Control of Power Converters in Low-Inertia Power Systems

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Acknowledgements


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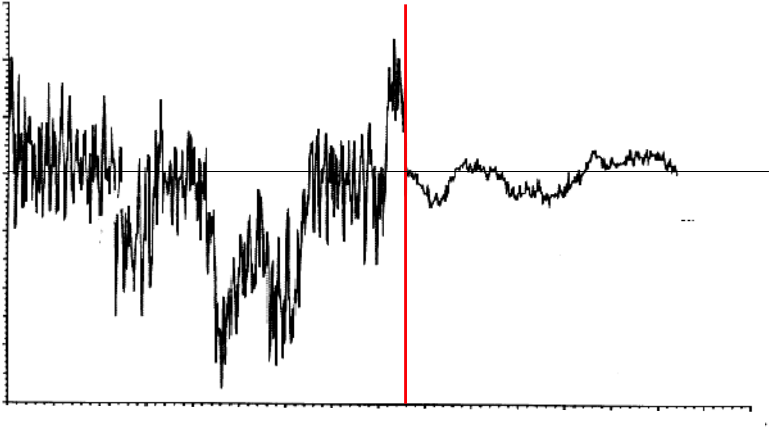
HORIZON 2020

ABB

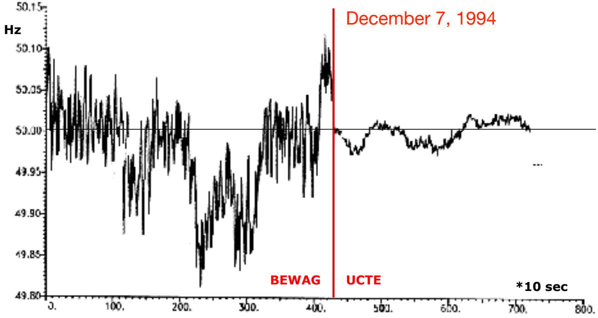
AIT Austrian Institute of Technology

Further: Gab-Su Seo, Brian Johnson, Mohit Sinha, & Sairaj Dhople

What do we see here ?



Frequency of West Berlin re-connecting to Europe


December 7, 1994

before re-connection: islanded operation based on batteries & single boiler

afterwards connected to European grid based on synchronous generation

The foundation of today's power system

Synchronous machines with rotational ***inertia***

$$M \frac{d}{dt}\omega \approx P_{\text{generation}} - P_{\text{demand}}$$

Today's grid operation ***heavily relies*** on

1. kinetic energy $\frac{1}{2}M\omega^2$ as ***safeguard*** against disturbances
2. ***self-synchronization*** of machines ***through the grid***
3. ***robust*** stabilization of ***frequency*** and ***voltage*** by generator controls

We are ***replacing*** this solid ***foundation*** with . . .

Tomorrow’s clean and sustainable power system

synchronous machines

- \+ **large** rotational **inertia**
- \+ **kinetic energy** $\frac{1}{2}M\omega^2$ as **buffer**
- \+ **self-synchronize** through **grid**
- \+ **robust** control of **voltage** & **freq.**
- − **slow** primary **control**

renewables & power electronics

- − **no** rotational **inertia**
- − almost **no energy storage**
- − **no** inherent **self-synchronization**
- − **fragile** control of **voltage** & **freq.**
- \+ **fast** actuation & control

what could possibly go wrong?

The concerns are not hypothetical

issues broadly recognized by TSOs, device manufacturers, academia, agencies, etc.

UPDATE REPORT –
BLACK SYSTEM EVENT
IN SOUTH AUSTRALIA ON
28 SEPTEMBER 2016

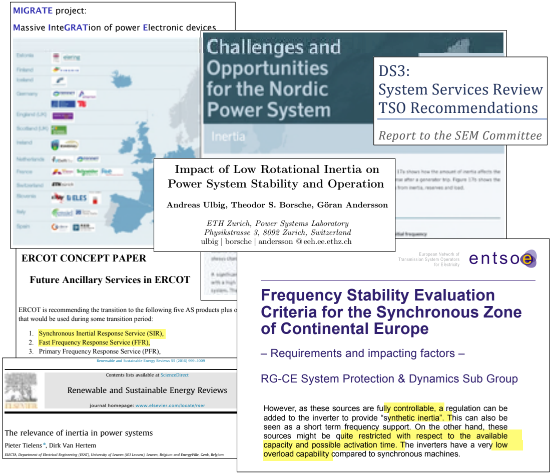
AN UPDATE TO THE PRELIMINARY OPERATING INCIDENT REPORT FOR THE NATIONAL ELECTRICITY MARKET.
DATA ANALYSIS AS AT 5.00 PM TUESDAY 11 OCTOBER 2016.

lack of robust control:

> *"Nine of the 13 wind farms online did not ride through the six voltage disturbances experienced during the event."*

between the lines:

conventional system would have been more resilient (?)



MIGRATE project:

Massive **I**nte**GRAT**ion of power **E**lectronic devices

Challenges and Opportunities for the Nordic Power System

Inertia

DS3:
System Services Review
TSO Recommendations

Report to the SEM Committee

Impact of Low Rotational Inertia on Power System Stability and Operation

Andreas Ulbig, Theodor S. Borsche, Göran Andersson

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ERCOT CONCEPT PAPER

Future Ancillary Services in ERCOT

ERCOT is recommending the transition to the following five AS products plus o
that would be used during some transition period:

1. Synchronous Inertial Response Service (SIR),
2. Fast Frequency Response Service (FFR),
3. Primary Frequency Response Service (PFR),

Renewable and Sustainable Energy Reviews 55 (2016) 999–1009

Contents lists available at ScienceDirect

Renewable and Sustainable Energy Reviews

journal homepage: www.elsevier.com/locate/rser

The relevance of inertia in power systems

Pieter Tielens*, Dirk Van Hertem

ELECTA, Department of Electrical Engineering (ESAT), University of Leuven (KU Leuven), Leuven, Belgium and EnergyVille, Genk, Belgium

Frequency Stability Evaluation Criteria for the Synchronous Zone of Continental Europe

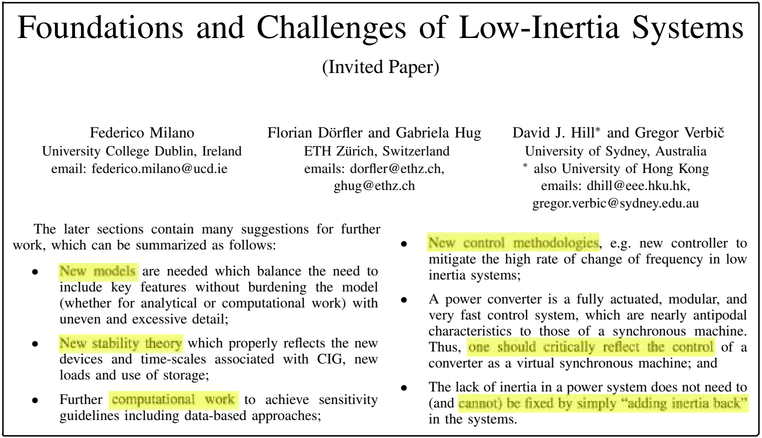
– Requirements and impacting factors –

RG-CE System Protection & Dynamics Sub Group

However, as these sources are fully controllable, a regulation can be added to the inverter to provide "synthetic inertia". This can also be seen as a short term frequency support. On the other hand, these sources might be quite restricted with respect to the available capacity and possible activation time. The inverters have a very low overload capability compared to synchronous machines.

Biblis A generator stabilizes the grid as a synchronous condenser

Critically re-visit system modeling/analysis/control



Foundations and Challenges of Low-Inertia Systems

(Invited Paper)

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gregor.verbic@sydney.edu.au

The later sections contain many suggestions for further work, which can be summarized as follows:

- New models are needed which balance the need to include key features without burdening the model (whether for analytical or computational work) with uneven and excessive detail;
- New stability theory which properly reflects the new devices and time-scales associated with CIG, new loads and use of storage;
- Further computational work to achieve sensitivity guidelines including data-based approaches;
- New control methodologies, e.g. new controller to mitigate the high rate of change of frequency in low inertia systems;
- A power converter is a fully actuated, modular, and very fast control system, which are nearly antipodal characteristics to those of a synchronous machine. Thus, one should critically reflect the control of a converter as a virtual synchronous machine; and
- The lack of inertia in a power system does not need to (and cannot) be fixed by simply "adding inertia back" in the systems.

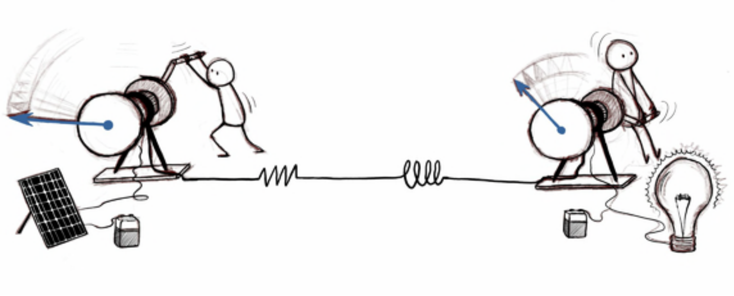
a key unresolved challenge: control of power converters in low-inertia grids

→ industry is willing to explore **green-field approach** (see MIGRATE project)

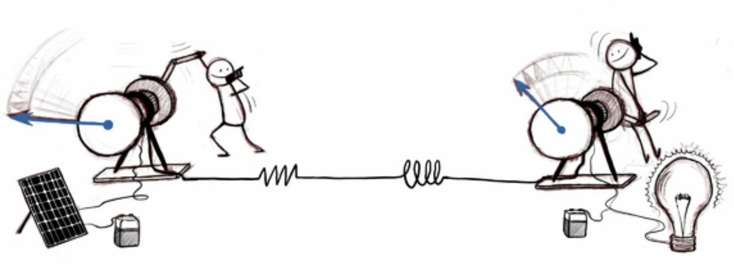
[Milano, Dörfler, Hug, Hill, & Verbic, PSCC' 18]

Cartoon summary of today’s approach

Conceptually, inverters are oscillators that have to synchronize

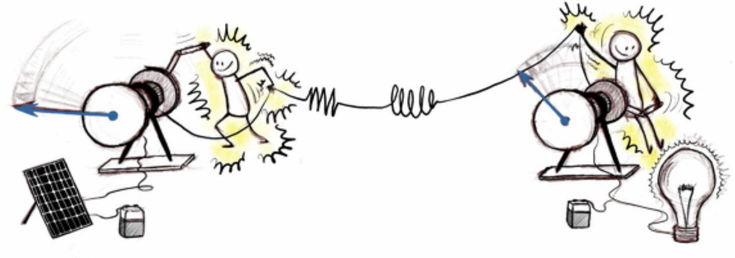


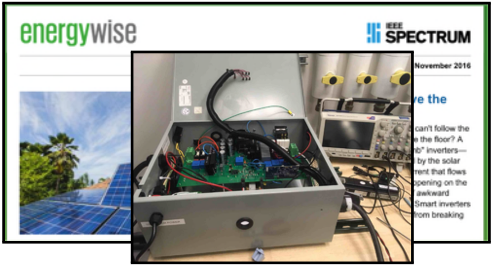
Hypothetically, they could sync by communication (not feasible)



Cartoon summary of today’s approach

Colorful idea: inverters sync through physics & clever local control





energywise

IEEE SPECTRUM

November 2016

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can't follow the
e the floor? A
nb" inverters—
l by the solar
rrent that flows
pening on the
' awkward
Smart inverters
from breaking

theory: sync of coupled oscillators & nonlinear decentralized control

power systems/electronics ***experiments*** @NREL show superior performance

Outline

Introduction: Low-Inertia Power Systems

Problem Setup: Modeling and Specifications

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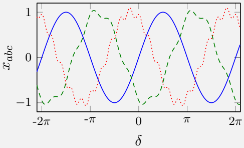
Modeling: signal space in 3-phase AC circuits

three-phase AC

$$\begin{bmatrix} x_a(t) \\ x_b(t) \\ x_c(t) \end{bmatrix} = \begin{bmatrix} x_a(t+T) \\ x_b(t+T) \\ x_c(t+T) \end{bmatrix}$$

periodic with 0 average

$$\frac{1}{T}\int_0^T x_i(t)dt = 0$$

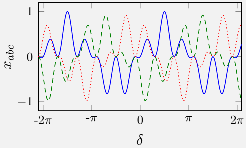


balanced (nearly true)

$$= A(t)\begin{bmatrix} \sin(\delta(t)) \\ \sin(\delta(t) - \frac{2\pi}{3}) \\ \sin(\delta(t) + \frac{2\pi}{3}) \end{bmatrix}$$

so that

$$x_a(t) + x_b(t) + x_c(t) = 0$$

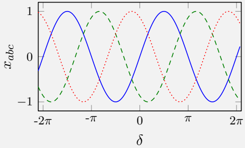


synchronous (desired)

$$= A\begin{bmatrix} \sin(\delta_0 + \omega_0 t) \\ \sin(\delta_0 + \omega_0 t - \frac{2\pi}{3}) \\ \sin(\delta_0 + \omega_0 t + \frac{2\pi}{3}) \end{bmatrix}$$

const. freq & amp

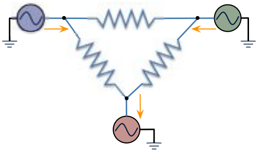
$\Rightarrow$ const. in rot. frame



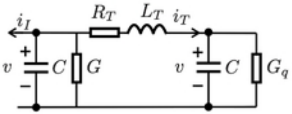
assumption : balanced $\Rightarrow$ 2d-coordinates $x(t) = [x_\alpha(t)\, x_\beta(t)]$ or $x(t) = A(t)\mathrm{e}^{\mathrm{i}\delta(t)}$

from currents/voltages to powers : active $p = v^\top i$ and reactive $q = v^T \underbrace{R(\tfrac{\pi}{2})}_{90^\circ \text{ rotation}} i$

Modeling: the network



interconnecting lines via Π-models & ODEs



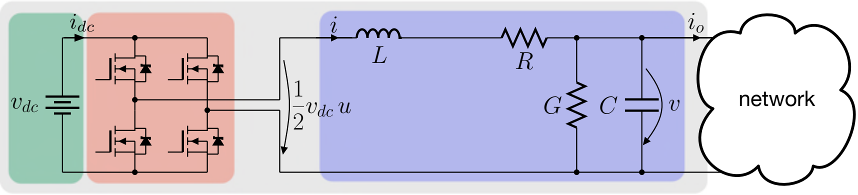
- quasi-steady state ***algebraic model*** $\sim$ diffusive (synchronizing) coupling

$$\underbrace{\begin{bmatrix} i_1 \\ \vdots \\ i_n \end{bmatrix}}_{\text{nodal injections}} = \underbrace{\begin{bmatrix} \vdots & \ddots & \vdots & \iddots & \vdots \\ -y_{k1} I_2 & \cdots & \sum_{j=1}^{n} y_{kj} I_2 & \cdots & -y_{kn} I_2 \\ \vdots & \iddots & \vdots & \ddots & \vdots \end{bmatrix}}_{\text{Laplacian } \otimes I_2 \text{ with } y_{kj} = 1 \text{ / complex impedance}} \underbrace{\begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}}_{\text{nodal potentials}}$$

- salient feature: ***local*** measurement reveal ***global*** information

$$\underbrace{i_k}_{\text{local variable}} = \underbrace{\sum\nolimits_j y_{kj} \left(v_k - v_j\right)}_{\text{global information}}$$

Modeling: the power converter



DC port modulation control (3-phase) LC output filter AC port to power grid

- passive ***DC port*** port (i_{dc}, v_{dc}) for energy balance control

→ details neglected today: assume v_{dc} to be stiffly regulated

- ***modulation*** $\equiv$ lossless signal transformer (averaged)

→ controlled switching voltage $\frac{1}{2}v_{dc}u$ with $u \in [-1, 1]$

- ***LC filter*** to smoothen harmonics with R, G modeling filter/switching losses

well actuated, modular, & fast control system ≈ ***controllable voltage source***

Control objectives in the stationary frame

1. ***synchronous frequency:***

$$\frac{d}{dt}\,v_k = \begin{bmatrix} 0 & -\omega_0 \\ \omega_0 & 0 \end{bmatrix} v_k \quad \forall\, k \in \mathcal{V} := \{1, \ldots, N\}$$

$\sim$ stabilization at **harmonic oscillation** with **synchronous frequency** ω_0

2. ***voltage amplitude:***

$$\|v_k\| = v^\star \quad \forall\, k \in \mathcal{V} \quad \text{(for ease of presentation)}$$

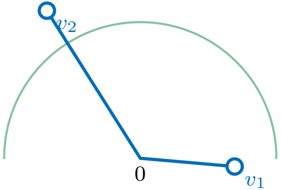
$\sim$ stabilization of voltage **amplitude** $\|v_k\|$

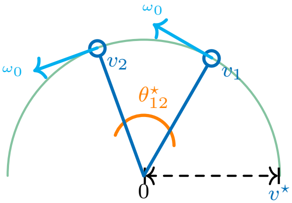
3. ***prescribed power flow:***

$$v_k^\top\, i_{o,k} = p_k^\star\,, \quad v_k^\top \underbrace{R(\tfrac{\pi}{2})}_{90^\circ\,\text{rotation}} i_{o,k} = q_k^\star \quad \forall\, k \in \mathcal{V}$$

$\sim$ steady-state **active & reactive power** injections $\{p_k^\star, q_k^\star\}$

Main control challenges





- ϟ ***nonlinear objectives*** $(v_k^{\star}, p_k^{\star}, q_k^{\star})$ & stabilization of a ***limit cycle***
- ϟ ***decentralized control:*** only local measurements $(v_k, i_{o,k})$ available
- ϟ ***time-scale separation*** between slow sources & fast network may not hold
- **+** ***fully controllable*** voltage sources & stable ***linear network dynamics***

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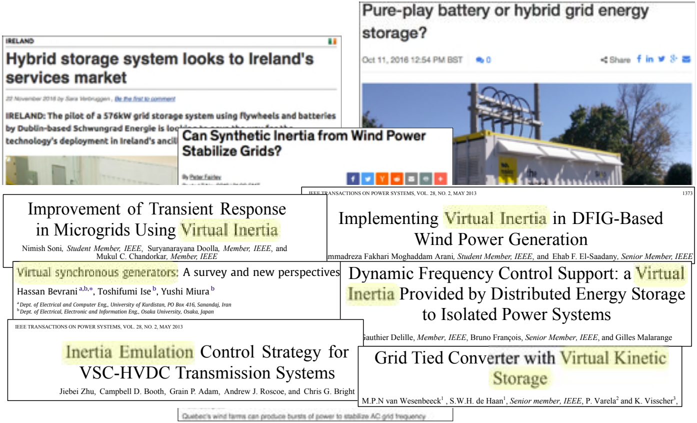
Baseline: virtual synchronous machine emulation



IRELAND

Hybrid storage system looks to Ireland's services market

22 November 2016 by Sara Verbruggen , Be the first to comment

IRELAND: The pilot of a 576kW grid storage system using flywheels and batteries by Dublin-based Schwungrad Energie is loo

technology's deployment in Ireland's ancil

Pure-play battery or hybrid grid energy storage?

Oct 11, 2016 12:54 PM BST 0 Share

Can Synthetic Inertia from Wind Power Stabilize Grids?

By Peter Fairley

IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 28, NO. 2, MAY 2013 1373

Implementing Virtual Inertia in DFIG-Based Wind Power Generation

mmadreza Fakhari Moghaddam Arani, *Student Member, IEEE*, and Ehab F. El-Saadany, *Senior Member, IEEE*

Improvement of Transient Response in Microgrids Using Virtual Inertia

Nimish Soni, *Student Member, IEEE*, Suryanarayana Doolla, *Member, IEEE*, and Mukul C. Chandorkar, *Member, IEEE*

Virtual synchronous generators: A survey and new perspectives

Hassan Bevrani[a,b,*], Toshifumi Ise[b], Yushi Miura[b]

[a] Dept. of Electrical and Computer Eng., University of Kurdistan, PO Box 416, Sanandaj, Iran
[b] Dept. of Electrical, Electronic and Information Eng., Osaka University, Osaka, Japan

Dynamic Frequency Control Support: a Virtual Inertia Provided by Distributed Energy Storage to Isolated Power Systems

iauthier Delille, *Member, IEEE*, Bruno François, *Senior Member, IEEE*, and Gilles Malarange

IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 28, NO. 2, MAY 2013

Inertia Emulation Control Strategy for VSC-HVDC Transmission Systems

Jiebei Zhu, Campbell D. Booth, Grain P. Adam, Andrew J. Roscoe, and Chris G. Bright

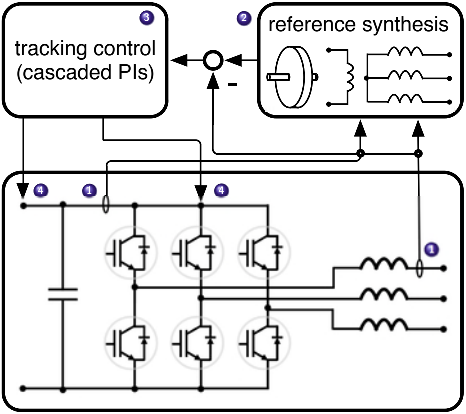
Grid Tied Converter with Virtual Kinetic Storage

M.P.N van Wesenbeeck[1] , S.W.H. de Haan[1], *Senior member, IEEE*, P. Varela[2] and K. Visscher[3],

Quebec's wind farms can produce bursts of power to stabilize AC grid frequency

- ***PD control*** on $\omega(t)$: $\boldsymbol{M}\,\frac{d}{dt}\,\omega(t) + \boldsymbol{D}\,(\omega(t) - \omega_0) \;=\; P_{\text{generation}}(t) - P_{\text{demand}}(t)$
- there are ***smarter implementations*** at the cost of algorithmic complexity

Standard power electronics control approach to virtual machine emulation would continue by



1. acquiring & processing of ***AC measurements***
2. synthesis of ***references*** (voltage/current/power)

   "how would a synchronous generator respond now ?"
3. ***track*** error signals at converter terminals
4. ***actuation*** via modulation and DC-side supply

Droop as simplest reference model

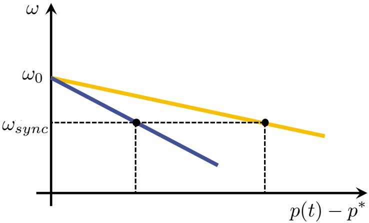
[Chandorkar, Divan, Adapa, '93]

- ***frequency control*** by mimicking $p - \omega$ droop property of synchronous machine:

$$D\left(\omega - \omega_0\right) \;=\; p - p^\star$$

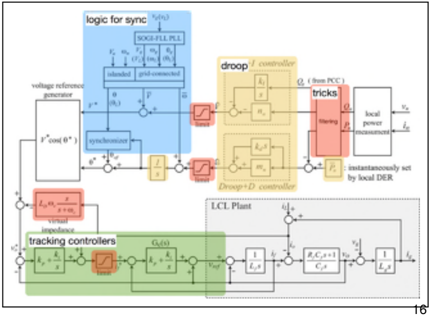
- ***voltage control*** via $q - \|v\|$ droop heuristic:

$$\tfrac{d}{dt}\|v\| = -c_1(\|v\| - v^\star) - c_2(q - q^\star)$$

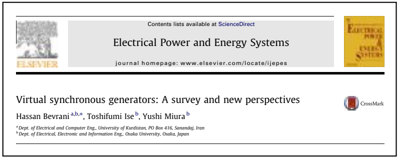


→ direct control of (p, ω) and (q, v) **assuming they are independent** (true only near steady state)

→ requires **tricks in implementation** : low-pass filters for dissipation, virtual impedances for saturation, limiters,…



Challenges in power converter implementations



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Virtual synchronous generators: A survey and new perspectives

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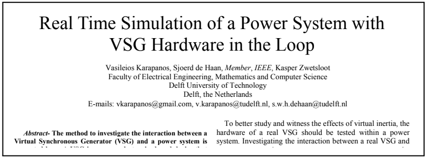

Real Time Simulation of a Power System with VSG Hardware in the Loop

Vasileios Karapanos, Sjoerd de Haan, *Member*, *IEEE*, Kasper Zwetsloot
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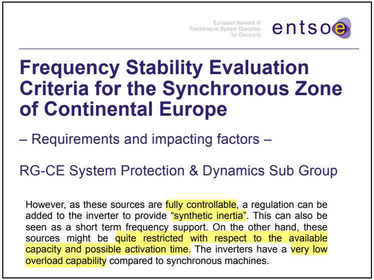
***Abstract*- The method to investigate the interaction between a Virtual Synchronous Generator (VSG) and a power system is**

To better study and witness the effects of virtual inertia, the hardware of a real VSG should be tested within a power system. Investigating the interaction between a real VSG and

1. ***delays*** in measurement acquisition, signal processing, & actuation
2. ***constraints*** on currents & voltages
3. ***performance*** improvement via “tricks”
4. ***certificates*** on stability & robustness



European Network of
Transmission System Operators
for Electricity
entsoe

Frequency Stability Evaluation Criteria for the Synchronous Zone of Continental Europe

– Requirements and impacting factors –

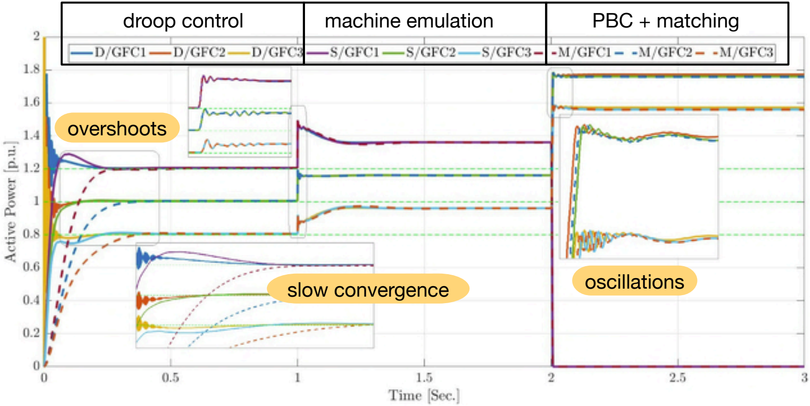
RG-CE System Protection & Dynamics Sub Group

However, as these sources are fully controllable, a regulation can be added to the inverter to provide “synthetic inertia”. This can also be seen as a short term frequency support. On the other hand, these sources might be quite restricted with respect to the available capacity and possible activation time. The inverters have a very low overload capability compared to synchronous machines.

→ proper implementation (internal model + matching + PBC) alleviates some issues

[Jouini, Arghir, & Dörfler, Automatica ’17]

Comparison of droop/emulation/matching @AIT



droop control

machine emulation

PBC + matching

overshoots

slow convergence

oscillations

[Tayebi, Dörfler, Kupzog, Miletic, & Hribernik, CIRED '18]

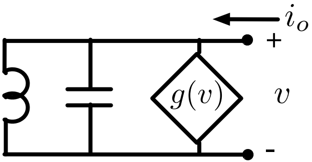
- all controllers perform fine near steady-state and under nominal conditions
- all show poor transient performance unless augmented with various "tricks"

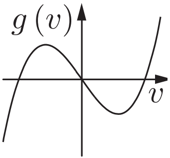
→ none appears suitable for post-fault stabilization in a low-inertia power system

Virtual Oscillator Control (VOC)

nonlinear & open limit cycle oscillator as reference model for terminal voltage (1-phase):

$$\ddot{v} + \omega_0^2 v + g(v) = i_o$$





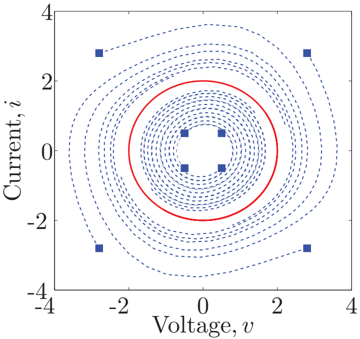
- history: [Torres, Hespanha, Moehlis, '11], [Johnson, Dhople, Krein, '13], [Dhople, Johnson, Dörfler, Hamadeh, '14], [Kim, Persis, '17]

- simplified model amenable to theoretic analysis

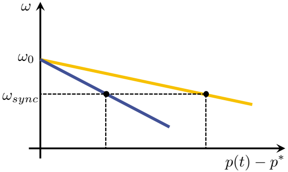
→ ***almost global synchronization*** & ***local droop***

- in practice proven to be ***robust mechanism*** with performance superior to droop & others

→ ***problem***: cannot be controlled(?) to meet specifications on amplitude & power injections

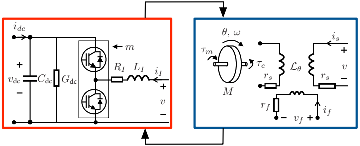


Comparison of grid-forming control strategies



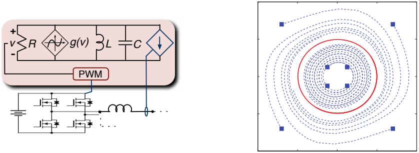
droop control

+ good performance near steady state
– relies on decoupling & small attraction basin



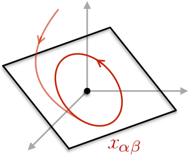
synchronous machine emulation

+ backward compatible in “nominal” case
– poor performance & needs hacks to work



virtual oscillator control (VOC)

+ robust & almost globally stable sync
– cannot meet amplitude/power specifications



today: foundational control approach

[Colombino, Groß, Brouillon, & Dörfler, ’17, ’18]
[Seo, Subotic, Johnson, Colombino, Groß, & Dörfler, ’18]

Outline

Introduction: Low-Inertia Power Systems

Problem Setup: Modeling and Specifications

State of the Art: Comparison & Critical Evaluation

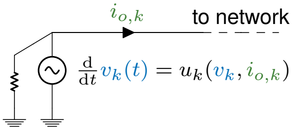
Dispatchable Virtual Oscillator Control

Experimental Validation

Conclusions

Recall problem setup

1. simplifying assumptions (will be removed later)



$$\frac{\mathrm{d}}{\mathrm{d}t} v_k(t) = u_k(v_k, i_{o,k})$$

- converter ≈ controllable voltage source
- grid ≈ quasi-static: $\ell \frac{d}{dt} i + ri \approx \big(\mathrm{j}\, \ell \omega_0 + r\big) i$
- lines ≈ homogeneous $\kappa = \tan(\ell_{kj}/r_{kj})\ \forall k, j$

2. fully decentralized control of converter terminal voltage & current

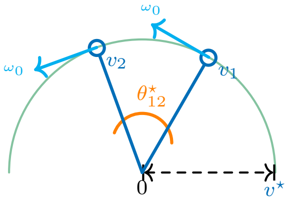
- ⚡ set-points for relative angles $\{\theta^\star_{jk}\}$
- ⚡ nonlocal measurements v_j
- ⚡ grid & load parameters

- ✓ local measurements $(v_k, i_{o,k})$
- ✓ local set-points $(v^\star_k, p^\star_k, q^\star_k)$

3. control objective

stabilize desired quasi steady state

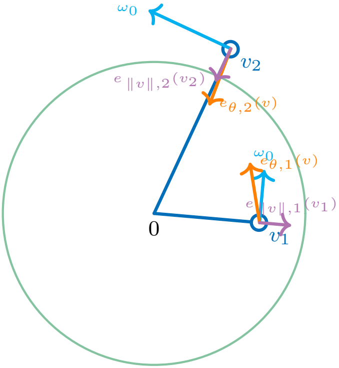
(synchronous, 3-phase-balanced,
and meet set-points in nominal case)



Colorful idea for closed-loop target dynamics

objectives: frequency, phase, and voltage stability

$$\frac{d}{dt}v_k = \underbrace{\begin{bmatrix}0 & -\omega_0\\ \omega_0 & 0\end{bmatrix} v_k}_{\text{rotation at }\omega_0} + c_1 \cdot \underbrace{e_{\theta,k}(v)}_{\text{synchronization}} + c_2 \cdot \underbrace{e_{\|v\|,k}(v_k)}_{\text{magnitude regulation}}$$



synchronization:

$$e_{\theta,k}(v) = \sum_{j=1}^{n} w_{jk}\left(v_j - R(\theta^{\star}_{jk})v_k\right)$$

amplitude regulation:

$$e_{\|v\|,k}(v_k) = \left(v^{\star 2} - \|v_k\|^2\right) v_k$$

Decentralized implementation of target dynamics

$$e_{\theta,k}(v) = \underbrace{\sum_j w_{jk}(v_j - R(\theta^\star_{jk})v_k)}_{\text{need to know } w_{jk},\, v_j,\, v_k \text{ and } \theta^\star_{jk}} = \underbrace{\sum_j w_{jk}(v_j - v_k)}_{\text{“Laplacian” feedback}} + \underbrace{\sum_j w_{jk}(I - R(\theta^\star_{jk}))v_k}_{\text{local feedback: } \mathcal{K}_k(\theta^\star)v_k}$$

insight I: non-local measurements from ***communication through physics***

$$\underbrace{R(\kappa)\, i_{o,k}}_{\text{local feedback } (\kappa = \ell/r \text{ uniform})} = \underbrace{\sum_j \|y_{jk}\|(v_j - v_k)}_{\text{distributed Laplacian feedback with } w_{jk} = \|y_{kj}\|}$$

insight II: angle set-points & line-parameters from ***power flow equations***

$$\left.\begin{aligned} p^\star_k &= v^{\star 2} \sum_j \frac{r_{jk}(1-\cos(\theta^\star_{jk})) - \omega_0 \ell_{jk} \sin(\theta^\star_{jk})}{r^2_{jk} + \omega_0^2 \ell^2_{jk}} \\ q^\star_k &= -v^{\star 2} \sum_j \frac{\omega_0 \ell_{jk}(1-\cos(\theta^\star_{jk})) + r_{jk} \sin(\theta^\star_{jk})}{r^2_{jk} + \omega_0^2 \ell^2_{jk}} \end{aligned}\right\} \Rightarrow \underbrace{\mathcal{K}_k(\theta^\star)}_{\text{global parameters}} = \underbrace{\frac{1}{v^{\star 2}} R(\kappa) \begin{bmatrix} q^\star_k & p^\star_k \\ -p^\star_k & q^\star_k \end{bmatrix}}_{\text{local parameters}}$$

Main results

1. desired target dynamics can be realized via ***fully decentralized control***:

$$\frac{d}{dt}v_k = \underbrace{\begin{bmatrix} 0 & -\omega_0 \\ \omega_0 & 0 \end{bmatrix} v_k}_{\text{rotation at } \omega_0} + c_1 \cdot \underbrace{\sum_{j=1}^{n} w_{jk}\left(v_j - R(\theta^{\star}_{jk})v_k\right)}_{\text{synchronization with global knowledge}} + c_2 \cdot \underbrace{\left({v^{\star}}^2 - \|v_k\|^2\right) v_k}_{\text{local amplitude regulation}}$$

$$= \underbrace{\begin{bmatrix} 0 & -\omega_0 \\ \omega_0 & 0 \end{bmatrix} v_k}_{\text{rotation at } \omega_0} + c_1 \cdot \underbrace{R\left(\kappa\right)\left(\frac{1}{{v^{\star}}^2}\begin{bmatrix} q_k^{\star} & p_k^{\star} \\ -p_k^{\star} & q_k^{\star} \end{bmatrix} v_k - i_{o,k}\right)}_{\text{synchronization through physics}} + c_2 \cdot \underbrace{\left({v^{\star}}^2 - \|v_k\|^2\right) v_k}_{\text{local amplitude regulation}}$$

2. ***almost global stability*** result:

If the $\boxed{\dots}$ condition holds, the system is **almost globally asymptotically stable** with respect to a **limit cycle** corresponding to a **pre-specified** solution of the **AC power-flow** equations at a **synchronous** frequency ω_0.

Main results cont'd

3. certifiable, sharp, and intuitive ***stability conditions***:

- consistent $v^\star$, $p_k^\star$, and $q_k^\star$ satisfy **AC power flow equations**
- **magnitude control** slower than **synchronization control**
- **power transfer** "small enough" compared to **network connectivity**

e.g., for resistive grid: $\frac{1}{2}\lambda_2(L) > \max_k \sum_{j=1}^{n} \frac{1}{v^{\star 2}} |p_{j,k}| + c_2\, v^\star$

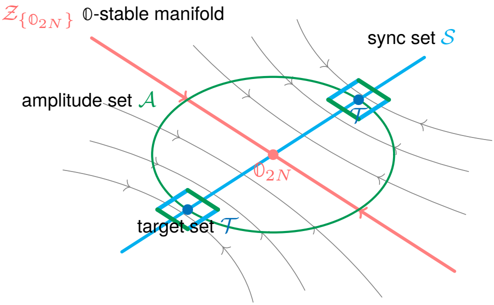
4. connection to ***droop control*** revealed in polar coordinates (for inductive grid):

$$\frac{d}{dt}\theta_k = \omega_0 + c_1 \left(\frac{p_k^\star}{v^{\star 2}} - \frac{p_k}{\|v_k\|^2} \right) \underset{\|v_k\|\approx 1}{\approx} \omega_0 + c_1 \left(p_k^\star - p_k \right) \qquad (p-\omega \text{ droop})$$

$$\frac{d}{dt}\|v_k\| \underset{\|v_k\|\approx 1}{\approx} c_1\, (q_k^\star - q_k) + c_2\, (v^\star - \|v_k\|) \qquad (q - \|v\| \text{ droop})$$

Proof sketch for algebraic grid: Lyapunov & center manifold

Lyapunov function: $V(v) = \frac{1}{2}\text{dist}(v, \mathcal{S})^2 + \frac{c_2}{v^{\star 2}} \sum_k \left(v^{\star 2} - \|v_k\|^2\right)^2$



$\mathcal{T} \cup \mathbb{0}_{2N}$ is globally attractive

$$\lim_{t\to\infty} \|v(t)\|_{\mathcal{T}\cup\mathbb{0}_{2N}} = 0$$

$\mathcal{T}$ is stable

$$\|v(t)\|_{\mathcal{T}} \leq \chi_2(\|v_0\|_{\mathcal{T}})$$

$\mathcal{T}$ is almost globally attractive

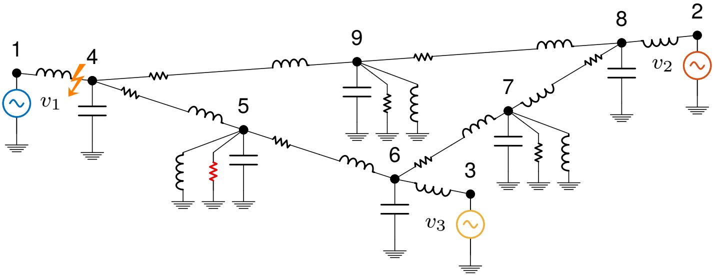
$\mathbb{0}_{2N}$ exponentially unstable

$\Longrightarrow$ $\mathcal{Z}_{\{\mathbb{0}_{2N}\}}$ has measure zero

$$\forall v_0 \notin \mathcal{Z}_{\{\mathbb{0}_{2N}\}} : \lim_{t\to\infty} \|v(t)\|_{\mathcal{T}} = 0$$

stability & almost global attractivity $\Longrightarrow$ ***almost global asymptotic stability***

Case study: IEEE 9 Bus system



t = 0 s: black start of three inverters

- **initial state:** $\|v_k(0)\| \approx 10^{-3}$
- **convergence** to **set-point**

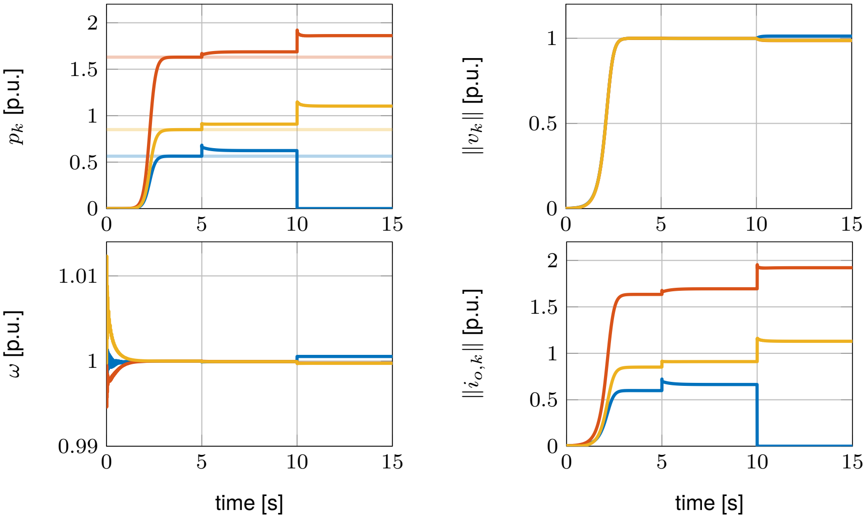
t = 5 s: load step-up

- 20% **load increase** at bus 5
- consistent **power sharing**

t = 10 s: loss of inverter 1

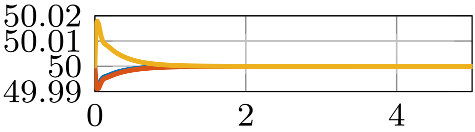
- the **remaining** inverters **synchronize**
- they **supply** the load **sharing power**

Simulation of IEEE 9 Bus system

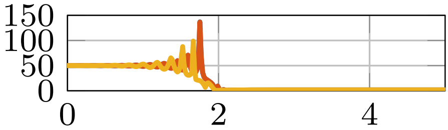


Dropping assumptions: dynamic lines

control gains $\sim 1.8 \cdot 10^{-4}$



control gains $\sim 1.8 \cdot 10^{-3}$



observations

- inverter control **interferes** with the line dynamics
- controller needs to be artificially **slowed down**
- recognized problem

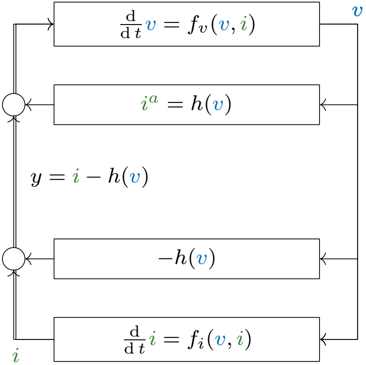
  [Vorobev, Huang, Hosaini, & Turitsyn,'17]

"networked control" reason

- **communication through currents** to infer voltages
- very inductive lines **delay** the information transfer
- the controller must be **slow** in very inductive networks

re-do the math leading to updated condition:
magnitude control slower than **sync control**
slower than line dynamics

Proof sketch for dynamic grid: perturbation-inspired Lyapunov



Individual Lyapunov functions

- slow system: $V(v)$ for $\frac{\mathrm{d}}{\mathrm{d}t}v = f_v(v, h(v))$
- fast system: $W(y)$ for $\frac{\mathrm{d}}{\mathrm{d}t}y = f_i(v, y + h(v))$
  where $\frac{\mathrm{d}}{\mathrm{d}t}v = 0$ & coordinate $y = i - h(v)$

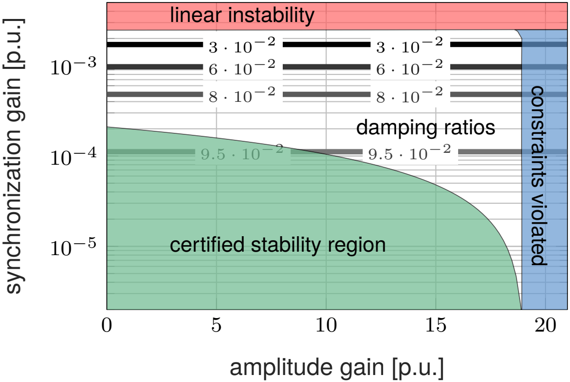
Lyapunov function for the full system

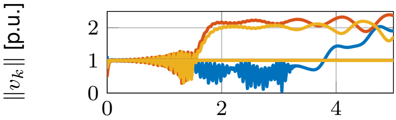
- $\nu(x) = dW(i - h(v)) + (1-d)V(v)$
  where $d \in [0,1]$ is free convex coefficient
- $\frac{\mathrm{d}}{\mathrm{d}t}\nu(x)$ is decaying under stability condition

Almost global asymptotic stability

- $\mathcal{T}' \cup \{\mathbb{0}_n\}$ globally attractive & $\mathcal{T}'$ stable
- $\mathcal{Z}_{\{\mathbb{0}_n\}}$ has measure zero

Evaluation of stability conditions





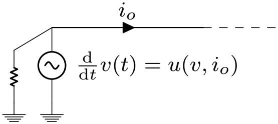
increase of control gains by factor 10

⇒ oscillations, overshoots, & instability

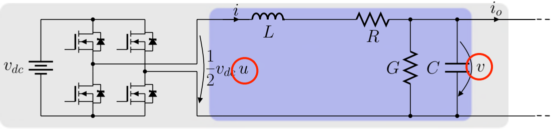
⇒ ***conditions are highly accurate***

Dropping assumptions: detailed converter model

voltage source model:



detailed converter model with LC filter:



- ***idea:*** **invert** LC filter so that $v \approx \frac{1}{2}v_{dc}u$

→ ***control:*** perform robust inversion of LC filter via **cascaded PI**

- ***analysis:*** repeat proof via **singular perturbation Lyapunov** functions

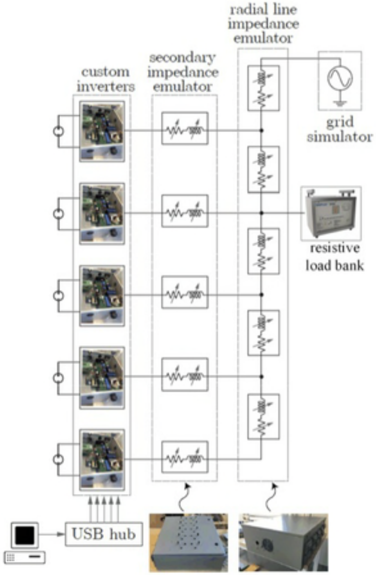
→ ***almost global stability*** for sufficient time scale separation (quantifiable)

$$\boxed{\text{VOC model} < \text{line dynamics} < \text{voltage PI} < \text{current PI}}$$

[Subotic, ETH Zürich Master thesis '18]

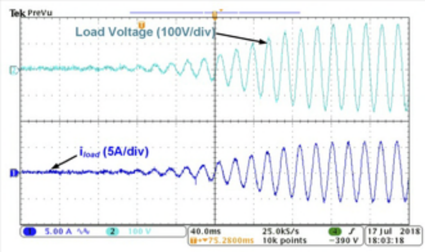
- . . . similar steps for control of v_{dc} in a more detailed model

Experimental setup @ NREL

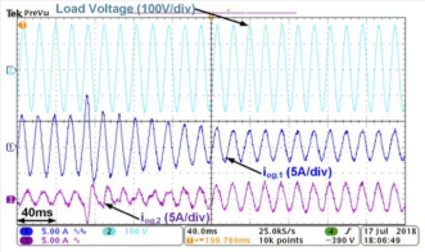


Experimental results

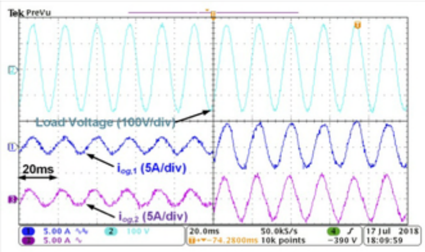
[Seo, Subotic, Johnson, Colombino, Groß, & Dörfler, APEC'18]



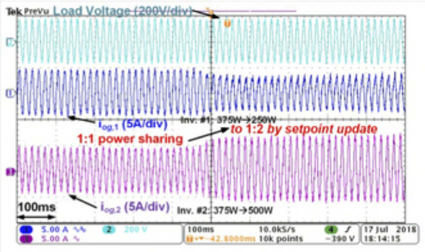
black start of inverter #1 under 500 W load (making use of almost global stability)



connecting inverter #2 while inverter #1 is regulating the grid under 500 W load



250 W to 750 W load transient with two inverters active



change of setpoint: $p^\star$ of inverter #2 updated from 250 W to 500 W

Conclusions

Summary

- challenges of low-inertia systems
- dispatchable virtual oscillator control
- theoretic analysis & experiments

Marcello Colombino

Ongoing & future work

- theoretical questions: robustness & regulation
- practical issue: compatibility with legacy system
- experimental validations @ ETH, NREL, AIT

Dominic Groß

Main references

D. Groß, M Colombino, J.S. Brouillon, & F. Dörfler. *The effect of transmission-line dynamics on grid-forming dispatchable virtual oscillator control*.

M. Colombino, D. Groß, J.S. Brouillon, & F. Dörfler. *Global phase and magnitude synchronization of coupled oscillators with application to the control of grid-forming power inverters*.

POWER IS NOTHING WITHOUT CONTROL