Dynamics and Control in Power Grids and Complex Oscillator Networks

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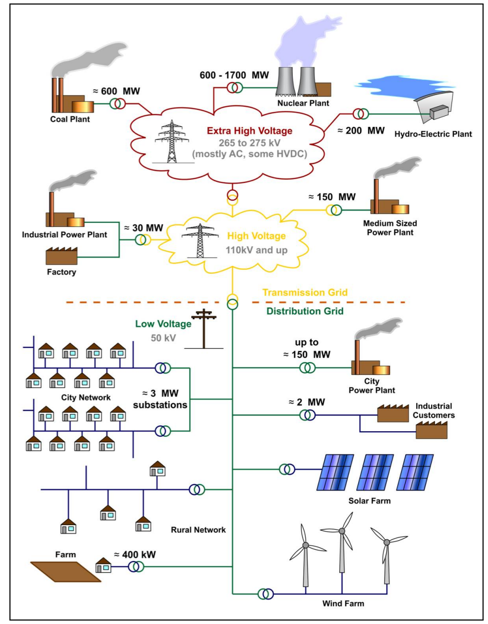
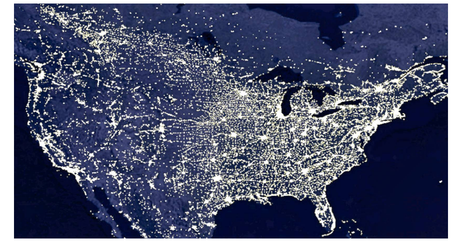
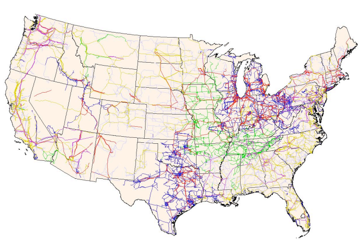
Electric Energy & Power Networks





- **Electric energy** is critical for our technological civilization
- Purpose of electric **power grid**: generate/transmit/distribute
- Op **challenges**: multiple scales, nonlinear, & complex

Trends, Advances, & Tomorrow's Power Grid



1. increasing renewables & deregulation
2. growing demand & operation at capacity

⇒ increasing volatility & complexity, decreasing robustness margins

Rapid technological and scientific advances:

1. re-instrumentation: PMUs & FACTS
2. complex & cyber-physical systems

⇒ cyber-coordination layer for smart grid

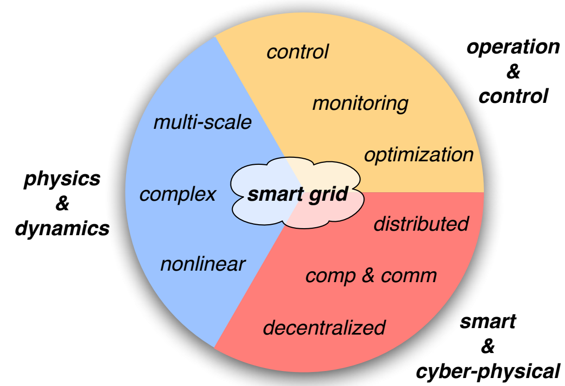
The Envisioned Power Grid

complex, cyber-physical, & "smart"

⇒ smart grid **keywords**

⇒ **interdisciplinary**:
power, control, comm, optim, comp, physics, . . . industry, & society

⇒ **research themes**:
"understanding & taming complexity"

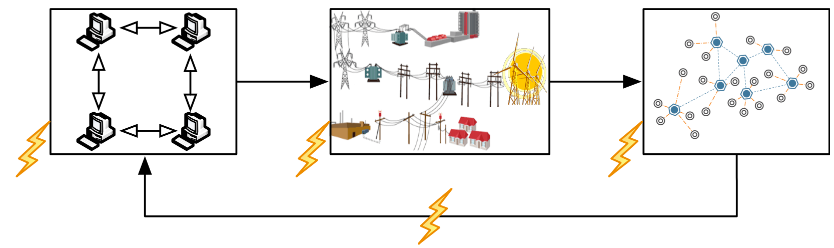


Outline

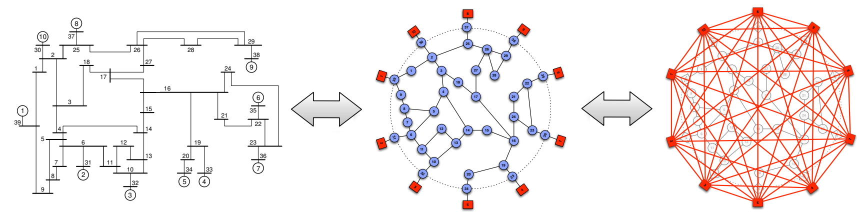
1. **Introduction and motivation**
   - Project Samples in Complex Systems Control ∩ Smart Grids
2. **Synchronization in power networks & coupled oscillators**
   - Relating power networks and coupled oscillator models
3. **Synchronization analysis & conditions**
   - Synchronization in a complete graph
   - Synchronization in a sparse graph
4. **Applications & experiments**
   - Comp & Opt: Power Flow Approximation
   - Monitoring: Contingency Screening
   - Distributed Control in Micrograids
5. **Conclusions**

Project Samples I

1. Cyber-physical security (with F. Pasqualetti & F. Bullo)

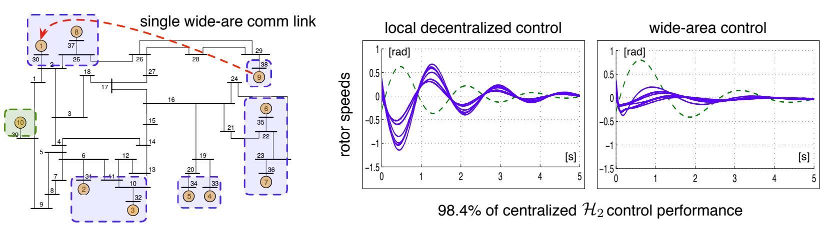


2. Coarse-graining of networks (with D. Romeres, I. Dobson, & F. Bullo)



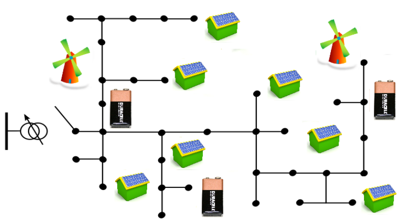
Project Samples II

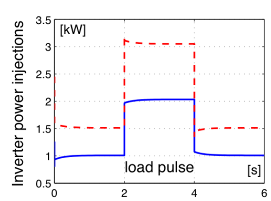
3. Distributed wide-area control (with M. Jovanovic, M. Chertkov, & F. Bullo)



98.4% of centralized $\mathcal{H}_2$ control performance

4. Inverters in micrograids (with J. Simpson-Porco, J.M. Guerrero, & F. Bullo)



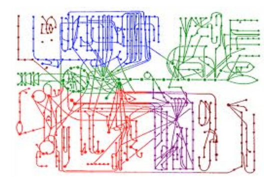


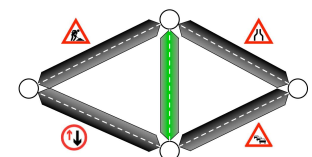
Power Grids as Prototypical Complex Networks

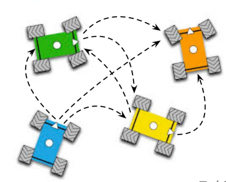
⇒ Similar challenges & tools in

- biochemical reaction networks
- social networks & epidemics
- transportation networks
- robotic coordination & sensor ntkws
- ⋮

⇒ Plenty of synergies and cross-fertilization

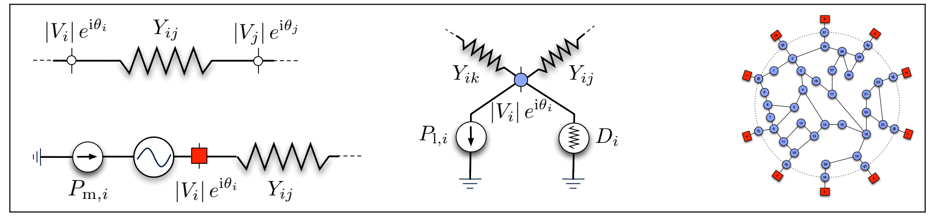






Outline

Mathematical Model of Power Transmission Network



- active power flow on line $i \rightsquigarrow j$: $\underbrace{|V_i||V_j||Y_{ij}|}_{a_{ij}=\text{max power transfer}} \cdot \sin(\theta_i - \theta_j)$
- power balance at node i: $\underbrace{P_i}_{\text{power injection}} = \sum_j a_{ij} \sin(\theta_i - \theta_j)$
- (DAE) power network dynamics [A. Bergen & D. Hill '81]:

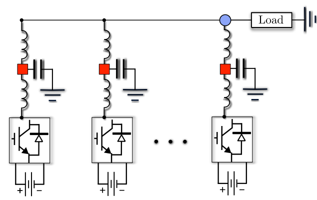
■: swing eq with $P_i > 0$ $\quad M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i - \sum_j a_{ij}\sin(\theta_i - \theta_j)$

●: $P_i < 0$ and $D_i \geq 0$ $\quad D_i\dot{\theta}_i = P_i - \sum_j a_{ij}\sin(\theta_i - \theta_j)$

Models of DC Sources with Inverters & Load Models

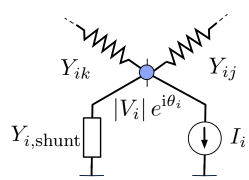
- DC source with droop-controlled DC/AC power converter [M.C. Chandorkar et. al. '93]:

$$D_i^{(\text{droop})}\dot{\theta}_i = P_i^{(\text{setpoint})} - \sum_j a_{ij}\sin(\theta_i - \theta_j)$$



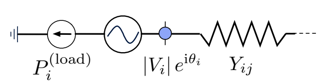
- constant current and admittance loads in Kron-reduced network [F. Dörfler et al. '13]:

$$M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i^{(\text{red})} - \sum_j a_{ij}^{(\text{red})}\sin(\theta_i - \theta_j)$$



- constant motor loads [P. Kundur '94]:

$$M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i^{(\text{load})} - \sum_j a_{ij}\sin(\theta_i - \theta_j)$$



Synchronization in Power Networks

Sync is crucial for the functionality and operation of the AC power grid. Generators have to swing in sync despite fluctuations/faults/contingencies.

Def: $\dot{\theta}_i = \dot{\theta}_j \quad \& \quad |\theta_i - \theta_j|$ bounded $\forall$ branches $\{i,j\}$

= sync'd frequencies & constrained active power flows

Given: network parameters & topology and load & generation profile

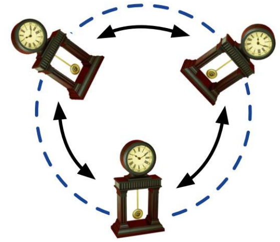
Q: "$\exists$ an optimal, stable, and robust synchronous operating point ?"

1. Security analysis [Araposthatis et al. '81, Wu et al. '80 & '82, Ilić '92, ...]
2. Load flow feasibility [Chiang et al. '90, Dobson '92, Lesieutre et al. '99, ...]
3. Optimal generation dispatch [Lavaei et al. '12, Bose et al. '12, ...]
4. Transient stability [Sastry et al. '80, Bergen et al. '81, Hill et al. '86, ...]
5. Inverters in microgrids [Chandorkar et. al. '93, Guerrero et al. '09, Zhong '11, ...]

Synchronization in Complex Oscillator Networks

Pendulum clocks & *"an odd kind of sympathy"*
[C. Huygens, Horologium Oscillatorium, 1673]

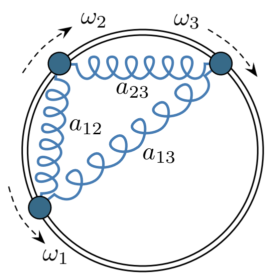
Today's canonical coupled oscillator model
[A. Winfree '67, Y. Kuramoto '75]



Coupled oscillator model:

$$\dot{\theta}_i = \omega_i - \sum\nolimits_{j=1}^{n} a_{ij} \sin(\theta_i - \theta_j)$$

- n **oscillators** with phase $\theta_i \in \mathbb{S}^1$
- **non-identical** natural frequencies $\omega_i \in \mathbb{R}^1$
- elastic **coupling** with strength $a_{ij} = a_{ji}$
- undirected & connected **graph** $G(\mathcal{V}, \mathcal{E}, A)$

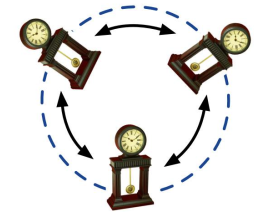


Synchronization in Complex Oscillator Networks

applications

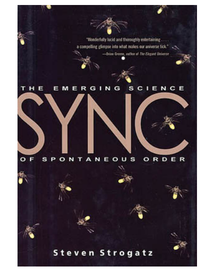
Coupled oscillator model:

$$\dot{\theta}_i = \omega_i - \sum\nolimits_{j=1}^{n} a_{ij} \sin(\theta_i - \theta_j)$$



A few related applications:

- Sync in Josephson junctions
  [S. Watanabe et. al '97, K. Wiesenfeld et al. '98]
- Sync in a population of fireflies
  [G.B. Ermentrout '90, Y. Zhou et al. '06]
- Canonical model of coupled limit cycle oscillators
  [F.C. Hoppensteadt et al. '97, E. Brown et al. '04]
- Countless sync phenomena in sciences/bio/tech.
  [S. Strogatz '00, J. Acebrón '05 et al., F. Dörfler et al. '13]

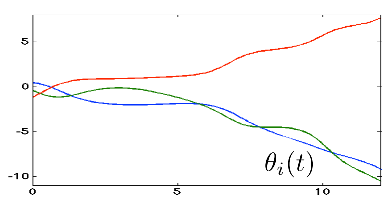


Synchronization in Complex Oscillator Networks

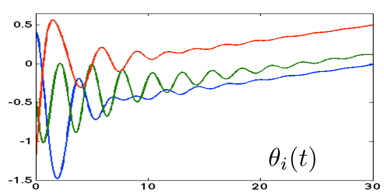
phenomenology and challenges

Synchronization is a **trade-off:**
coupling vs. heterogeneity

$$\dot{\theta}_i = \omega_i - \sum\nolimits_{j=1}^{n} a_{ij} \sin(\theta_i - \theta_j)$$



coupling small & $|\omega_i - \omega_j|$ large
$\Rightarrow$ incoherence



coupling large & $|\omega_i - \omega_j|$ small
$\Rightarrow$ frequency sync

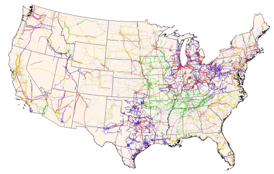
A central question: quantify "coupling" vs. "heterogeneity"
[S. Strogatz '01, A. Arenas et al. '08, S. Boccaletti et al. '06]

Outline

Relating power networks and coupled oscillator models

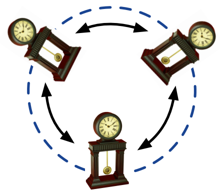
(1) Power network model:

$$M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i - \sum_j a_{ij}\sin(\theta_i - \theta_j)$$
$$D_i\dot{\theta}_i = P_i - \sum_j a_{ij}\sin(\theta_i - \theta_j)$$



(2.1) Variation of coupled oscillator model:

$$\dot{\theta}_i = P_i - \sum_j a_{ij}\sin(\theta_i - \theta_j)$$



(2.2) Add decoupled frequency dynamics:

$$\ddot{\theta}_i = -\dot{\theta}_i$$

Homotopy: construct continuous interpolation between (1) and (2)

Relating power networks and coupled oscillator models
main result

Family of dynamical system $\mathcal{H}_\lambda$:

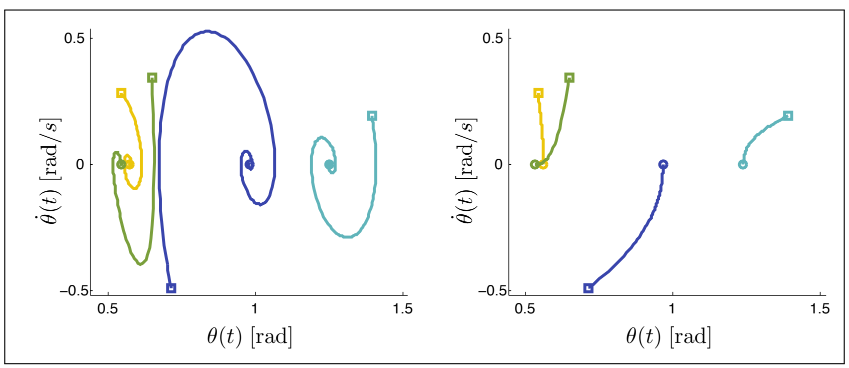
$$\frac{d}{dt}\begin{bmatrix}\theta \\ \dot{\theta}\end{bmatrix} = (1-\lambda)\cdot(1) + \lambda\cdot(2), \qquad \lambda \in [0,1]$$

Theorem: Properties of the $\mathcal{H}_\lambda$ family [F. Dörfler & F. Bullo '11]

1. **Invariance of equilibria:** For all $\lambda \in [0,1]$ the equilibria are
$$\left\{(\theta,\dot{\theta}) : \dot{\theta}_i = 0\ ,\ P_i = \sum_j a_{ij}\sin(\theta_i - \theta_j)\right\}.$$
2. **Invariance of local stability:** For all equilibria and $\lambda \in [0,1]$, the Jacobian has constant number of stable/unstable/zero eigenvalues.

Relating power networks and coupled oscillator models
topological equivalence interpretation

$\Rightarrow$ near the equilibrium manifolds (1) synchronizes $\Leftrightarrow$ (2) synchronizes



$\Rightarrow$ **main message:** "w.l.o.g." focus on coupled oscillator model

Outline

Synchronization in a Complete & Homogeneous Graph

Classic **Kuramoto model**:
[Y. Kuramoto '75]

$$\dot{\theta}_i = \omega_i - \frac{K}{n} \sum_{j=1}^{n} \sin(\theta_i - \theta_j)$$

Theorem: Explicit sync condition [F. Dörfler & F. Bullo '11]

The following statements are equivalent:

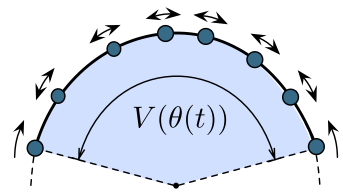
1. Coupling dominates heterogeneity, i.e., $K > K_{\text{critical}} \triangleq \omega_{\max} - \omega_{\min}$.
2. Kuramoto models with $\{\omega_1, \dots, \omega_n\} \subseteq [\omega_{\min}, \omega_{\max}]$ synchronize.

Strictly improves existing cond's [F. de Smet et al. '07, N. Chopra et al. '09, G. Schmidt et al. '09, A. Jadbabaie et al. '04, S.J. Chung et al. '10, J.L. van Hemmen et al. '93, A. Franci et al. '10, S.Y. Ha et al. '10, G.B. Ermentrout '85, A. Acebron et al. '00]

Synchronization in a Complete & Homogeneous Graph

main proof ideas

1. **Arc invariance**: $\theta(t)$ in γ arc $\Leftrightarrow$ arc-length $V(\theta(t))$ is non-increasing



$$\Leftrightarrow \begin{cases} V(\theta(t)) = \max_{i,j \in \{1,\dots,n\}} |\theta_i(t) - \theta_j(t)| \\ D^+ V(\theta(t)) \leq 0 \end{cases}$$

true if $K \sin(\gamma) \geq K_{\text{critical}}$

$\Rightarrow$ Binary synchronization condition: $K > K_{\text{critical}}$

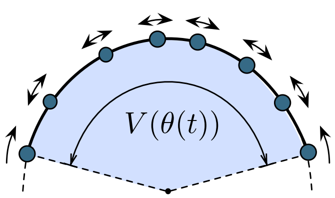
$\Rightarrow$ Bounds on transient dynamics: $K_{\text{critical}}/K = \sin(\gamma_{\min}) = \sin(\gamma_{\max})$

- **region of attraction** includes angles $\theta(t=0)$ in $\gamma_{\max}$ arc, &
- **asymptotic cohesiveness** of angles $\theta(t \to \infty)$ in $\gamma_{\min}$ arc

Synchronization in a Complete & Homogeneous Graph

main proof ideas

1. **Arc invariance**: $\theta(t)$ in γ arc $\Leftrightarrow$ arc-length $V(\theta(t))$ is non-increasing



$$\Leftrightarrow \begin{cases} V(\theta(t)) = \max_{i,j \in \{1,\dots,n\}} |\theta_i(t) - \theta_j(t)| \\ D^+ V(\theta(t)) \leq 0 \end{cases}$$

true if $K \sin(\gamma) \geq K_{\text{critical}}$

2. **Frequency synchronization** $\Leftrightarrow$ linear time-varying system (consensus)

$$\frac{d}{dt}\dot{\theta}_i = -\sum_{j=1}^{n} a_{ij}(t)\left(\dot{\theta}_i - \dot{\theta}_j\right),$$

where $a_{ij}(t) = \frac{K}{n}\cos(\theta_i(t) - \theta_j(t))$ becomes positive in finite time

Outline

Primer on Algebraic Graph Theory

Laplacian matrix $L =$ "degree matrix" $-$ "adjacency matrix"

$$L = L^T = \begin{bmatrix} \vdots & \ddots & \vdots & \iddots & \vdots \\ -a_{i1} & \cdots & \sum_{j=1}^n a_{ij} & \cdots & -a_{in} \\ \vdots & \iddots & \vdots & \ddots & \vdots \end{bmatrix} \geq 0$$

Notions of connectivity

- spectral: 2nd smallest eigenvalue of L is "algebraic connectivity" $\lambda_2(L)$
- topological: degree $\sum_{j=1}^n a_{ij}$ or degree distribution

Notions of heterogeneity

$\|\omega\|_{\mathcal{E},\infty} = \max_{\{i,j\}\in\mathcal{E}} |\omega_i - \omega_j|$, $\quad\|\omega\|_{\mathcal{E},2} = \left(\sum_{\{i,j\}\in\mathcal{E}} |\omega_i - \omega_j|^2\right)^{1/2}$

Synchronization in Sparse Graphs

a brief overview

$$\dot\theta_i = \omega_i - \sum_{j=1}^n a_{ij}\sin(\theta_i - \theta_j)$$

1. **necessary sync condition**: $\sum_{j=1}^n a_{ij} \geq |\omega_i| \quad \Leftarrow \quad$ sync
   [C. Tavora and O.J.M. Smith '72]
2. **sufficient sync condition**: $\lambda_2(L) > \|\omega\|_{\mathcal{E},2} \quad \Rightarrow \quad$ sync
   [F. Dörfler and F. Bullo '12]

$\Rightarrow$ $\exists$ similar conditions with diff. metrics on coupling & heterogeneity

$\Rightarrow$ **Problem:** sharpest general conditions are conservative

A Nearly Exact Synchronization Condition

main result

Theorem: Sharp sync condition [F. Dörfler, M. Chertkov, & F. Bullo '12]

Under one of following assumptions:

1) **extremal topologies:** trees, homogeneous graphs, or $\{3,4\}$ rings
2) **extremal parameters:** $L^\dagger\omega$ is bipolar, small, or symmetric (for rings)
3) arbitrary one-connected **combinations** of 1) and 2)

If $\left\|L^\dagger\omega\right\|_{\mathcal{E},\infty} < 1$

$\Rightarrow$ $\exists$ a unique & locally exponentially stable synchronous solution $\theta^* \in \mathbb{T}^n$ satisfying $|\theta_i^* - \theta_j^*| \leq \arcsin\left(\left\|L^\dagger\omega\right\|_{\mathcal{E},\infty}\right)$ for all $\{i,j\}\in\mathcal{E}$

...and result is **"statistically correct"** .

A Nearly Exact Synchronization Condition

statistical accuracy for power networks

Randomized power network test cases
with 50 % randomized loads and 33 % randomized generation

Randomized test case (1000 instances)	Numerical worst-case angle differences: $\max_{\{i,j\}\in\mathcal{E}} \lvert\theta_i^* - \theta_j^*\rvert$	Analytic prediction of angle differences: $\arcsin(\|L^\dagger\omega\|_{\mathcal{E},\infty})$	Accuracy of condition: $\arcsin(\|L^\dagger\omega\|_{\mathcal{E},\infty}) - \max_{\{i,j\}\in\mathcal{E}} \lvert\theta_i^* - \theta_j^*\rvert$
9 bus system	0.12889 rad	0.12893 rad	$4.1218 \cdot 10^{-5}$ rad
IEEE 14 bus system	0.16622 rad	0.16650 rad	$2.7995 \cdot 10^{-4}$ rad
IEEE RTS 24	0.22309 rad	0.22480 rad	$1.7089 \cdot 10^{-3}$ rad
IEEE 30 bus system	0.16430 rad	0.16456 rad	$2.6140 \cdot 10^{-4}$ rad
New England 39	0.16821 rad	0.16828 rad	$6.6355 \cdot 10^{-5}$ rad
IEEE 57 bus system	0.20295 rad	0.22358 rad	$2.0630 \cdot 10^{-2}$ rad
IEEE RTS 96	0.24593 rad	0.24854 rad	$2.6076 \cdot 10^{-3}$ rad
IEEE 118 bus system	0.23524 rad	0.23584 rad	$5.9959 \cdot 10^{-4}$ rad
IEEE 300 bus system	0.43204 rad	0.43257 rad	$5.2618 \cdot 10^{-4}$ rad
Polish 2383 bus system (winter peak 1999/2000)	0.25144 rad	0.25566 rad	$4.2183 \cdot 10^{-3}$ rad

$\Rightarrow$ similar results have been reproduced by SIEMENS Los Alamos NATIONAL LABORATORY EST.1943

A Nearly Exact Synchronization Condition
comments

- **Monte Carlo studies:** for range of random topologies & parameters

$\Rightarrow$ with high prob & accuracy: sync "for almost all" $G(\mathcal{V}, \mathcal{E}, A)$ & ω

- Possibly thin sets of degenerate **counter-examples** for large rings

- **Intuition:** the condition $\| L^{\dagger} \omega \|_{\mathcal{E},\infty} < 1$ is equivalent to

$$\left\| \left[\text{eigenvectors of L}\right] \begin{bmatrix} 0 & 0 & \dots & \dots & 0 \\ 0 & \frac{1}{\lambda_2(L)} & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & \dots & 0 & \frac{1}{\lambda_n(L)} \end{bmatrix} \left[\text{eigenvectors of L}\right]^T \omega \right\|_{\mathcal{E},\infty} < 1$$

$\Rightarrow$ includes previous conditions on $\lambda_2(L)$ and degree ($\approx \lambda_n(L)$)

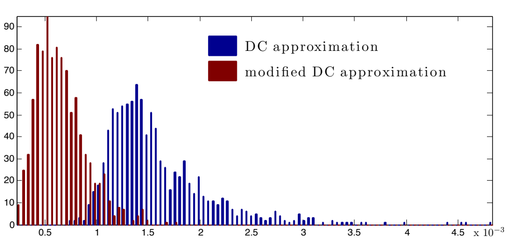
Outline

Power Flow Approximation

1. AC power flow: $P_i = \sum_{j=1}^{n} a_{ij} \sin(\theta_i - \theta_j)$
2. DC power flow: $P_i = \sum_{j=1}^{n} a_{ij} (\delta_i - \delta_j)$

$\Rightarrow$ Conventional DC approximation: $\theta_i^* - \theta_j^* \approx \delta_i^* - \delta_j^*$

$\Rightarrow$ Our modified DC approximation: $\theta_i^* - \theta_j^* \approx \arcsin(\delta_i^* - \delta_j^*)$



Error histograms for 1000 samples of randomized IEEE 118 system

$\Rightarrow$ apps: convexify OPF, planning, contingency screening, etc.

Power Flow Approximation
Security-Constrained Power Flow

AC power flow with security constraints

$$P_i = \sum_{j=1}^{n} a_{ij} \sin(\theta_i - \theta_j)\,, \qquad |\theta_i - \theta_j| < \gamma_{ij} \quad \forall \{i,j\} \in \mathcal{E}$$

DC power flow with security constraints

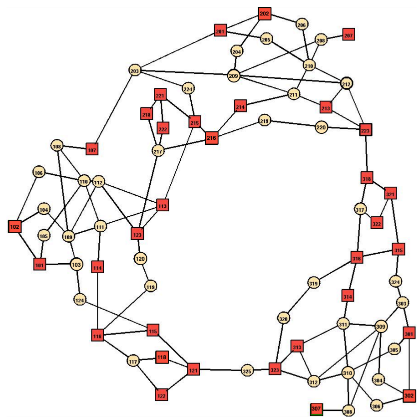
$$P_i = \sum_{j=1}^{n} a_{ij} (\delta_i - \delta_j)\,, \qquad |\delta_i - \delta_j| < \gamma_{ij} \quad \forall \{i,j\} \in \mathcal{E}$$

Novel test

$$P_i = \sum_{j=1}^{n} a_{ij} (\delta_i - \delta_j)\,, \qquad |\delta_i - \delta_j| < \sin(\gamma_{ij}) \quad \forall \{i,j\} \in \mathcal{E}$$

Proof of equivalence for a tree: $\theta_i^* - \theta_j^* = \arcsin(\delta_i^* - \delta_j^*)$ ∎

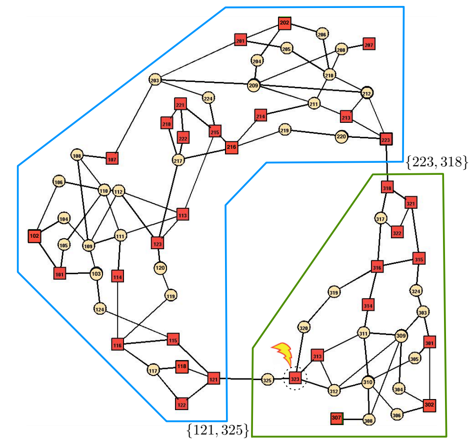
Contingency Analysis



IEEE Reliability Test System '96 at nominal operating point

Contingency Analysis

two contingencies

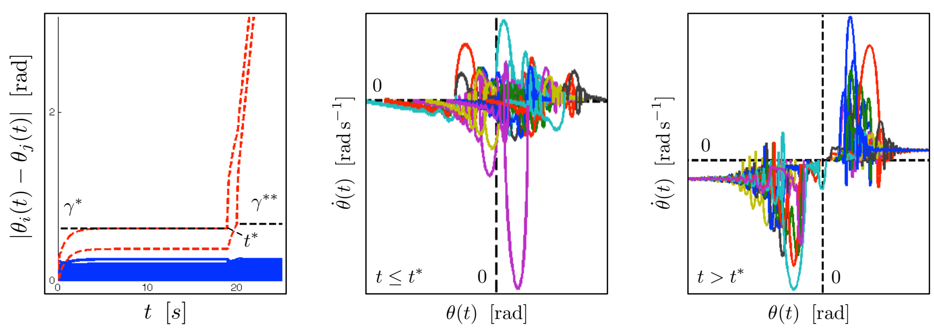


#1: increase generation & increase loads

#2: generator 323 is tripped

Contingency Analysis

predicting transition to instability



Continuously increase loads:

$\Rightarrow$ condition $\arcsin(\|L^\dagger \omega\|_{\mathcal{E},\infty}) < \gamma^*$ predicts that thermal limit γ^* of line $\{121, 325\}$ is violated at 22.23 % of additional loading

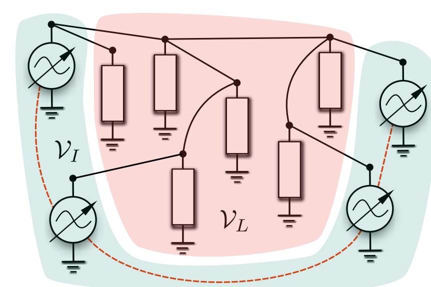
$\Rightarrow$ line $\{121, 325\}$ is tripped at 22.24% of additional loading

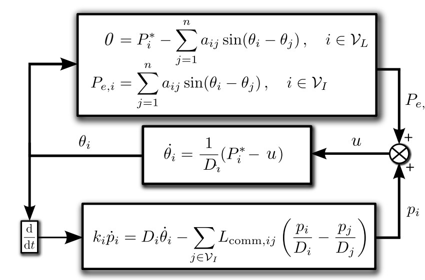
Distributed Averaging PI Droop Control in Microgrids

design based on coupled oscillator insights

Microgrid modeled as network of loads and inverters

Distributed & **A**veraging **PI** droop-controller (DAPI)





Decentralized primary control

$\Rightarrow$ sync: $\dot{\theta}_i(t) \to \omega_{\text{sync}}$

Distributed secondary control

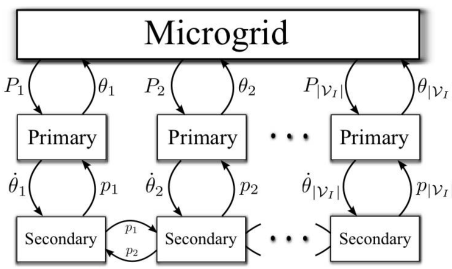
$\Rightarrow$ frequency regulation: $\omega_{\text{sync}} \to 0$

Distributed Averaging PI Droop Control in Microgrids
theoretic guarantees

Theorem (Properties DAPI control)
[J. Simpson-Porco, F. Dörfler, & F. Bullo, '12]

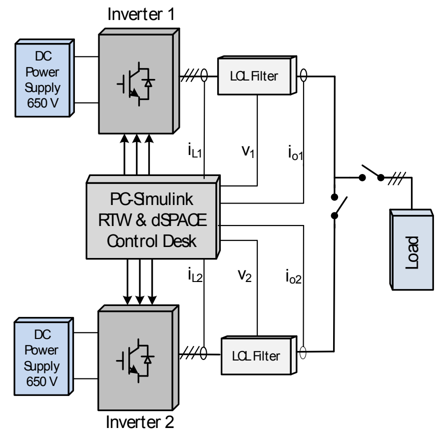
1. unique & exponentially stable closed-loop sync manifold;
2. frequency regulation & optimal power sharing;
3. robustness to voltage variations, losses, & uncertainties;
4. plug'n'play & arbitrary tuning.

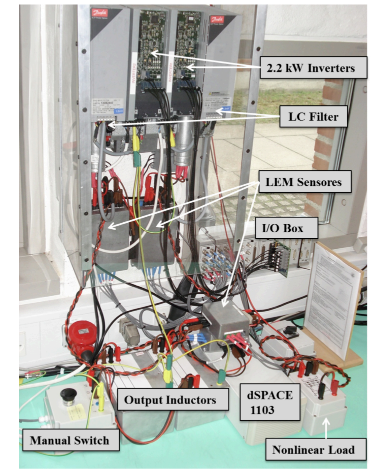
Distributed & **A**veraging **PI** droop-controller (DAPI)



Distributed Averaging PI Droop Control in Microgrids
Practical implementation at Aalborg University, Denmark

Implementation (together with Q. Shafiee & J.M. Guerrero)





Experimental results are remarkable: off-the-shelf, robust, small transients

Distributed Averaging PI Droop Control in Microgrids
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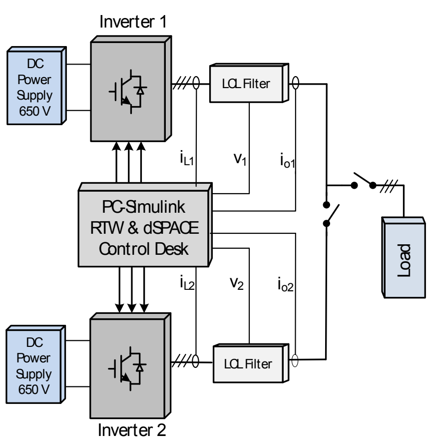
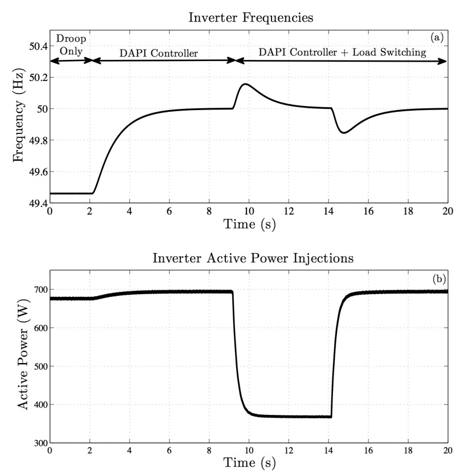


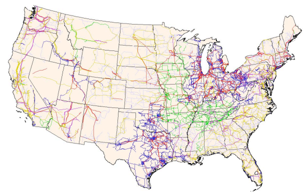


Experimental results are remarkable: off-the-shelf, robust, small transients

Outline

1. **Introduction and motivation**
   - Project Samples in Complex Systems Control ∩ Smart Grids
2. **Synchronization in power networks & coupled oscillators**
   - Relating power networks and coupled oscillator models
3. **Synchronization analysis & conditions**
   - Synchronization in a complete graph
   - Synchronization in a sparse graph
4. **Applications & experiments**
   - Comp & Opt: Power Flow Approximation
   - Monitoring: Contingency Screening
   - Distributed Control in Microgrids
5. **Conclusions**

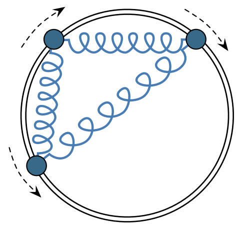
Summary



Lessons learned today:

- power networks are coupled oscillators
- sync if "coupling > heterogeneity"
- necessary, sufficient, & sharp sync cond's
- theory is useful, robust & applicable

Further **results** & **applications** (not shown)



Related **ongoing and future work**:

- more complete theory & more detailed models
- from analysis to control synthesis: cont. control design, hybrid remedial action schemes, computation & optimization

Related Publications

- F. Dörfler and F. Bullo. Synchronization in Complex Oscillator Networks: A Survey. In *Automatica*, April 2013, Note: submitted.
- F. Dörfler, M. Chertkov, and F. Bullo. Synchronization in Complex Oscillator Networks and Smart Grids. In *Proceedings of the National Academy of Sciences*, February 2013.
- F. Dörfler, F. Pasqualetti and F. Bullo. Continuous-Time Distributed Observers with Discrete Communication. In *IEEE Journal of Selected Topics in Signal Processing*, March 2013.
- J.W. Simpson-Porco, F. Dörfler, and F. Bullo. Synchronization and Power-Sharing for Droop-Controlled Inverters in Islanded Microgrids. In *Automatica*, Februay 2013, Note: provisionally accepted.
- F. Dörfler and F. Bullo. Kron Reduction of Graphs with Applications to Electrical Networks. In *IEEE Transactions on Circuits and Systems I.*, January 2013.
- F. Pasqualetti, F. Dörfler, and F. Bullo. Attack Detection and Identification in Cyber-Physical Systems. In *IEEE Transactions on Automatic Control*, December 2012, Note: to appear.
- F. Dörfler and F. Bullo. Synchronization and Transient Stability in Power Networks and Non-uniform Kuramoto Oscillators. In *SIAM Journal on Control and Optimization*, June 2012.
- F. Dörfler and F. Bullo. On the Critical Coupling for Kuramoto Oscillators. In *SIAM Journal on Applied Dynamical Systems*, September 2011.

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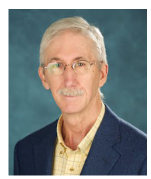

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