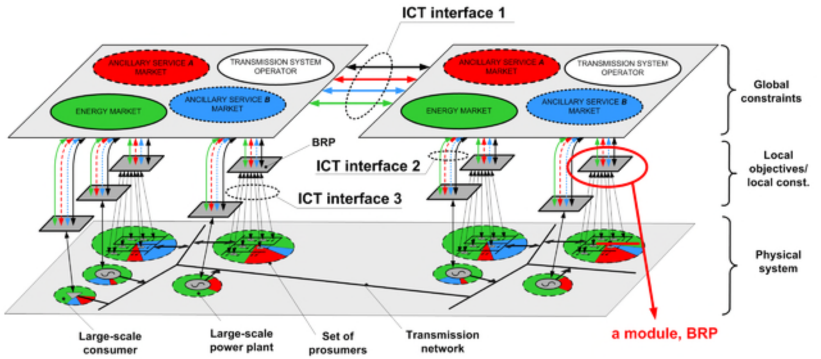
$$x_{do,s}^{AS} \le x_p^{PX}, \ s=1,\dots,L, \tag{6.4g}$$

$$x_p^{PX} \ge 0, \tag{6.4h}$$

$$x_{up,s}^{AS} \ge 0, \ s=1,\dots,L, \tag{6.4i}$$

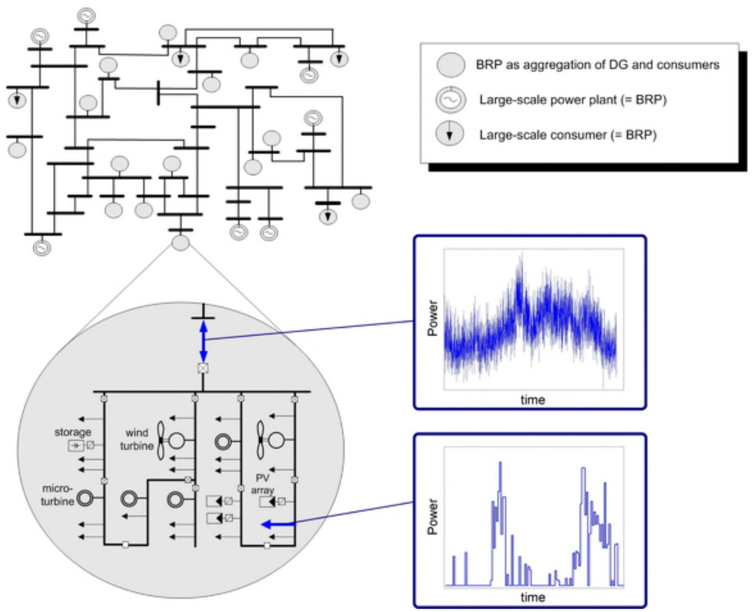
$$x_{do,s}^{AS} \ge 0, \ s=1,\dots,L. \tag{6.4j}$$

Bids as well defined protocol



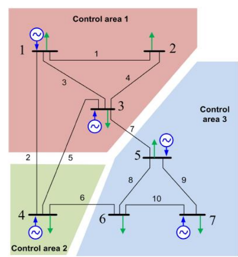
- All that matters are interfaces and protocols on them
- Heterogeneity, local complexities.... all "hidden" behind the interface (*Interface 2*)
- *Interface 2* requires decoupling of coupled problems (e.g. no 2D bids are allowed): enforcing manageable simplicity on the higher level

Outline

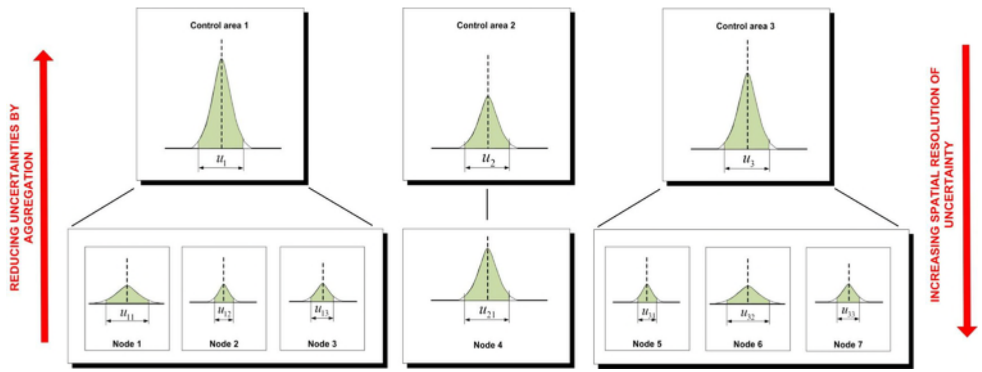


What is the added value of aggregation? Can the rest of network do a better job than my neighbour?

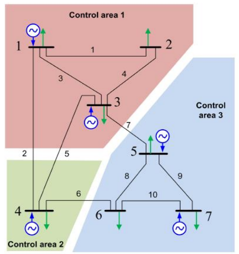
Spatial resolution of uncertainty



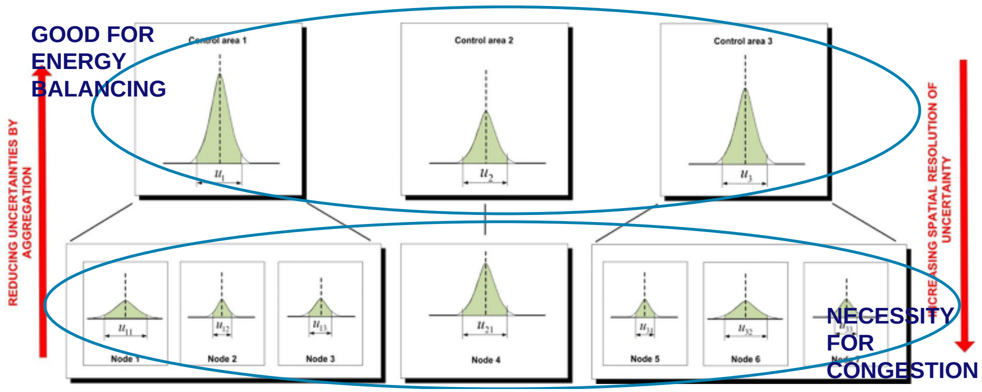
Spatial distribution of uncertainties is crucial in defining uncertainties in power flows



Spatial resolution of uncertainty

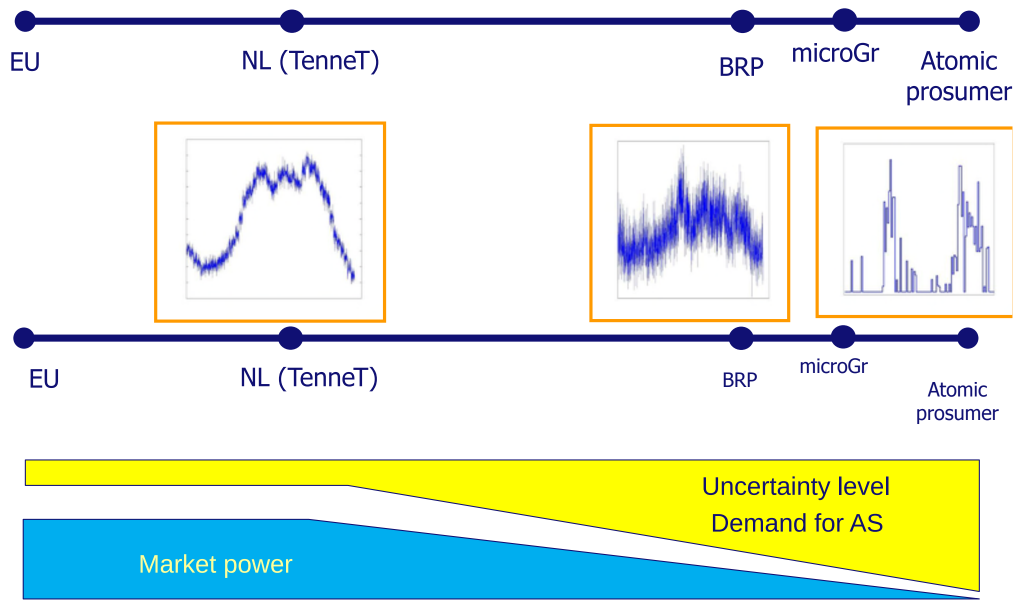


Spatial distribution of uncertainties is crucial in defining uncertainties in power flows

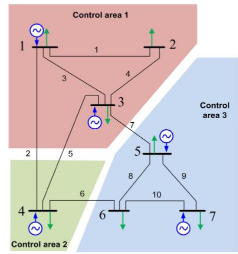


Spatial information

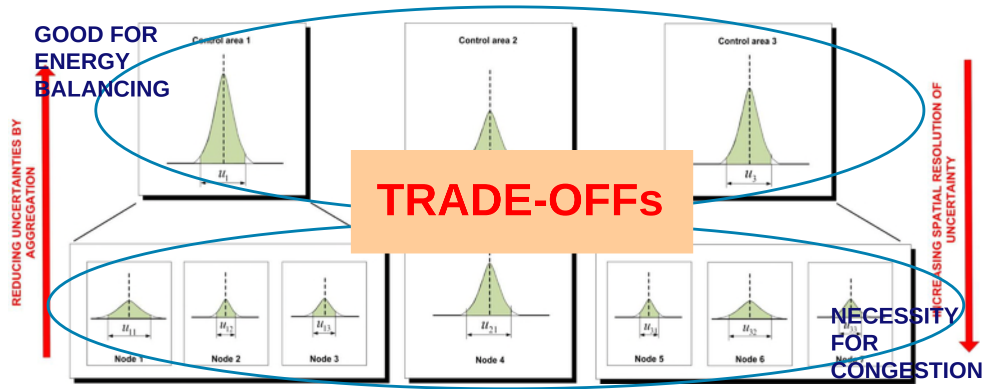
$$\int \Delta f(t) + \beta \Delta P_{NL}(t) \qquad \Delta E(15\,\text{min}) = \int \Delta P(t)$$



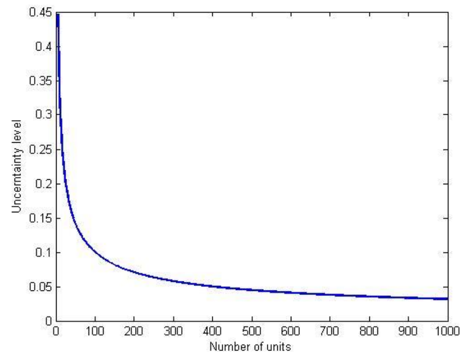
Spatial resolution of uncertainty

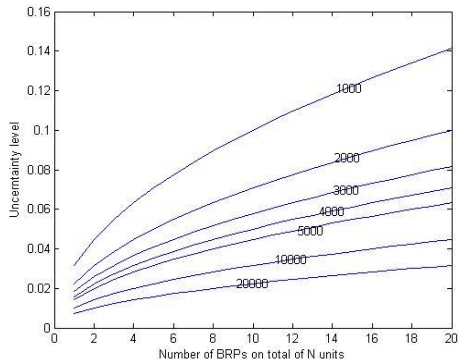


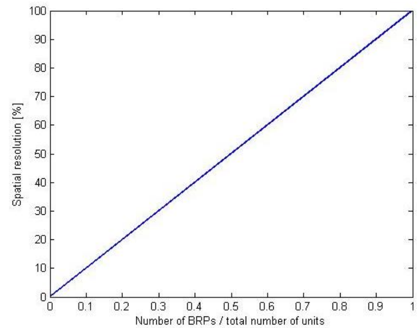
Spatial distribution of uncertainties is crucial in defining uncertainties in power flows



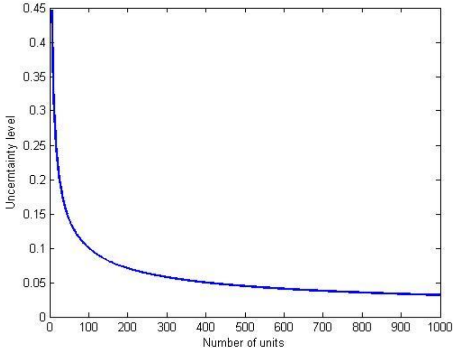
Trade-offs

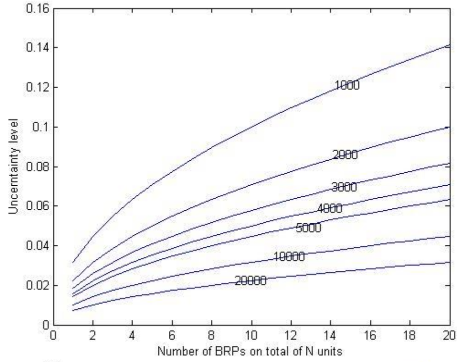


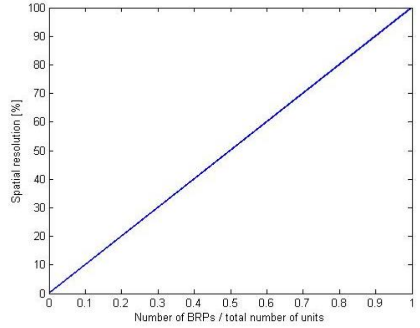




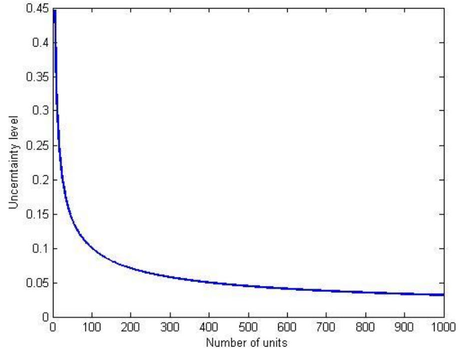
Trade-offs

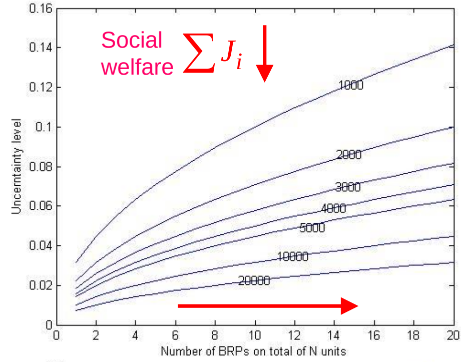


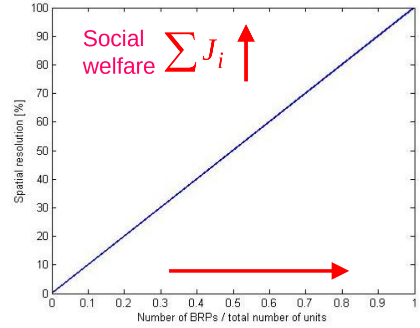




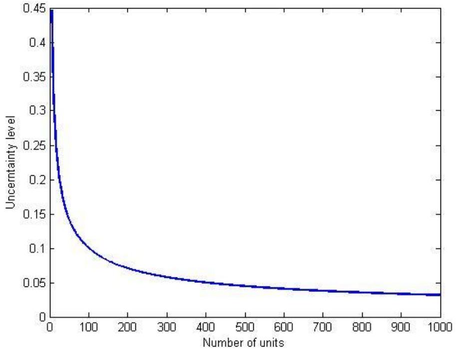
Trade-offs

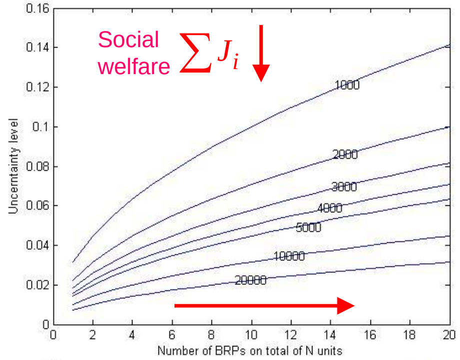




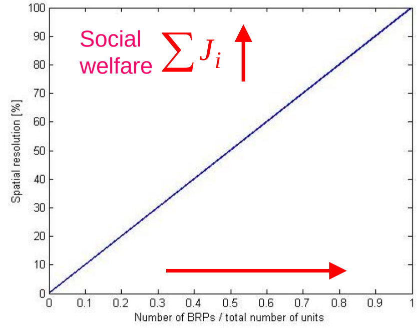


Trade-offs

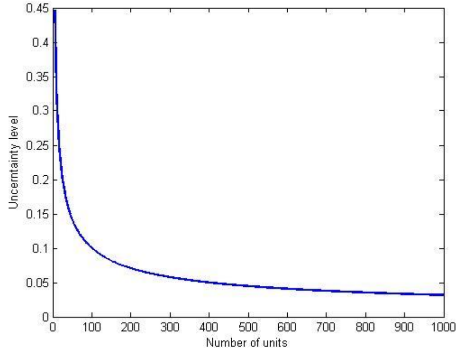


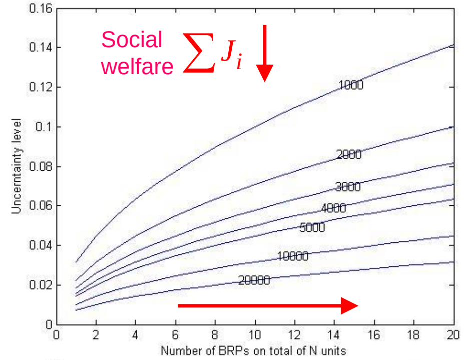


Number of BRP's ↑ $\Rightarrow$ Market power ↓ $\Rightarrow$ Social welfare $\sum J_i$ ↓



Trade-offs

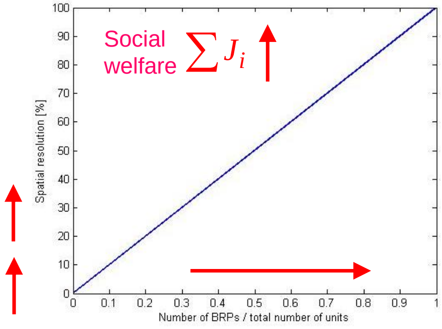


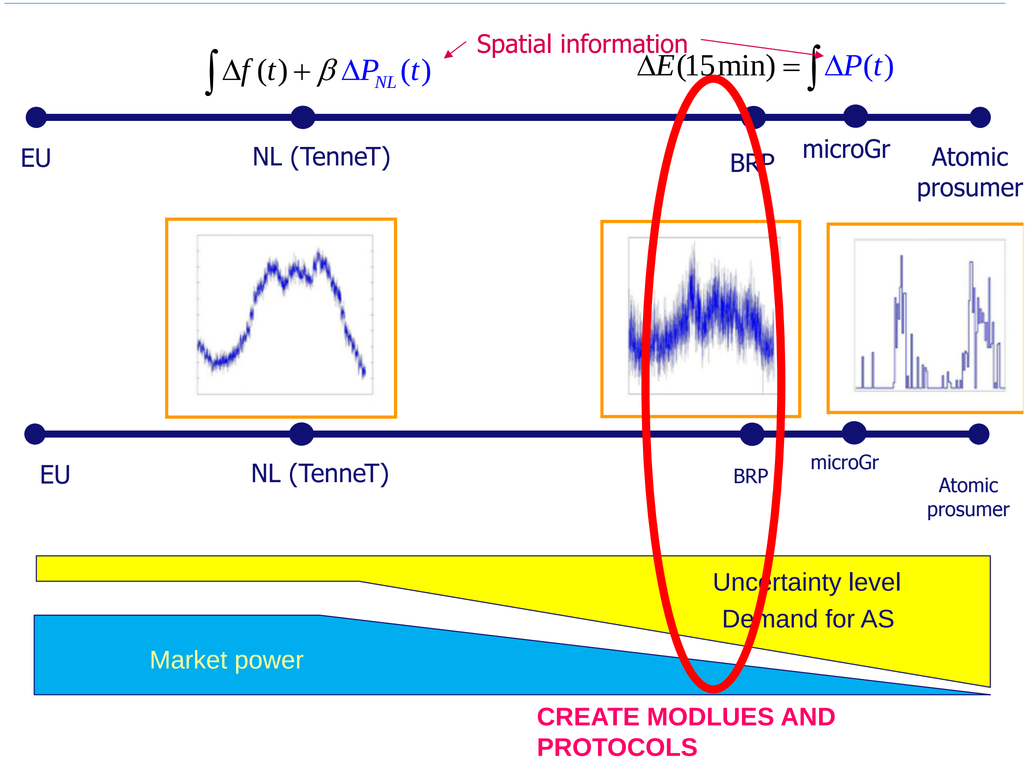


Number of BRP's ↑ $\Rightarrow$ Market power ↓ $\Rightarrow$ Social welfare $\sum J_i$ ↓

Duration of trading interval ↓ $\Rightarrow$ Unicerntaity level ↓ $\Rightarrow$ Social welfare $\sum J_i$ ↑

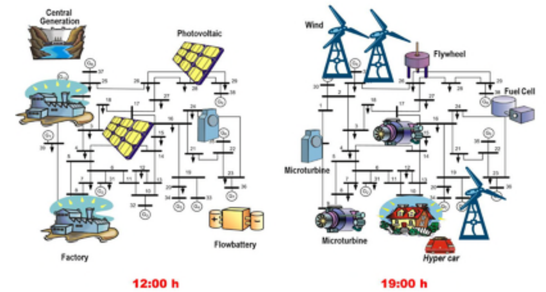
$\Rightarrow$ Coupling economy-physics ↑

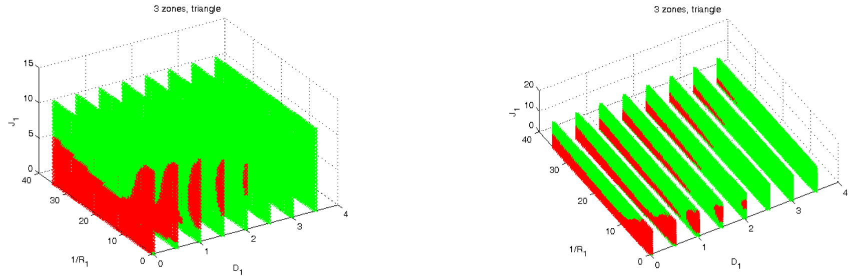




CHALLENGE

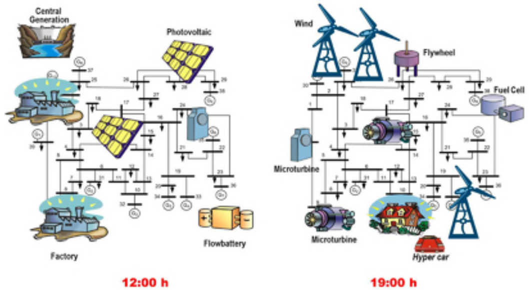
Accumulating /adapting proper amount of gains (AS) for time-varying system

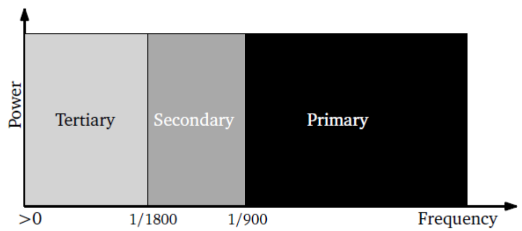




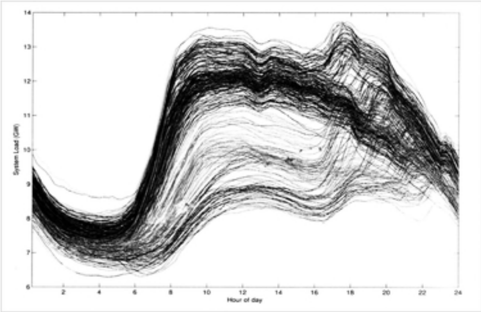
CHALLENGE

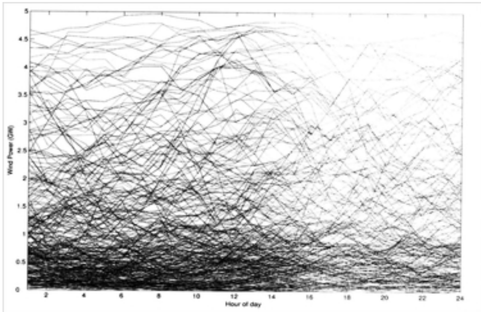
Accumulating /adapting proper amount of gains (AS) for time-varying system





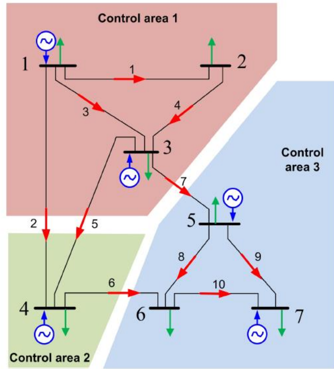
Outline

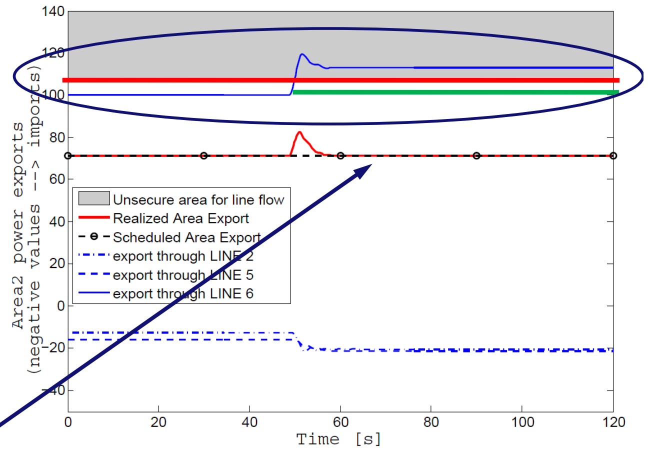




NOW **FUTURE**

- Increased uncertainties → Tight coupling economy (markets), physics and RT control
- Uncertain spatial distribution of uncertainties → uncertain power flows
- In today's systems efficiency largely relies on repetitiveness
- Put economic optimization in closed loop; care of congestion constraints





In current system, reliability is accounted for in "aggregated" form here

RELIABILITY MARGIN

Size of reliability margin: reliability vs. efficiency trade-off

Economically optimal working point is often on the border of feasible region

Distributed, real-time, price-based control

Optimal nodal pricing problem

$$\min_{\lambda,\delta} \sum_{i=1}^{n} J_i(\gamma_i(\lambda_i))$$

$$\text{subject to} \quad \gamma(\lambda) - B\delta + \hat{p} = 0,$$
$$b_{ij}(\delta_i - \delta_j) \leq \overline{p}_{ij}, \ \forall (i, j \in I(N_i)),$$

Distributed, real-time, price-based control

Optimal power flow problem

$$\min_{p,\delta} \sum_i J_i(p_i)$$

$$\text{subject to} \quad p - B\delta + \hat{p} = 0,$$
$$L\delta \leq \overline{e}_c,$$
$$\underline{p} \leq p \leq \overline{p},$$

KKT conditions

$$p - B\delta + \hat{p} = 0,$$
$$B\lambda + L^{\top}\mu = 0,$$
$$\nabla J(p) - \lambda + \nu^{+} - \nu^{-} = 0,$$
$$0 \leq \ (-L\delta + \overline{e}_c) \ \perp \ \mu \ \geq 0,$$
$$0 \leq \ (-p + \overline{p}) \ \perp \ \nu^{+} \ \geq 0,$$
$$0 \leq \ (p + \underline{p}) \ \perp \ \nu^{-} \ \geq 0,$$

Distributed, real-time, price-based control

$\Delta p_L = L\delta - \bar{e}_c$

Nodal pricing controller

$$\begin{pmatrix} \dot{x}_\lambda \\ \dot{x}_\mu \end{pmatrix} = \begin{pmatrix} -K_\lambda B & -K_\lambda L^\top \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \end{pmatrix} + \begin{pmatrix} -K_f & 0 \\ 0 & K_p \end{pmatrix} \begin{pmatrix} \Delta f \\ \Delta p_L + w \end{pmatrix},$$
$$0 \leq \ w \ \perp \ K_o x_\mu + \Delta p_L + w \ \geq \ 0,$$
$$\lambda = \begin{pmatrix} I_n & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \end{pmatrix},$$

$$\begin{aligned} p - B\delta + \hat{p} &= 0, \\ B\lambda + L^\top \mu &= 0, \\ \nabla J(p) - \lambda + \nu^+ - \nu^- &= 0, \\ 0 \leq \ (-L\delta + \bar{e}_c) \ \perp \ \mu \ &\geq 0, \\ 0 \leq \ (-p + \overline{p}) \ \perp \ \nu^+ \ &\geq 0, \\ 0 \leq \ (p + \underline{p}) \ \perp \ \nu^- \ &\geq 0, \end{aligned}$$

$$B\lambda + L^\top \mu + \Delta f^\star \mathbf{1} = 0,$$
$$\mathbf{1}^\top \begin{pmatrix} B & L^\top \end{pmatrix} = 0 \implies \mathbf{1} \notin \operatorname{Im} \begin{pmatrix} B & L^\top \end{pmatrix},$$
$$\implies \Delta f = 0, \ B\lambda + L^\top \mu = 0$$

Distributed, real-time, price-based control

$\Delta p_L = L\delta - \bar{e}_c$

Nodal pricing controller

$$\begin{pmatrix} \dot{x}_\lambda \\ \dot{x}_\mu \end{pmatrix} = \begin{pmatrix} -K_\lambda B & -K_\lambda L^\top \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \end{pmatrix} + \begin{pmatrix} -K_f & 0 \\ 0 & K_p \end{pmatrix} \begin{pmatrix} \Delta f \\ \Delta p_L + w \end{pmatrix},$$
$$0 \leq \ w \ \perp \ K_o x_\mu + \Delta p_L + w \ \geq \ 0,$$
$$\lambda = \begin{pmatrix} I_n & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \end{pmatrix},$$

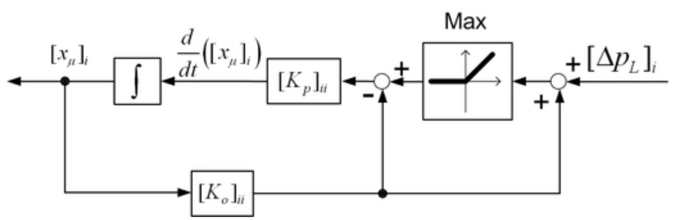
- no knowledge of cost/benefit functions of producers/consumers required
- required no knowledge of actual power injections
- required: B and L
- preserves the structure of B and L

Distributed, real-time, price-based control

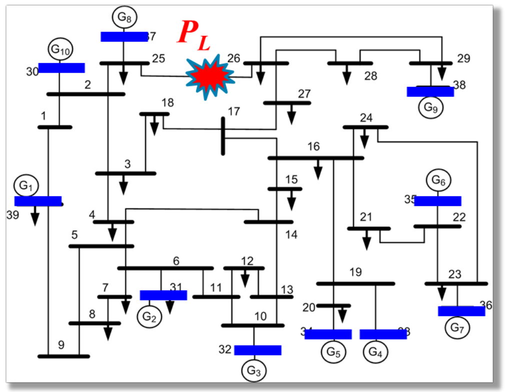
$\Delta p_L = L\delta - \bar{e}_c$

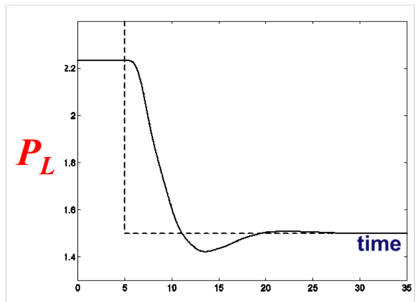
Nodal pricing controller

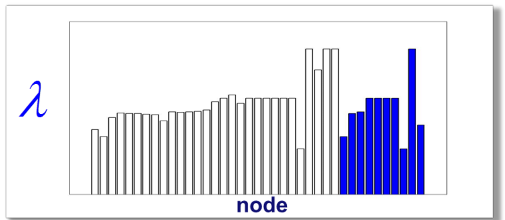
$$\begin{pmatrix} \dot{x}_\lambda \\ \dot{x}_\mu \end{pmatrix} = \begin{pmatrix} -K_\lambda B & -K_\lambda L^\top \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \end{pmatrix} + \begin{pmatrix} -K_f & 0 \\ 0 & K_p \end{pmatrix} \begin{pmatrix} \Delta f \\ \Delta p_L + w \end{pmatrix},$$
$$0 \leq \ w \ \perp \ K_o x_\mu + \Delta p_L + w \ \geq \ 0,$$
$$\lambda = \begin{pmatrix} I_n & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \end{pmatrix},$$

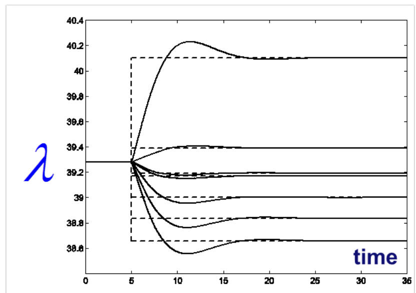


max-based complementarity integrator



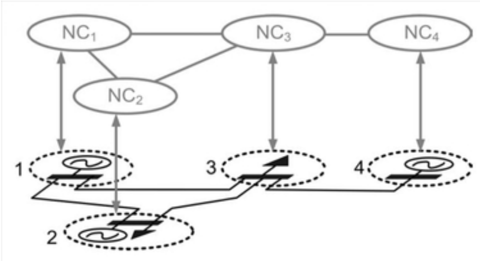






Distributed, real-time, price-based control

REAL-TIME MARKET AND CONGESTION CONTROL



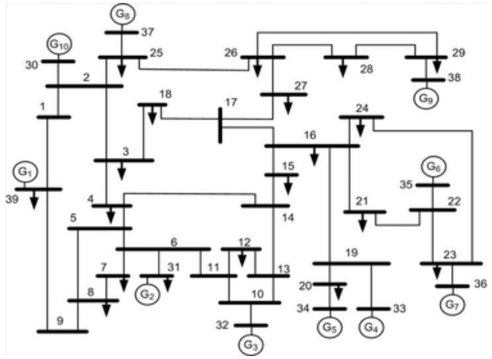
$B\lambda + L^{\top}\mu = 0$, λ prices for local balance, μ prices for not overloanding the lines

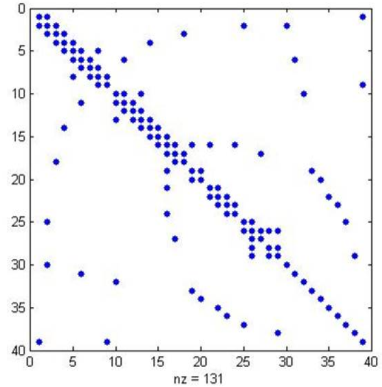
$$\left(\begin{array}{cccc|cc} b_{12,13} & -b_{12} & -b_{13} & 0 & b_{12} & b_{13} \\ -b_{12} & b_{12,23} & -b_{23} & 0 & -b_{12} & 0 \\ -b_{13} & -b_{23} & b_{13,23,34} & -b_{34} & 0 & -b_{13} \\ 0 & 0 & -b_{34} & b_{34} & 0 & 0 \end{array}\right) \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \hline \mu_{12} \\ \mu_{13} \end{pmatrix} = 0,$$

Distributed, real-time, price-based control

REAL-TIME MARKET AND CONGESTION CONTROL

$$B\lambda + L^{\top}\mu = 0$$





Distributed, real-time, price-based control

SEPARATING BALANCING PRICING FROM CONGESTION PRICING

$$B = \begin{pmatrix} * & * \\ * & B_{\Delta} \end{pmatrix} \quad L = \begin{pmatrix} * & L \end{pmatrix}$$

Modified price-based controller

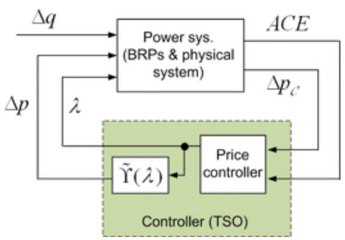
$$\begin{pmatrix} \dot{x}_{\lambda_0} \\ \dot{x}_{\Delta\lambda} \\ \dot{x}_{\mu} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -K_{\Delta}B_{\Delta} & -K_{\Delta}L_{\Delta}^{\top} \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_{\lambda_0} \\ x_{\Delta\lambda} \\ x_{\mu} \end{pmatrix} + \begin{pmatrix} -k_f \mathbf{1}_n^{\top} & 0 \\ 0 & 0 \\ 0 & K_p \end{pmatrix} \begin{pmatrix} \Delta f \\ \Delta p_L + w \end{pmatrix},$$

$$0 \leq w \perp K_o x_{\mu} + \Delta p_L + w \geq 0,$$

$$\lambda = \begin{pmatrix} 1 & 0 & 0 \\ \mathbf{1}_{n-1} & I_{n-1} & 0 \end{pmatrix} \begin{pmatrix} x_{\lambda_0} \\ x_{\Delta\lambda} \\ x_{\mu} \end{pmatrix},$$

Distributed, real-time, price-based control

PROVISION OF ANCILLARY SERVICES



Optimality conditions

$$\begin{aligned} \beta(p^{\star}) - \lambda^{\star} &= 0 \\ p^{\star} - B\theta^{\star} &= 0 \\ B\lambda^{\star} + L^{\top}\mu^{\star} &= 0 \\ 0 \leq (-L\theta^{\star} + \bar{e}_{\mathcal{E}}) \quad \perp \quad \mu^{\star} &\geq 0 \end{aligned}$$

Real-time nodal price based SC controller (each control area balanced separately)

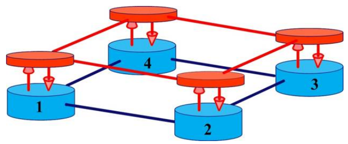
$$\begin{pmatrix} \dot{x}_{\lambda} \\ \dot{x}_{\mu} \\ \dot{x}_{\sigma} \end{pmatrix} = \begin{pmatrix} -K_{\lambda}B & -K_{\lambda}L^{\top} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_{\lambda} \\ x_{\mu} \\ x_{\sigma} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & K_{\mu} \\ -K_{\sigma} & 0 \end{pmatrix} \begin{pmatrix} ACE \\ \Delta p_C \end{pmatrix} + \begin{pmatrix} 0 \\ K_{\mu}w \\ 0 \end{pmatrix},$$

$$0 \leq w \perp K_0 x_{\mu} + \Delta p_C + w \geq 0,$$

$$\lambda = \begin{pmatrix} \boxed{I} & 0 & E \end{pmatrix} \begin{pmatrix} x_{\lambda} \\ x_{\mu} \\ x_{\sigma} \end{pmatrix}, \qquad \Delta p = \tilde{\Upsilon}(\lambda)$$

Distributed, real-time, price-based control

PROVISION OF ANCILLARY SERVICES



Optimality conditions

$$\beta(p^\star) - \lambda^\star = 0$$
$$p^\star - B\theta^\star = 0$$
$$B\lambda^\star + L^\top \mu^\star = 0$$
$$0 \le (-L\theta^\star + \overline{e}_\mathcal{E}) \quad \perp \quad \mu^\star \ge 0$$

Real-time nodal price based SC controller (each control area balanced separately)

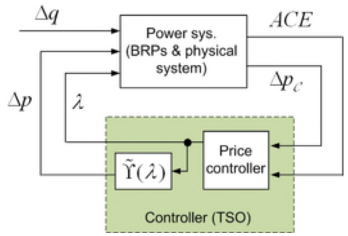
$$\begin{pmatrix} \dot{x}_\lambda \\ \dot{x}_\mu \\ \dot{x}_\sigma \end{pmatrix} = \begin{pmatrix} -K_\lambda B & -K_\lambda L^\top & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \\ x_\sigma \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & K_\mu \\ -K_\sigma & 0 \end{pmatrix} \begin{pmatrix} ACE \\ \Delta p_\mathcal{C} \end{pmatrix} + \begin{pmatrix} 0 \\ K_\mu w \\ 0 \end{pmatrix},$$

$$0 \le w \perp K_0 x_\mu + \Delta p_\mathcal{C} + w \ge 0,$$

$$\lambda = \begin{pmatrix} \boxed{I} & 0 & E \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \\ x_\sigma \end{pmatrix}, \qquad \Delta p = \tilde{\Upsilon}(\lambda)$$

Distributed, real-time, price-based control

PROVISION OF ANCILLARY SERVICES



Optimality conditions

$$\beta(p^\star) - \lambda^\star = 0$$
$$p^\star - B\theta^\star = 0$$
$$B\lambda^\star + L^\top \mu^\star = 0$$
$$0 \le (-L\theta^\star + \overline{e}_\mathcal{E}) \quad \perp \quad \mu^\star \ge 0$$

Real-time zonal price based SC controller (each control area balanced separately)

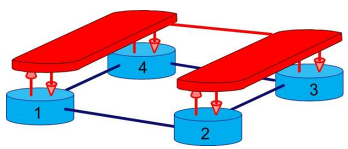
$$\begin{pmatrix} \dot{x}_\lambda \\ \dot{x}_\mu \\ \dot{x}_\sigma \end{pmatrix} = \begin{pmatrix} -K_\lambda B & -K_\lambda L^\top & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \\ x_\sigma \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & K_\mu \\ -K_\sigma & 0 \end{pmatrix} \begin{pmatrix} ACE \\ \Delta p_\mathcal{C} \end{pmatrix} + \begin{pmatrix} 0 \\ K_\mu w \\ 0 \end{pmatrix}$$

$$0 \le w \perp K_0 x_\mu + \Delta p_\mathcal{C} + w \ge 0$$

$$\lambda_\mathcal{Z} = \begin{pmatrix} \boxed{F(\cdot)} & 0 & E \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \\ x_\sigma \end{pmatrix}, \qquad \Delta p = \Upsilon(\lambda_\mathcal{Z})$$

Distributed, real-time, price-based control

PROVISION OF ANCILLARY SERVICES



Optimality conditions

$$\beta(p^\star) - \lambda^\star = 0$$
$$p^\star - B\theta^\star = 0$$
$$B\lambda^\star + L^\top \mu^\star = 0$$
$$0 \le (-L\theta^\star + \overline{e}_\mathcal{E}) \quad \perp \quad \mu^\star \ge 0$$

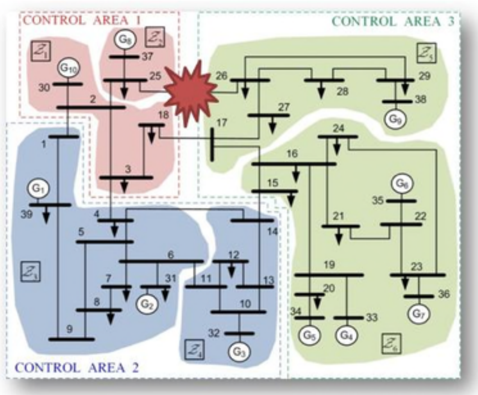
Real-time zonal price based SC controller (each control area balanced separately)

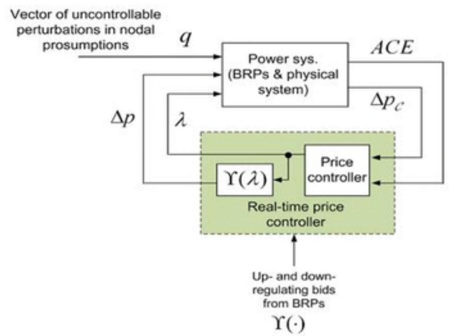
$$\begin{pmatrix} \dot{x}_\lambda \\ \dot{x}_\mu \\ \dot{x}_\sigma \end{pmatrix} = \begin{pmatrix} -K_\lambda B & -K_\lambda L^\top & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \\ x_\sigma \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & K_\mu \\ -K_\sigma & 0 \end{pmatrix} \begin{pmatrix} ACE \\ \Delta p_\mathcal{C} \end{pmatrix} + \begin{pmatrix} 0 \\ K_\mu w \\ 0 \end{pmatrix}$$

$$0 \le w \perp K_0 x_\mu + \Delta p_\mathcal{C} + w \ge 0$$

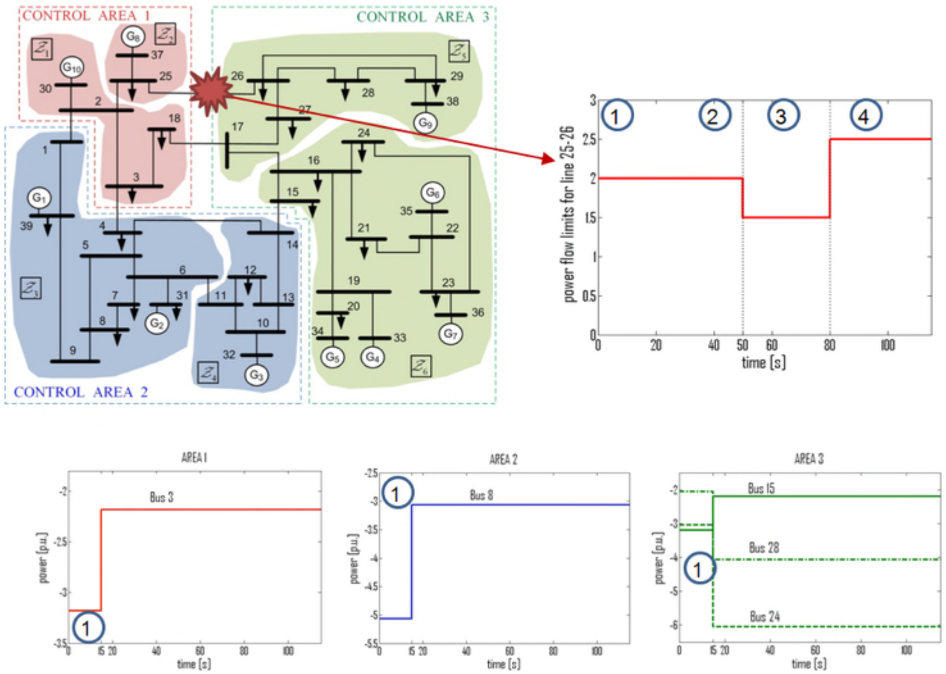
$$\lambda_\mathcal{Z} = \begin{pmatrix} \boxed{F(\cdot)} & 0 & E \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \\ x_\sigma \end{pmatrix}, \qquad \Delta p = \Upsilon(\lambda_\mathcal{Z})$$

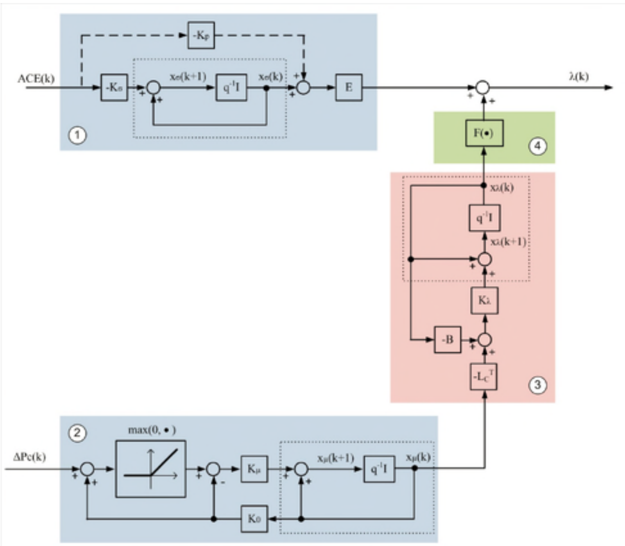
Distributed, real-time, price-based congestion control

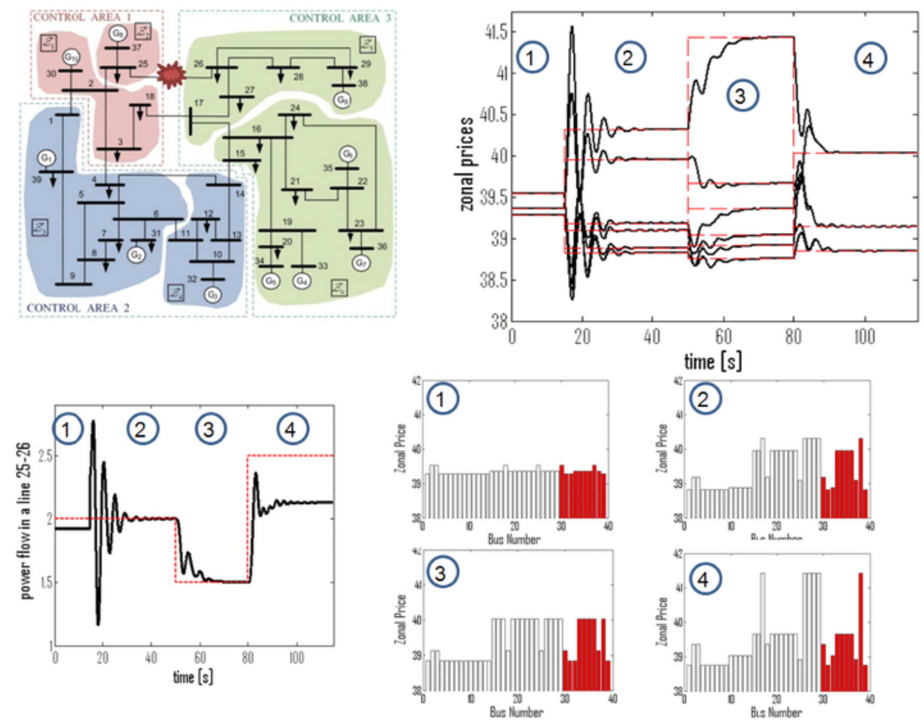




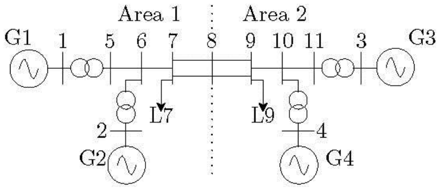
EXAMPLE

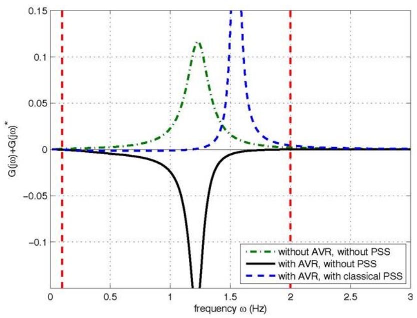


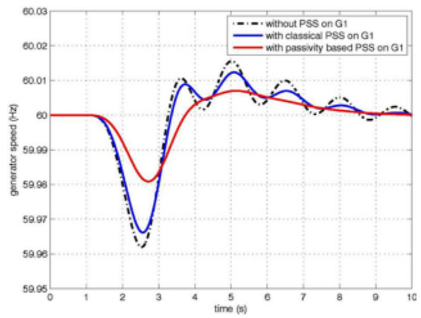


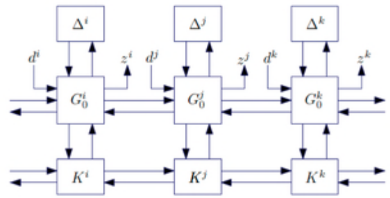


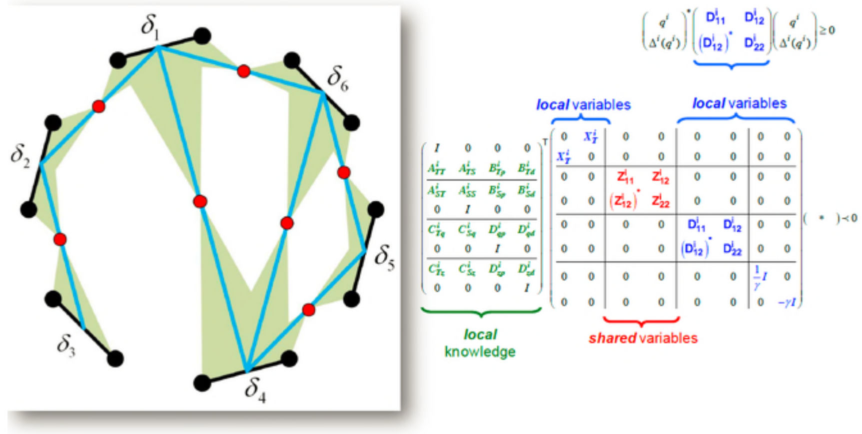
More on real-time distributed control

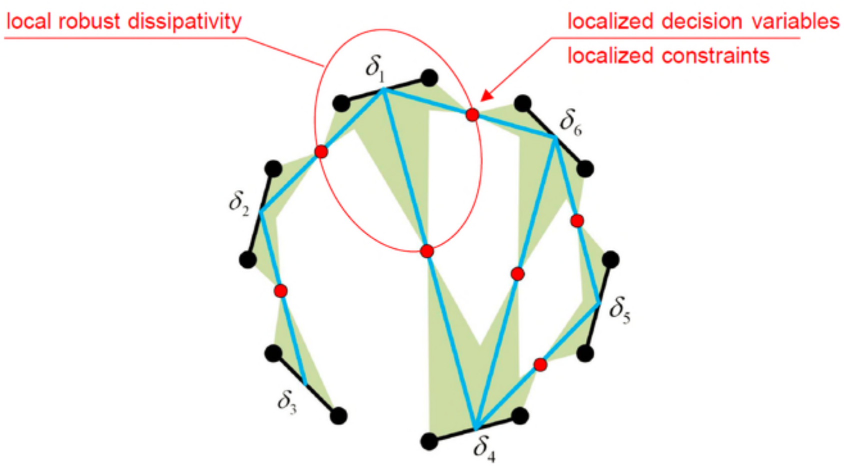




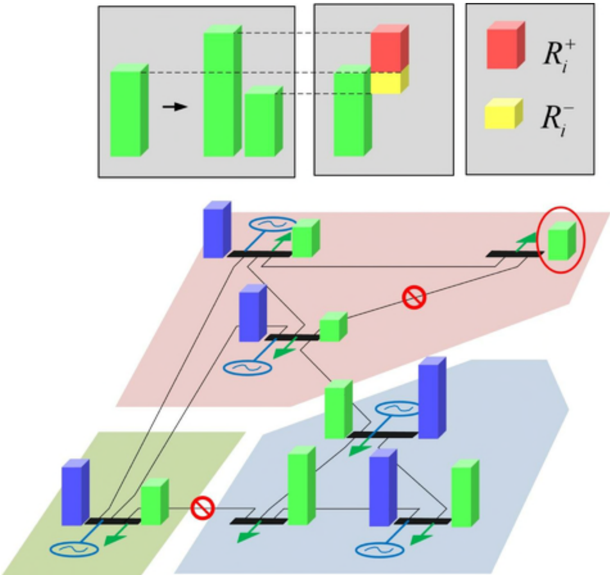


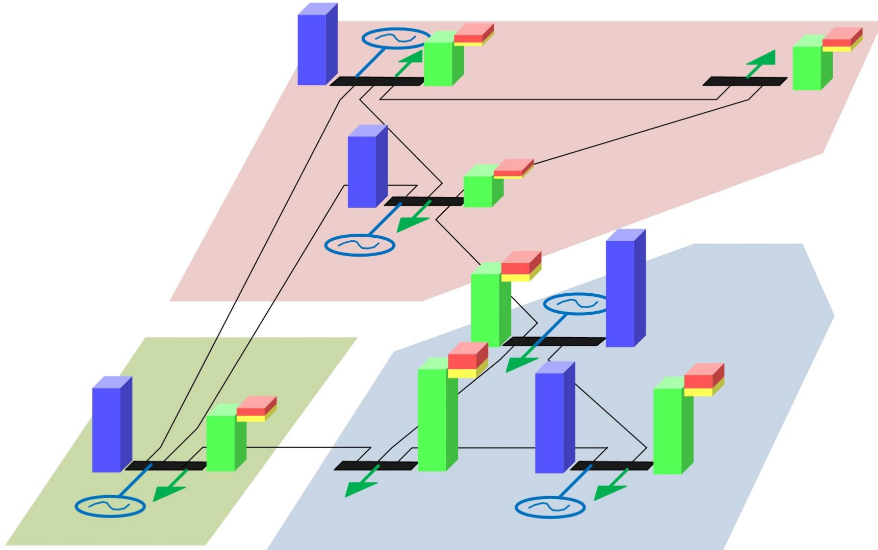


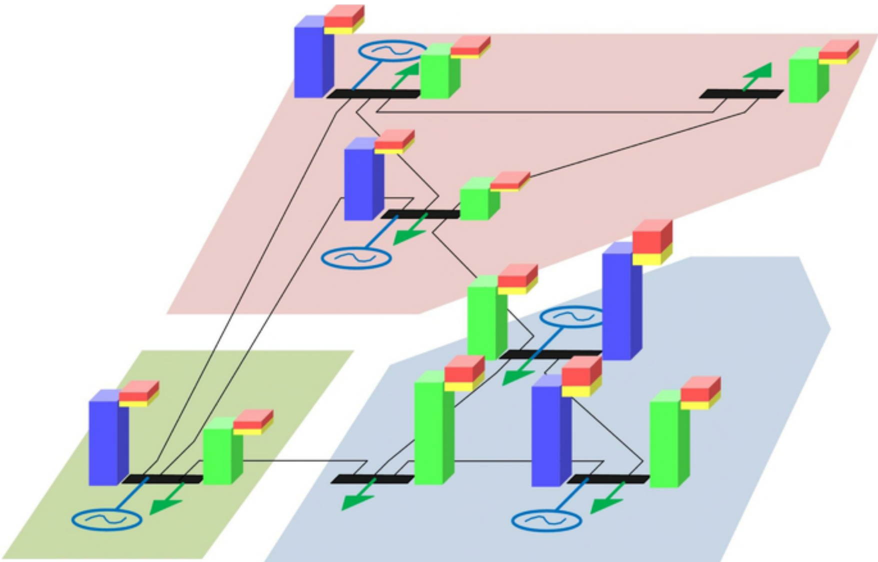


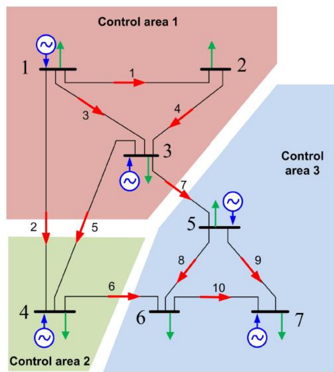


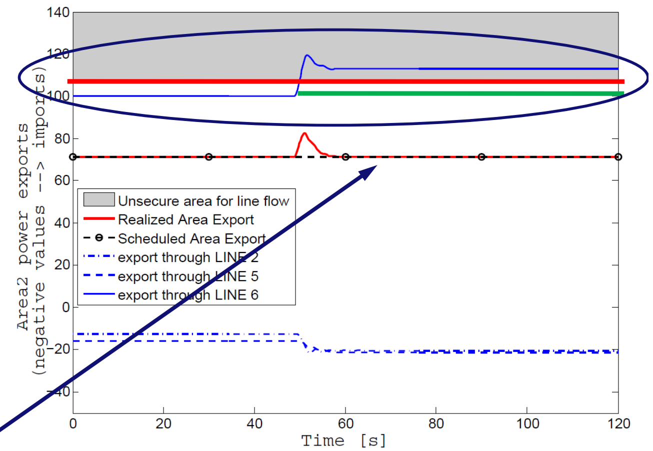
Market-based robust spatial distribution of ancillary services











In current system, reliability is accounted for in "aggregated" form here

RELIABILITY MARGIN

Size of reliability margin: reliability vs. efficiency trade-off

Economically optimal working point is often on the border of feasible region

Problem definition

Robust congestion constraints

The participation function

$$f(t) = \gamma(\tilde{a}^+(k), \tilde{a}^-(k), q(t))$$

$\tilde{a}^+(k)$ = purchased and allocated up-regulating AS
$\tilde{a}^-(k)$ = purchased and allocated down-regulating AS
$\tilde{a}^+(k)$ and $\tilde{a}^-(k)$ are **vectors defining spatial distribution of AS**

Uncertainty model

$$q(t) \in \tilde{\mathcal{Q}}(k) = \{ q \mid q = \tilde{R}(k)w, \ w \in \tilde{\mathcal{W}}(k) \subset \mathbb{R}^m \}$$
$$\tilde{\mathcal{W}}(k) = \text{conv}\{\tilde{w}_1(k), \ldots, \tilde{w}_T(k)\}, \qquad 0 \in \tilde{\mathcal{W}}(k)$$

Robust congestion constraints

$$L\delta \leq \Delta\tilde{l}(k) \qquad \text{for all} \quad \delta \in \tilde{\mathcal{D}}(k) \text{ where}$$
$$\tilde{\mathcal{D}}(k) := \{\delta \mid \begin{array}{l} \tilde{R}(k)w + \gamma\left(\tilde{a}^+(k), \tilde{a}^-(k), \tilde{R}(k)w\right) = B\delta, \\ w \in \tilde{\mathcal{W}}(k) \end{array} \}$$

The participation function $f(t) = \gamma(\tilde{a}^+(k), \tilde{a}^-(k), q(t))$

- **structure**: defined by the real-time secondary control scheme
- **parameters**: defined by $\tilde{a}^+(k), \tilde{a}^-(k)$ = the AS market clearing results

Example

Participation vectors:

$$\tilde{\alpha}^+(k) := \tilde{a}^+(k)\frac{1}{\sum_i \tilde{a}_i^+(k)}, \quad \tilde{\alpha}^-(k) := \tilde{a}^-(k)\frac{1}{\sum_i \tilde{a}_i^-(k)}$$

Real-time SC controller of a area:

$$f_{\mathcal{A}_i}(t) = \begin{cases} -\tilde{\alpha}^+_{\mathcal{A}_i} k_I \int ACE_i(t)dt & \text{for } \int ACE_i(t)dt \leq 0 \\ -\tilde{\alpha}^-_{\mathcal{A}_i} k_I \int ACE_i(t)dt & \text{for } \int ACE_i(t)dt > 0 \end{cases}$$

The participation function

$$f(t) = \gamma(\tilde{a}^+(k), \tilde{a}^-(k), q(t)) = -\tilde{\alpha}^+(k)\min(\mathbf{1}^\top q(t), 0) + \tilde{\alpha}^-(k)\max(\mathbf{1}^\top q(t), 0)$$

AS market clearing problem

For a time instant k on energy time scale

Input

- AS bids: $\beta_i^+(a_i^+, k)$, $\beta_i^-(a_i^-, k)$ $\rightarrow$ deduce objective functions
- Uncertainties (spatial distribution): $\mathcal{Q}(k)$

Market clearing problem (optimal spatial distribution of AS)

$$\min_{a^+, a^-, \{\delta_t\}_{t\in\{1,\ldots,T\}}} \sum_{i=1}^N \left(J_i^+(a_i^+) + J_i^-(a_i^-)\right), \qquad \text{(max socail welfare)}$$

subject to

$$\boxed{\gamma(a^+(k), a^-(k), q_t) + q_t = B\delta_t, \ t = 1, \ldots, T} \quad \textbf{(spatial info.)}$$
$$L\delta_t \leq \Delta l, \quad t = 1, \ldots, T \qquad \text{(robust congestion constraints)}$$
$$\sum_i a_i^+ = r^+ \qquad \text{(required AS+ accomulation)}$$
$$\sum_i a_i^- = r^- \qquad \text{(required AS- accomulation)}$$

Nodal prices solution

Lagrangian

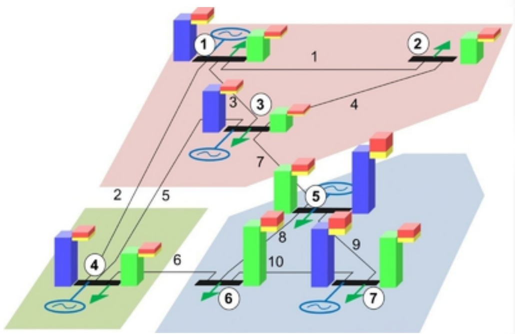
$$\begin{aligned}\mathcal{L} = & \sum_{i=1}^N \left(J_i^+(a_i^+) + J_i^-(a_i^-)\right) \\ & + \sum_{t=1}^T \mu_t^\top \left(L\delta_t - \Delta l\right) + \sum_{t=1}^T \tau_t^\top \left(\gamma(a^+(k), a^-(k), q_t) + q_t - B\delta_t\right) \\ & + (\sigma^+)^\top \left(\sum_i a_i^+ - r^+\right) + (\sigma^-)^\top \left(\sum_i a_i^- - r^-\right)\end{aligned}$$

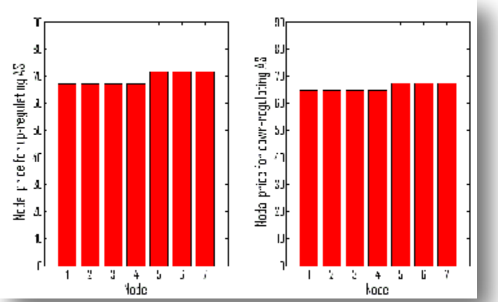
Optimal AS nodal prices

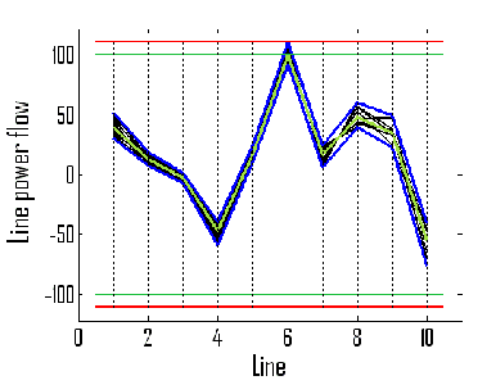
$$\overline{q}^+ := \min(\{\mathbf{1}^\top q_t\}_{t=1,\ldots,T}, 0), \ \overline{q}^- := \max(\{\mathbf{1}^\top q_t\}_{t=1,\ldots,T}, 0), \ z_t^+ := \mathbf{1}\frac{\overline{q}^+}{r^+}, \quad z_t^- := \mathbf{1}\frac{\overline{q}^-}{r^-}$$

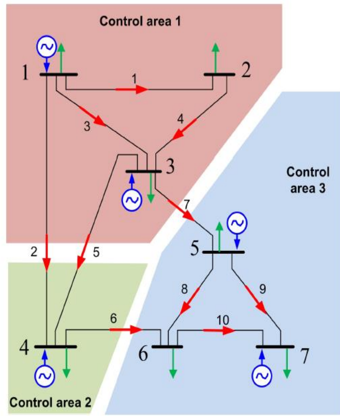
$$\lambda^+ = -\mathbf{1}\tilde{\sigma}^+ + \sum_{t=1}^T \tilde{\tau}_t \circ z_t^+, \quad \lambda^- = -\mathbf{1}\tilde{\sigma}^- + \sum_{t=1}^T \tilde{\tau}_t \circ z_t^-$$

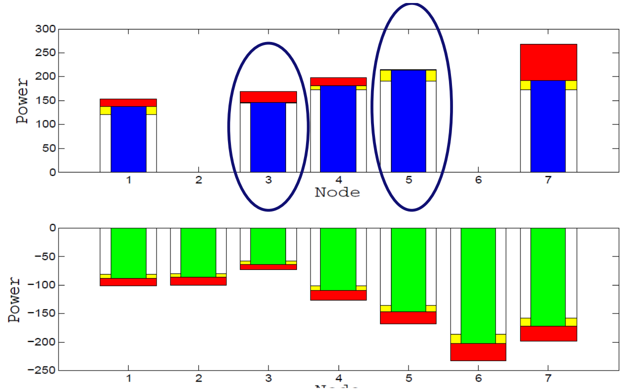
Robustly optimal AS spatial distribution: $\beta^+(a^+) = \lambda^+$, $\beta^-(a^-) = \lambda^-$.







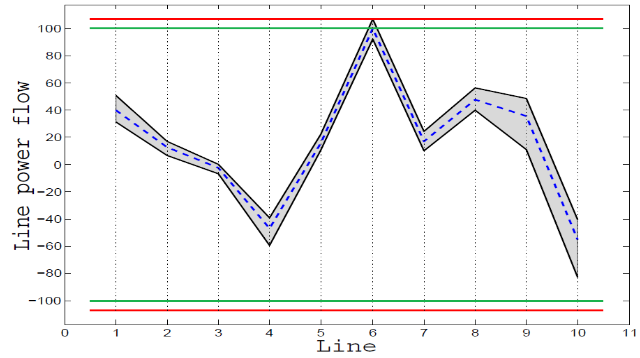


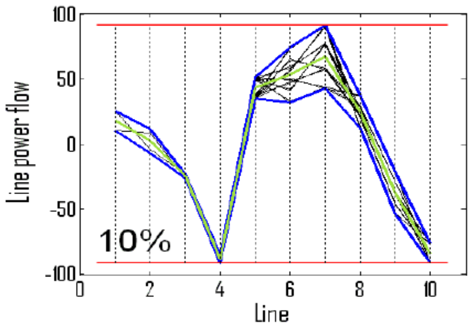


Spatial distribution of AS:
Shaping the “uncertainty tube”→

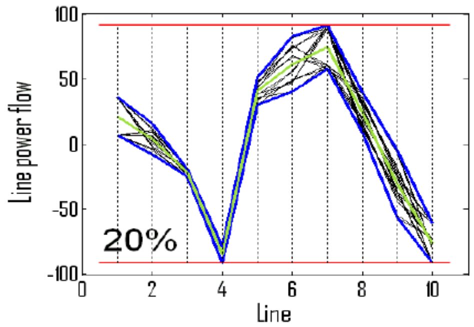
Get reliability for best costs

Possible to include optimal cooperation between control areas

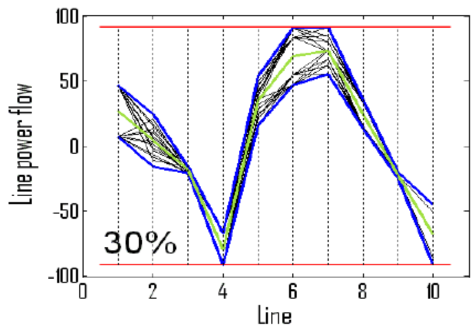




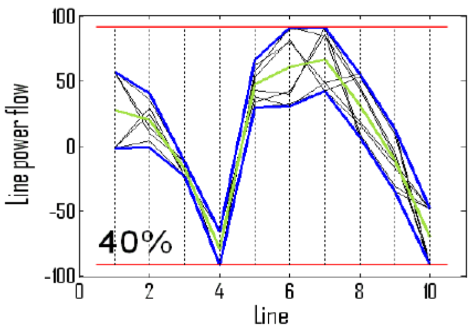
(a) Power flows for 10% uncertainty level.



(b) Power flows for 20% uncertainty level.

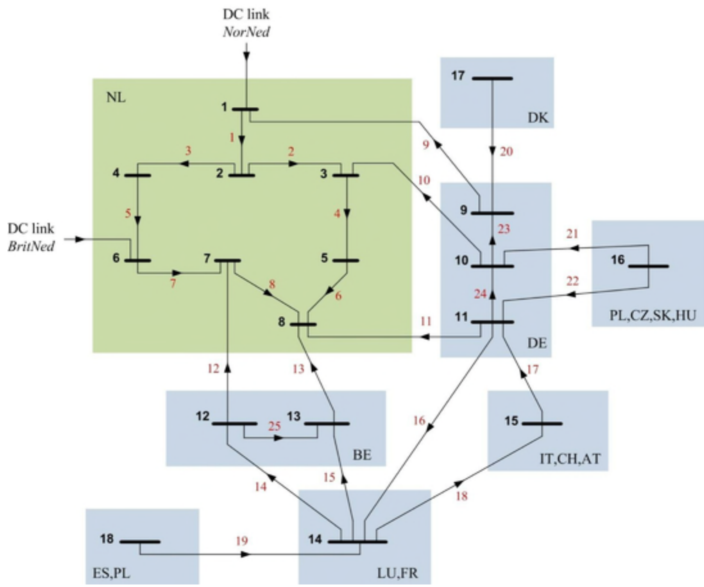


(c) Power flows for 30% uncertainty level.

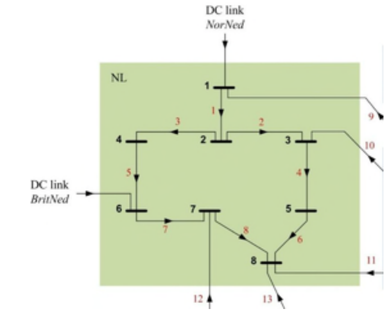


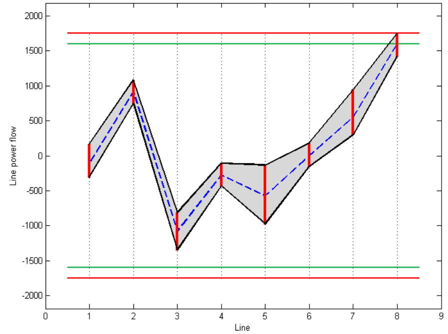
(d) Power flows for 40% uncertainty level.

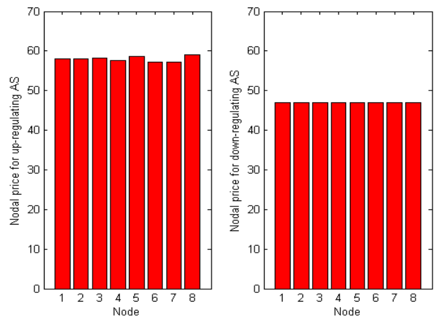
The E-Price benchmark model



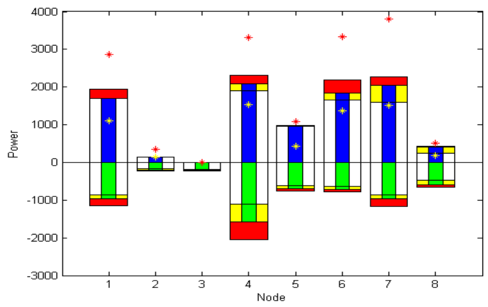
Locational prices for ancillary services

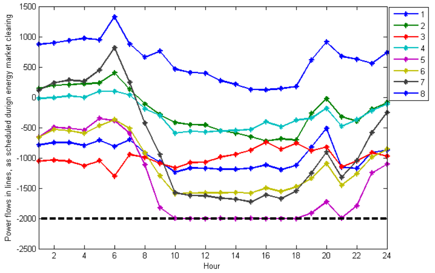


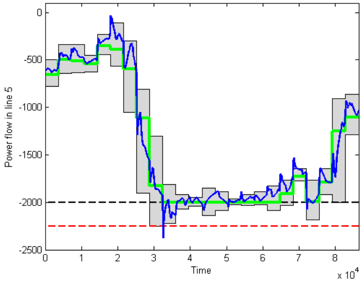


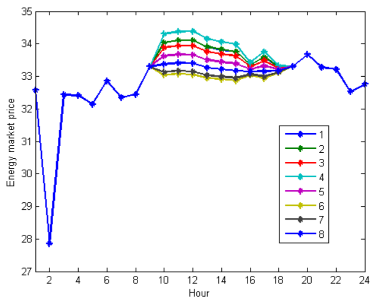


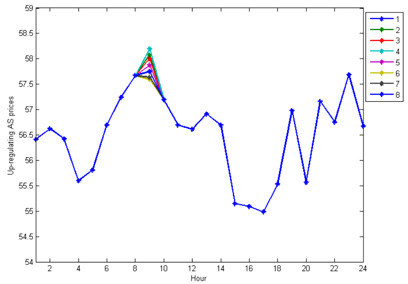
7



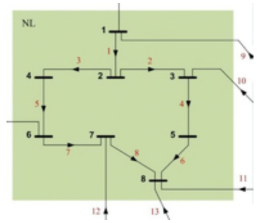


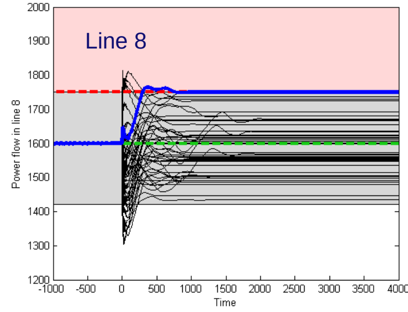


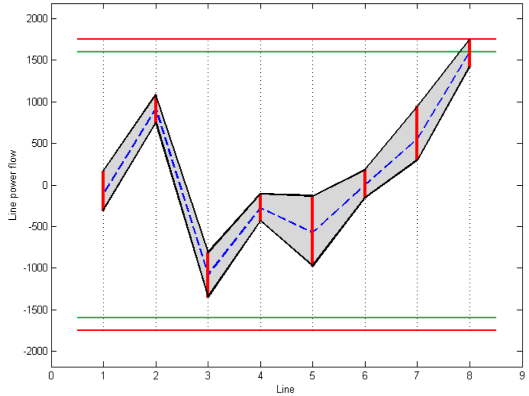


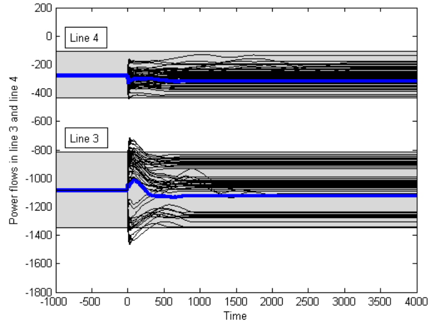


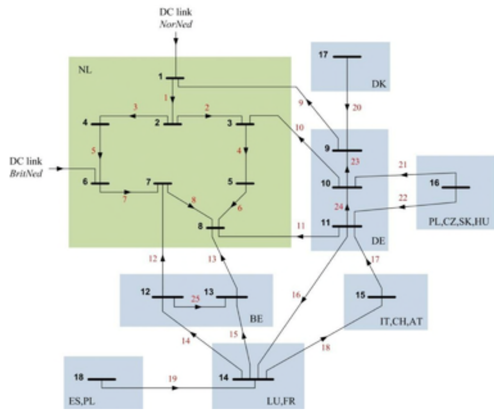
Optimized uncertainty in line power flows

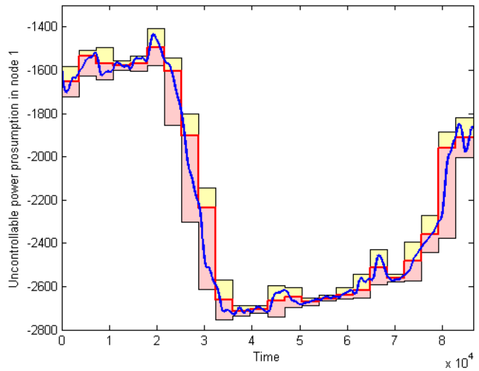


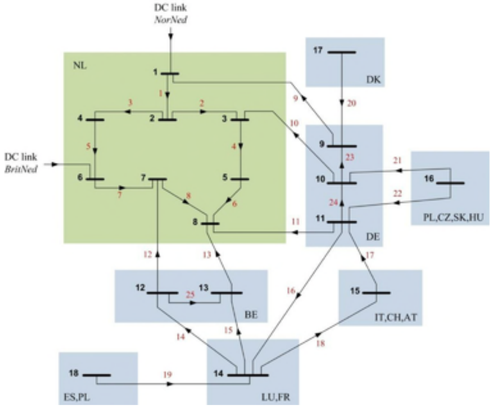


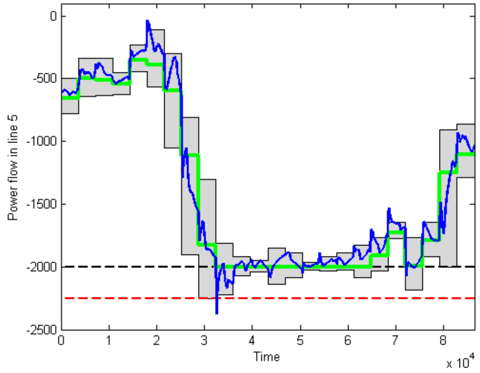


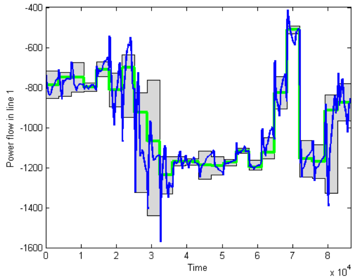


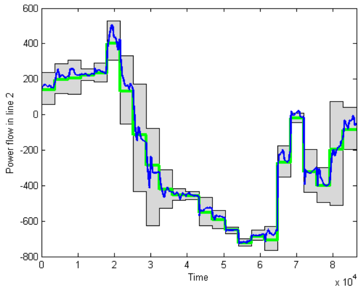




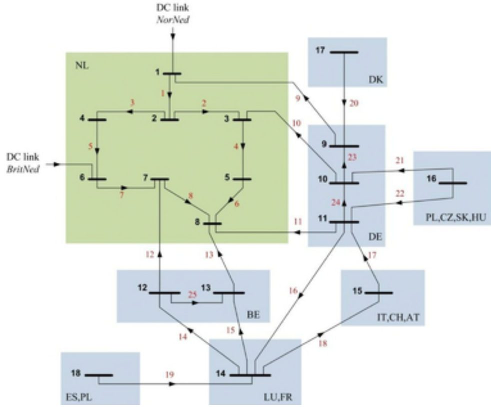


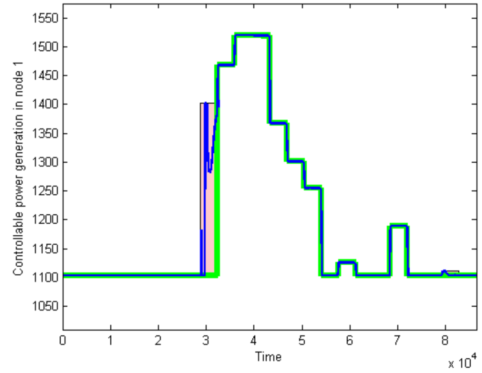


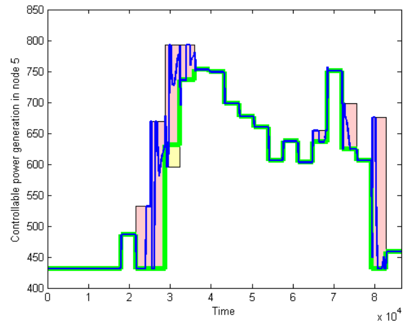


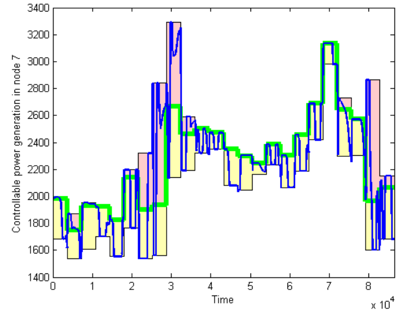


21

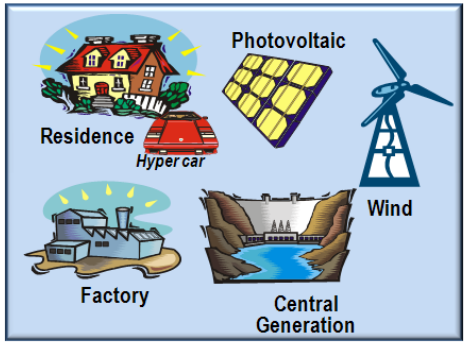


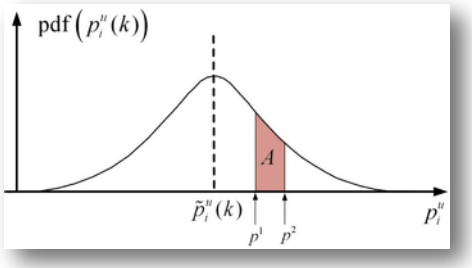


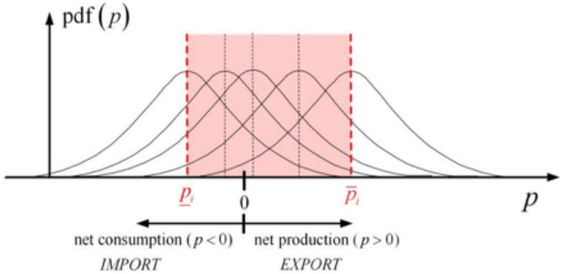




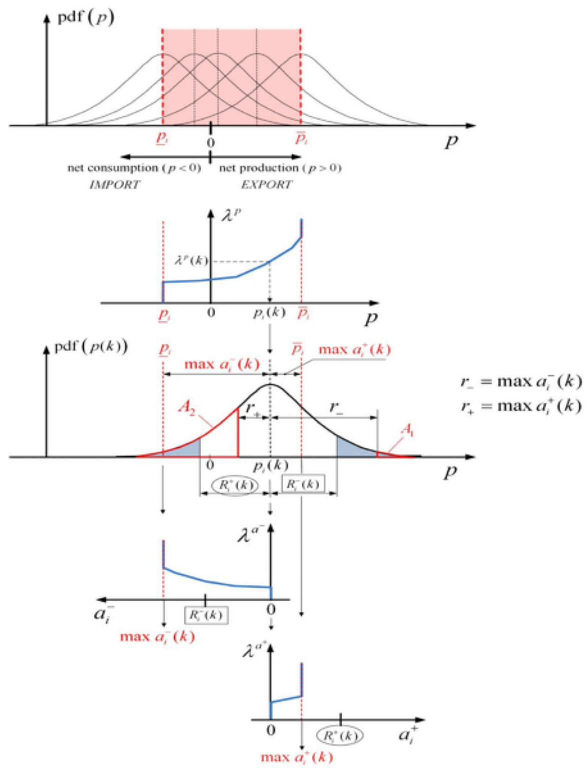
Double sided Ancillary Services (AS) markets







- Employ controllable prosumers in its own portfolio for keeping up the contracted prosumption level
- Buy/sell options on double-sided AS markets



Conclusions and messages

- Today's robustness: partly due to conservative engineering
- Future: increased complexity. Robustness (fragility?), efficiency, scalability?
- Exploit the networking! (often neglected in research)
- smart? better understood, explained: hidden (technology), invisible (hand of market)
- think in terms of modules (plug and play), protocols and architecture
- Optimization (duality!): holistic approach to market (and control)
- Huge area for important research (exciting parallel research in control systems field)

eprice www.e-price-project.eu

hrzz Hrvatska zaklada za znanost www.fsb.hr/ConDis