



Control of Power Converters in Low-Inertia Power Systems

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Acknowledgements



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FONDS NATIONAL SUISSE
SCHWEIZERISCHER NATIONALFONDS
FONDO NAZIONALE SVIZZERO
SWISS NATIONAL SCIENCE FOUNDATION

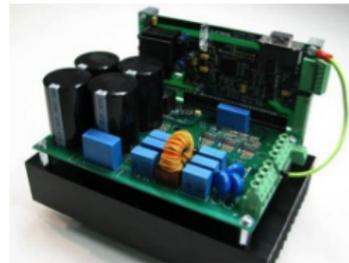


Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zürich



Further: Gab-Su Seo, Brian Johnson, Mohit Sinha, & Sairaj Dhople

Replacing the power system foundation



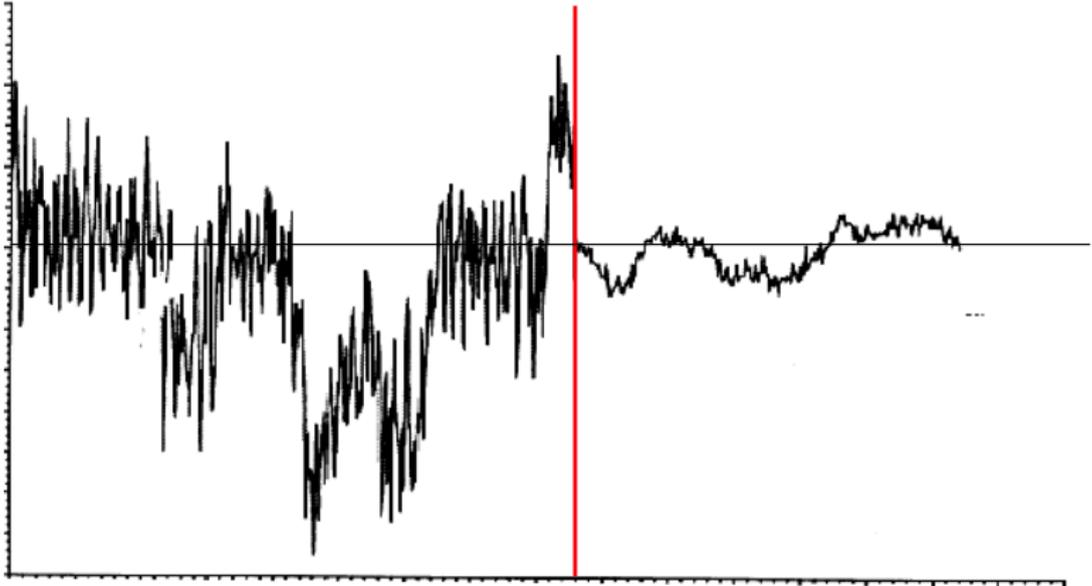
fuel & synchronous machines

- not sustainable
- + central & dispatchable generation
- + large rotational inertia as buffer
- + self-synchronize through the grid
- + resilient voltage / frequency control
- slow actuation & control

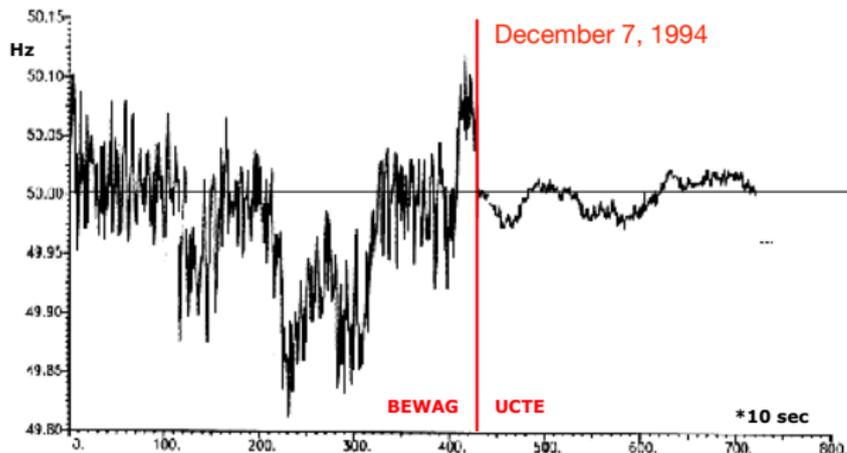
renewables & power electronics

- + sustainable
- distributed & variable generation
- almost no energy storage
- no inherent self-synchronization
- fragile voltage / frequency control
- + fast / flexible / modular control

What do we see here ?



Frequency of West Berlin re-connecting to Europe



before re-connection: islanded operation based on batteries & single boiler

afterwards connected to European grid based on synchronous generation

The concerns are not hypothetical

issues broadly recognized by TSOs, device manufacturers, academia, agencies, etc.

UPDATE REPORT –
BLACK SYSTEM EVENT
IN SOUTH AUSTRALIA ON
28 SEPTEMBER 2016



AN UPDATE TO THE PRELIMINARY OPERATING INCIDENT
REPORT FOR THE NATIONAL ELECTRICITY MARKET.
DATA ANALYSIS AS AT 5.00 PM TUESDAY 11 OCTOBER 2016.

lack of robust control:

*“Nine of the 13 wind farms
online did not ride through the
six voltage disturbances
experienced during the event.”*

between the lines:

conventional system would
have been more resilient (?)

obstacle to sustainability:
power electronics integration

MIGRATE project:

Massive InteGRATION of power Electronic devices

INDEPENDENT

News > World > Australasia

Tesla's new mega-battery in Australia reacts to outages in 'record' time

One of Australia's biggest power plants suffered a drop in
output - the new battery kicked in just 0.14 seconds later

ERCOT CONCEPT PAPER

Future

ERCOT

that would

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Critically re-visit modeling/analysis/control

Foundations and Challenges of Low-Inertia Systems

(Invited Paper)

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The later sections contain many suggestions for further work, which can be summarized as follows:

- **New models** are needed which balance the need to include key features without burdening the model (whether for analytical or computational work) with uneven and excessive detail;
- **New stability theory** which properly reflects the new devices and time-scales associated with CIG, new loads and use of storage;
- Further **computational work** to achieve sensitivity guidelines including data-based approaches;
- **New control methodologies**, e.g. new controller to mitigate the high rate of change of frequency in low inertia systems;
- A power converter is a fully actuated, modular, and very fast control system, which are nearly antipodal characteristics to those of a synchronous machine. Thus, **one should critically reflect the control** of a converter as a virtual synchronous machine; and
- The lack of inertia in a power system does not need to (and **cannot**) be fixed by simply "adding inertia back" in the systems.

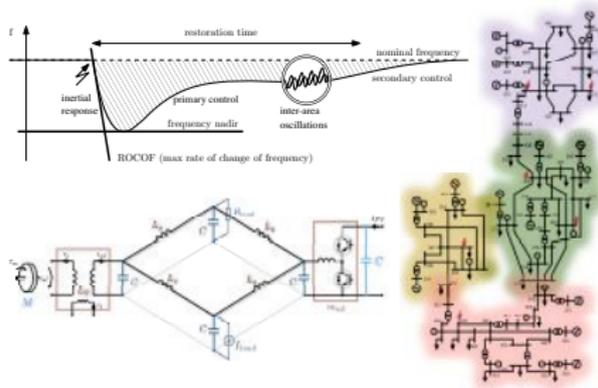
a key unresolved challenge: control of power converters in low-inertia grids

→ industry & power community willing to explore **green-field approach** (see MIGRATE) with **advanced control** methods & **theoretical certificates**

Our research agenda

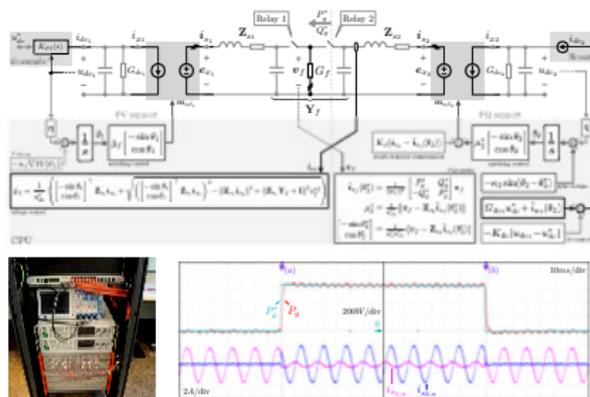
system-level

- low-inertia power system **models**, **stability**, & **performance** metrics
- **optimal allocation** of virtual inertia & fast-frequency response services

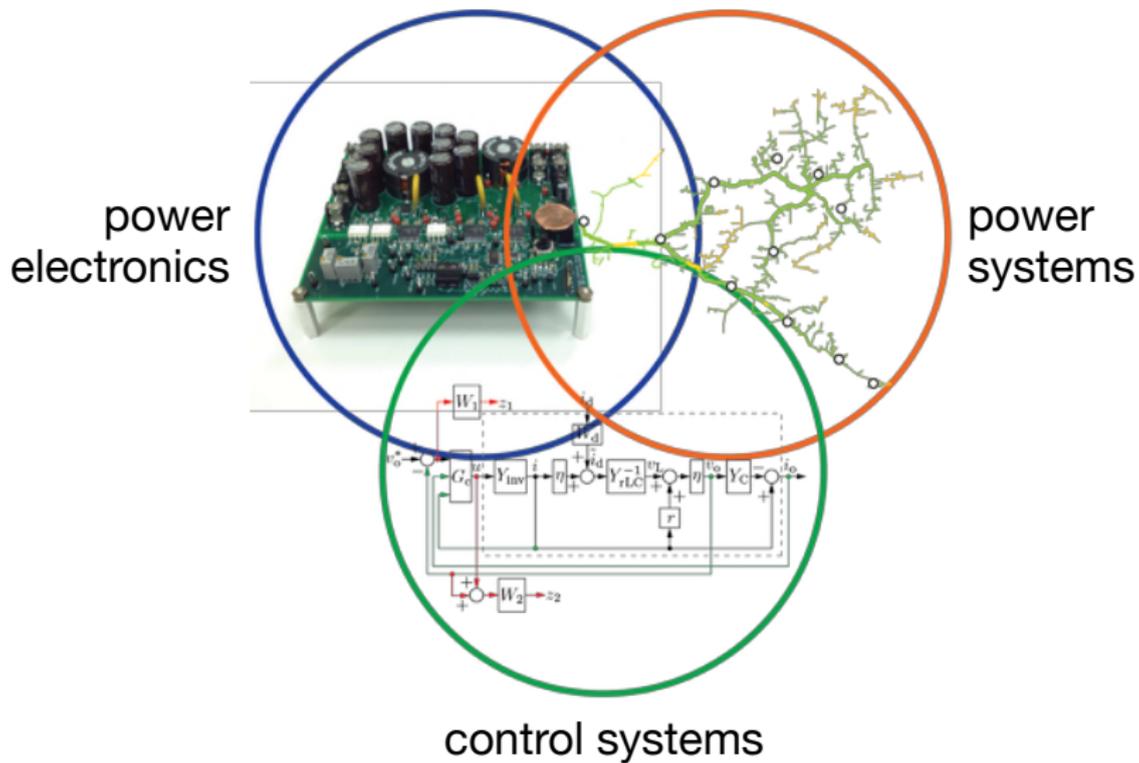


device-level (today)

- decentralized nonlinear **power converter control** strategies
- experimental **implementation**, cross-validation, & comparison



Exciting research domain bridging communities



Outline

Introduction: Low-Inertia Power Systems

Problem Setup: Modeling and Specifications

State of the Art: Comparison & Critical Evaluation

Dispatchable Virtual Oscillator Control

Experimental Validation

Conclusions

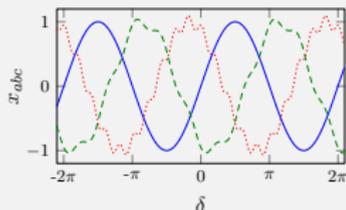
Modeling: signal space in 3-phase AC circuits

three-phase AC

$$\begin{bmatrix} x_a(t) \\ x_b(t) \\ x_c(t) \end{bmatrix} = \begin{bmatrix} x_a(t+T) \\ x_b(t+T) \\ x_c(t+T) \end{bmatrix}$$

periodic with 0 average

$$\frac{1}{T} \int_0^T x_i(t) dt = 0$$

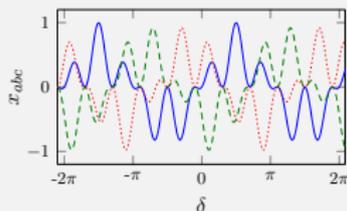


balanced (nearly true)

$$= A(t) \begin{bmatrix} \sin(\delta(t)) \\ \sin(\delta(t) - \frac{2\pi}{3}) \\ \sin(\delta(t) + \frac{2\pi}{3}) \end{bmatrix}$$

so that

$$x_a(t) + x_b(t) + x_c(t) = 0$$

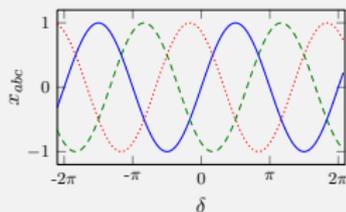


synchronous (desired)

$$= A \begin{bmatrix} \sin(\delta_0 + \omega_0 t) \\ \sin(\delta_0 + \omega_0 t - \frac{2\pi}{3}) \\ \sin(\delta_0 + \omega_0 t + \frac{2\pi}{3}) \end{bmatrix}$$

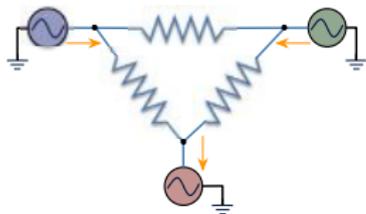
const. freq & amp

\Rightarrow const. in rot. frame

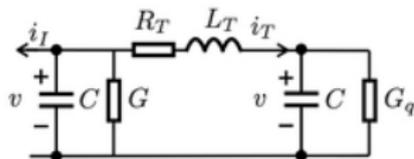


assumption: balanced \Rightarrow 2d-coordinates $x(t) = [x_\alpha(t) \ x_\beta(t)]$ or $x(t) = A(t)e^{i\delta(t)}$

Modeling: the network



interconnecting lines via Π -models & ODEs



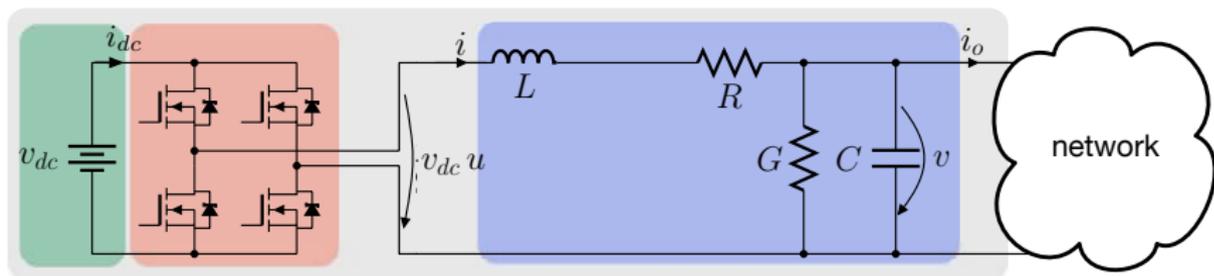
- quasi-steady state *algebraic model*

$$\underbrace{\begin{bmatrix} i_1 \\ \vdots \\ i_n \end{bmatrix}}_{\text{nodal injections}} = \underbrace{\begin{bmatrix} \vdots & \ddots & \vdots & \ddots & \vdots \\ -y_{k1} & \cdots & \sum_{j=1}^n y_{kj} & \cdots & -y_{kn} \\ \vdots & \ddots & \vdots & \ddots & \vdots \end{bmatrix}}_{\text{Laplacian matrix with } y_{kj} = 1 / \text{complex impedance}} \underbrace{\begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}}_{\text{nodal potentials}}$$

- salient feature: *local* measurement reveal *global* information

$$\underbrace{i_k}_{\text{local variable}} = \underbrace{\sum_j y_{kj} (v_k - v_j)}_{\text{global information}}$$

Modeling: the power converter



DC port modulation control (3-phase) LC output filter AC port to power grid

- ▶ passive **DC port** port (i_{dc}, v_{dc}) for energy balance control
→ details neglected today: assume v_{dc} to be stiffly regulated
- ▶ **modulation** \equiv lossless signal transformer (averaged)
→ controlled switching voltage $v_{dc}u$ with $u \in [-\frac{1}{2}, +\frac{1}{2}] \times [-\frac{1}{2}, +\frac{1}{2}]$
- ▶ **LC filter** to smoothen harmonics with R, G modeling filter/switching losses

well actuated, modular, & fast control system \approx **controllable voltage source**

Control objectives in the stationary frame

1. *synchronous frequency:*

$$\frac{d}{dt} v_k = \begin{bmatrix} 0 & -\omega_0 \\ \omega_0 & 0 \end{bmatrix} v_k \quad \forall k \in \mathcal{V} := \{1, \dots, N\}$$

~ stabilization at **harmonic oscillation** with **synchronous frequency** ω_0

2. *voltage amplitude:*

$$\|v_k\| = v^* \quad \forall k \in \mathcal{V} \quad (\text{for ease of presentation})$$

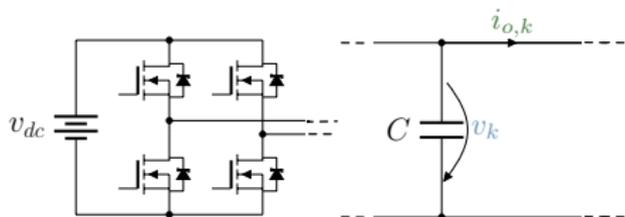
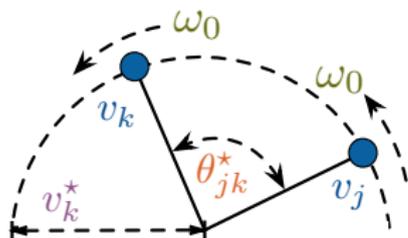
~ stabilization of voltage **amplitude** $\|v_k\|$

3. *prescribed power flow:*

$$v_k^\top i_{o,k} = p_k^*, \quad v_k^\top \begin{bmatrix} 0 & -1 \\ +1 & 0 \end{bmatrix} i_{o,k} = q_k^* \quad \forall k \in \mathcal{V}$$

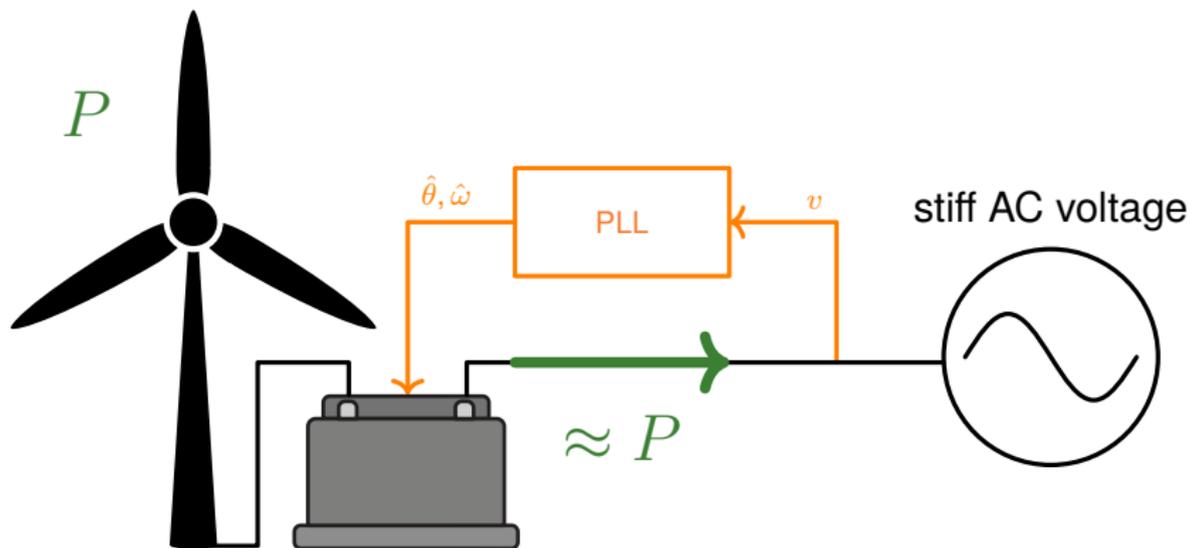
~ steady-state **active & reactive power** injections $\{p_k^*, q_k^*\}$

Main control challenges



- ⚡ **nonlinear objectives** (v_k^*, θ_{kj}^*) & stabilization of a **limit cycle**
- ⚡ **local set-points**: voltage/power (v_k^*, p_k^*, q_k^*) but no relative angles θ_{kj}^*
- ⚡ **decentralized control**: only local measurements $(v_k, i_{o,k})$ available
- ⚡ **converter physics not resilient**: no significant storage & state constraints
- ⚡ **no time-scale separation** between slow sources & fast network
- + **fully controllable** voltage sources & stable **linear network dynamics**

Limitations of grid-following control



- ▶ **is good for** transferring power to a strong grid (what if everyone follows?)
- ▶ **is not good for** providing a voltage reference, stabilization, or black start
- ▶ tomorrow's grid needs **grid-forming control** \equiv *emergence of synchronization*

Naive baseline solution: emulation of virtual inertia

IRELAND

Hybrid storage system looks to Ireland's services market

22 November 2016 by Sara Verbruggen · [Be the first to comment](#)

IRELAND: The pilot of a 576kW grid storage system using flywheels and batteries by Dublin-based Schwungrad Energie is the latest technology's deployment in Ireland's ancillary services market.



IRELAND: The pilot of a 576kW grid storage system using flywheels and batteries by Dublin-based Schwungrad Energie is the latest technology's deployment in Ireland's ancillary services market.

Pure-play battery or hybrid grid energy storage?

Oct 11, 2016 12:54 PM BST

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Can Synthetic Inertia from Wind Power Stabilize Grids?

By Peter Falster
Posted 7 Nov 2016 | 21:00 GMT



Improvement of Transient Response in Microgrids Using Virtual Inertia

Nimish Soni, *Student Member, IEEE*, Suryanarayana Doolla, *Member, IEEE*, and Mukul C. Chandorkar, *Member, IEEE*

Virtual synchronous generators: A survey and new perspectives

Hassan Bevrani^{a,b,v}, Toshifumi Ise^b, Yushi Miura^b

^aDept. of Electrical and Computer Eng., University of Kurdistan, PO Box 416, Sanandaj, Iran

^bDept. of Electrical, Electronic and Information Eng., Osaka University, Osaka, Japan

IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 28, NO. 2, MAY 2013

Inertia Emulation Control Strategy for VSC-HVDC Transmission Systems

Jiebei Zhu, Campbell D. Booth, Grain P. Adam, Andrew J. Roscoe, and Chris G. Bright

Implementing Virtual Inertia in DFIG-Based Wind Power Generation

Mahmadsreza Fakhari Moghaddam Arani, *Student Member, IEEE*, and Ehab F. El-Saadany, *Senior Member, IEEE*

Dynamic Frequency Control Support: a Virtual Inertia Provided by Distributed Energy Storage to Isolated Power Systems

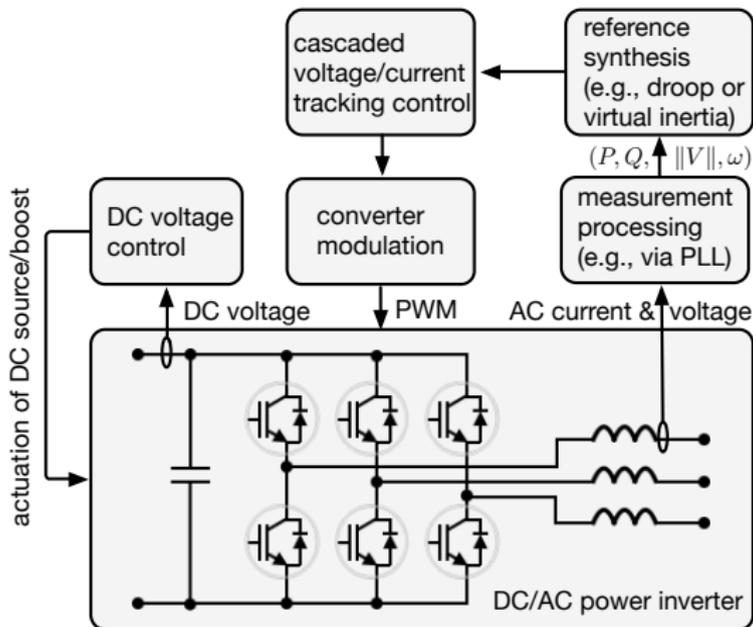
Gauthier Delille, *Member, IEEE*, Bruno François, *Senior Member, IEEE*, and Gilles Malarange

Grid Tied Converter with Virtual Kinetic Storage

M.P.N van Wessenbeeck¹, S.W.H. de Haan¹, *Senior member, IEEE*, P. Varella² and K. Visscher³,

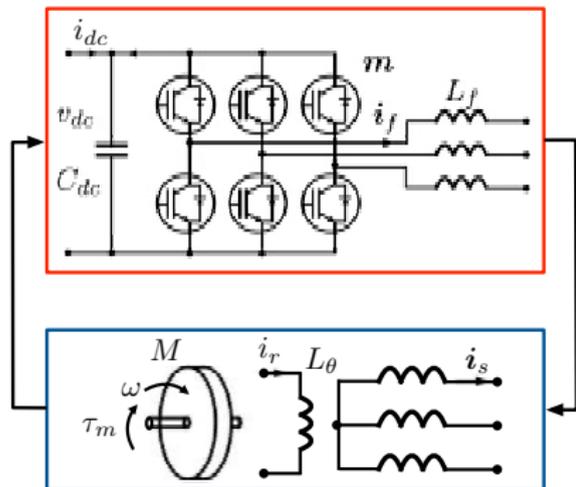
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Standard approach to converter control



1. acquiring & processing of **AC measurements**
 2. synthesis of **references** (voltage/current/power)
“how would a synchronous generator respond now ?”
 3. cascaded PI controllers to **track** references
 4. **actuation** via modulation
 5. **hidden assumption**: DC supply instantaneously provides unlimited power
- tight & fast DC-side control

Virtual synchronous machine \equiv flywheel emulation



- **reference model**: detailed model of synchronous generator + controls

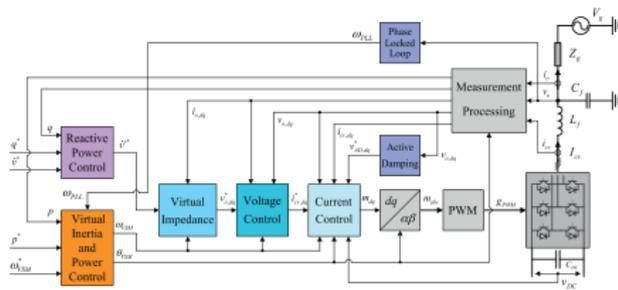
→ most commonly **accepted solution** in **industry** (backward compatibility)

→ robust implementation requires tricks

→ good **nominal performance** but poor post-fault behavior → **not resilient**

→ **poor fit**: converter \neq flywheel

- converter: **fast** actuation & **no significant energy storage**
- machine: **slow** actuation & **significant energy storage**



→ **over-parametrized** & ignores limits

→ issues can be partially alleviated via proper nonlinear control [Arghir et al. '17, '19]

Droop as simplest reference model

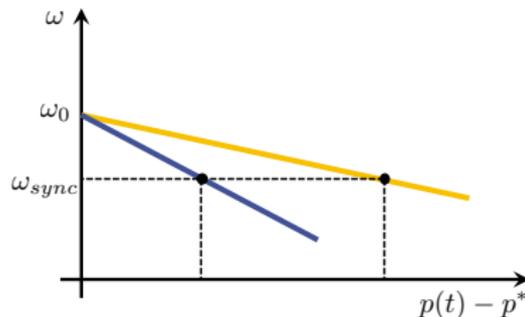
[Chandorkar, Divan, Adapa, '93]

- ▶ **frequency control** by mimicking $p - \omega$ droop property of synchronous machine:

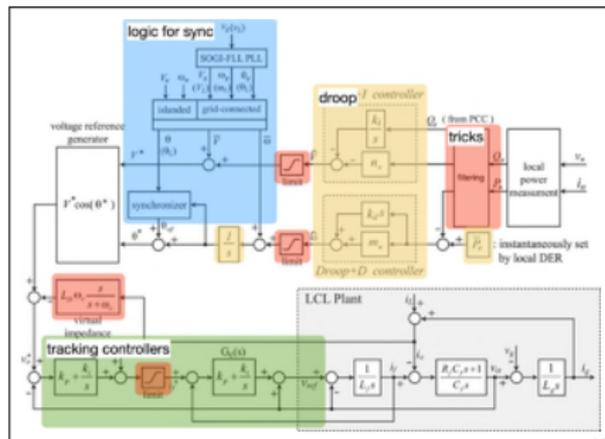
$$\omega - \omega_0 \propto p - p^*$$

- ▶ **voltage control** via $q - \|v\|$ droop control:

$$\frac{d}{dt} \|v\| = -c_1 (\|v\| - v^*) - c_2 (q - q^*)$$



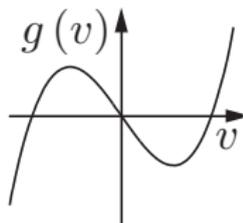
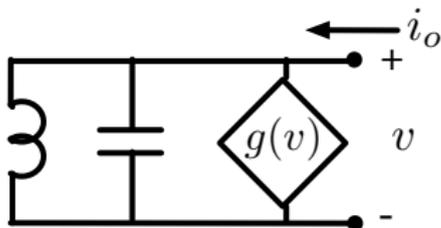
- direct control of (p, ω) and $(q, \|v\|)$ **assuming they are independent** (approx. true only near steady state)
- requires **tricks in implementation**: low-pass filters for dissipation, virtual impedances for saturation, limiters,...
- **performance**: good near steady state but narrow region of attraction



Virtual Oscillator Control (VOC)

nonlinear & open limit cycle oscillator as reference model for terminal voltage (1-phase):

$$\ddot{v} + \omega_0^2 v + g(v) = i_o$$

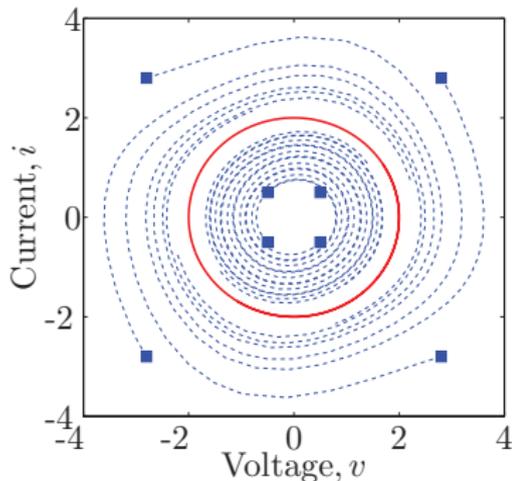


- simplified model amenable to theoretic analysis

→ **almost global synchronization** & **local droop**

- in practice proven to be **robust mechanism** with performance superior to droop & others

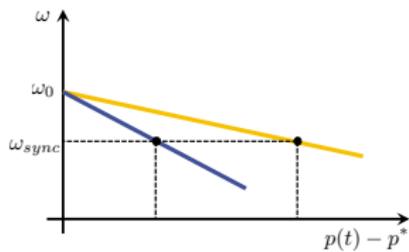
→ **problem**: cannot be controlled(?) to meet specifications on amplitude & power injections



[J. Aracil & F. Gordillo, '02], [Torres, Hespanha, Moehlis, '11],

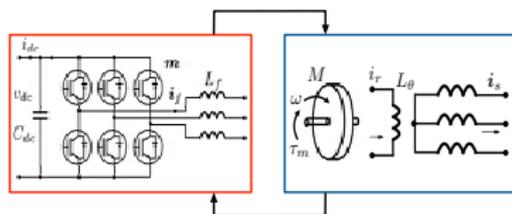
[Johnson, Dhople, Krein, '13], [Dhople, Johnson, Dörfler, '14]

Comparison of grid-forming control [Tayyebi et al., '19]



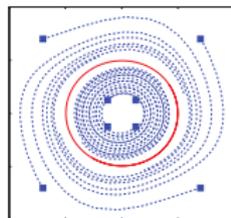
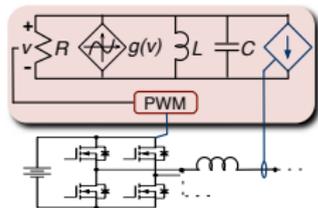
droop control

- + good performance near steady state
- relies on decoupling & small attraction basin



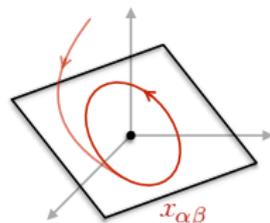
synchronous machine emulation

- + backward compatible in nominal case
- not resilient under large disturbances



virtual oscillator control (VOC)

- + robust & almost globally synchronization
- cannot meet amplitude/power specifications

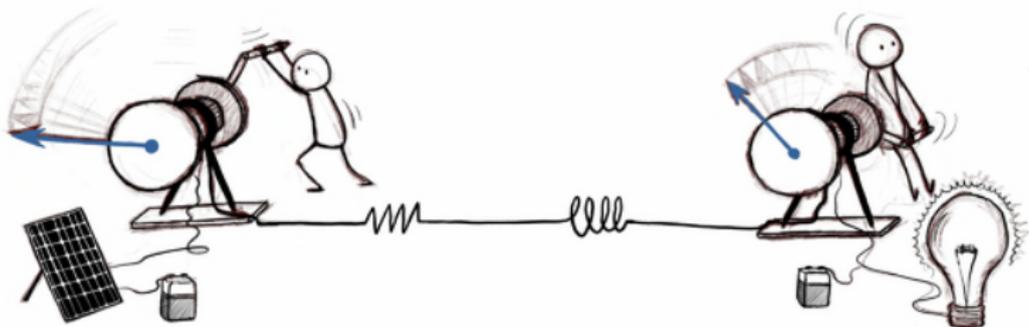


today: foundational control approach

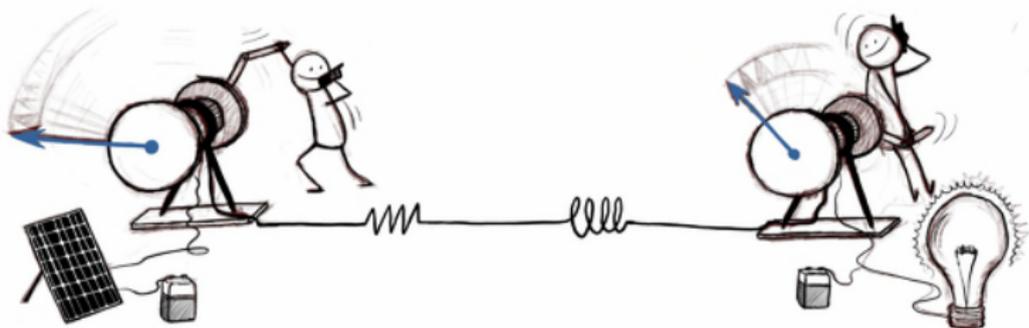
[Colombino, Groß, Brouillon, & Dörfler, '17, '18, '19]
[Seo, Subotic, Johnson, Colombino, Groß, & Dörfler, '18]

Cartoon summary of today's approach

Conceptually, inverters are oscillators that have to synchronize

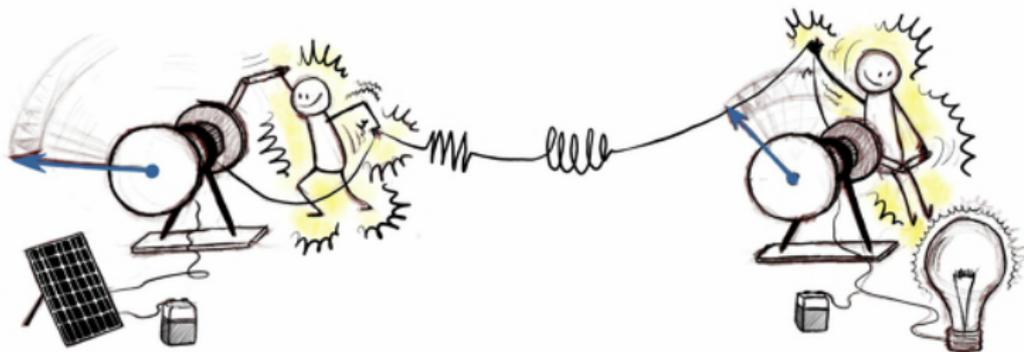


Hypothetically, they could sync by communication (not feasible)



Cartoon summary of today's approach

Colorful idea: inverters sync through physics & clever local control

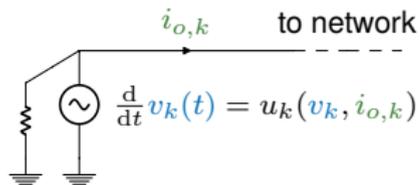


theory: sync of coupled oscillators & nonlinear decentralized control

power systems/electronics
experiments @NREL show superior performance

Recall problem setup

1. *simplifying assumptions* (will be removed later)



- converter \approx controllable voltage source
- grid \approx quasi-static: $\ell \frac{d}{dt} i + r i \approx (j \omega_0 \ell + r) i$
- lines \approx homogeneous $\kappa = \tan(\ell_{kj}/r_{kj}) \forall k, j$

2. *fully decentralized control* of converter terminal voltage & current

⚡ set-points for relative angles $\{\theta_{jk}^*\}$

⚡ nonlocal measurements v_j

⚡ grid & load parameters

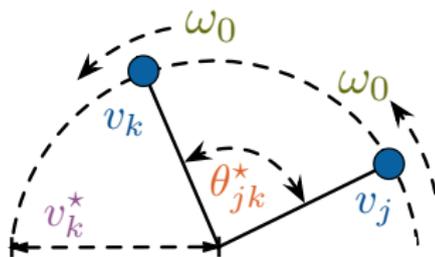
✓ local measurements $(v_k, i_{o,k})$

✓ local set-points (v_k^*, p_k^*, q_k^*)

3. *control objective*

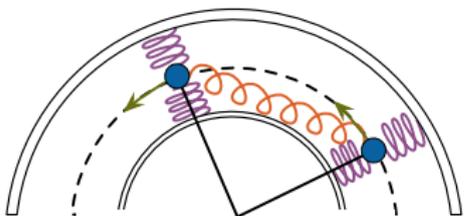
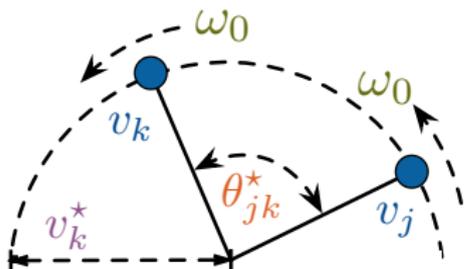
stabilize desired quasi steady state

(synchronous, 3-phase-balanced,
and meet set-points in nominal case)



Colorful idea for closed-loop target dynamics

$$\frac{d}{dt} v_k = \underbrace{\begin{bmatrix} 0 & -\omega_0 \\ \omega_0 & 0 \end{bmatrix} v_k}_{\text{rotation at } \omega_0} + \underbrace{c_1 \cdot e_{\theta,k}(v)}_{\text{synchronization}} + \underbrace{c_2 \cdot e_{\|v\|,k}(v_k)}_{\text{amplitude regulation}}$$



synchronization:

$$e_{\theta,k}(v) = \sum_{j=1}^n w_{jk} (v_j - R(\theta_{jk}^*) v_k)$$

amplitude regulation:

$$e_{\|v\|,k}(v_k) = (v^{*2} - \|v_k\|^2) v_k$$

Decentralized implementation of target dynamics

$$e_{\theta,k}(v) = \underbrace{\sum_j w_{jk}(v_j - R(\theta_{jk}^*)v_k)}_{\text{need to know } w_{jk}, v_j, v_k \text{ and } \theta_{jk}^*} = \underbrace{\sum_j w_{jk}(v_j - v_k)}_{\text{"Laplacian" feedback}} + \underbrace{\sum_j w_{jk}(I - R(\theta_{jk}^*))v_k}_{\text{local feedback: } \mathcal{K}_k(\theta^*)v_k}$$

insight I: non-local measurements from *communication through physics*

$$\underbrace{\dot{i}_{o,k}}_{\text{local feedback}} = \underbrace{\sum_j y_{jk}(v_j - v_k)}_{\text{distributed feedback with } w_{jk} = y_{kj} = \|y_{kj}\| R(1/\kappa)}$$

insight II: angle set-points & line-parameters from *power flow equations*

$$\left. \begin{aligned} p_k^* &= v^{*2} \sum_j \frac{r_{jk}(1 - \cos(\theta_{jk}^*)) - \omega_0 \ell_{jk} \sin(\theta_{jk}^*)}{r_{jk}^2 + \omega_0^2 \ell_{jk}^2} \\ q_k^* &= -v^{*2} \sum_j \frac{\omega_0 \ell_{jk}(1 - \cos(\theta_{jk}^*)) + r_{jk} \sin(\theta_{jk}^*)}{r_{jk}^2 + \omega_0^2 \ell_{jk}^2} \end{aligned} \right\} \Rightarrow \mathcal{K}_k(\theta^*) = \underbrace{\frac{1}{v^{*2}}}_{\text{global parameters}} R(\kappa) \underbrace{\begin{bmatrix} q_k^* & p_k^* \\ -p_k^* & q_k^* \end{bmatrix}}_{\text{local parameters}}$$

Main results

1. desired target dynamics can be realized via **fully decentralized control**:

$$\begin{aligned}
 \frac{d}{dt} v_k &= \underbrace{\begin{bmatrix} 0 & -\omega_0 \\ \omega_0 & 0 \end{bmatrix} v_k}_{\text{rotation at } \omega_0} + c_1 \cdot \underbrace{\sum_{j=1}^n w_{jk} (v_j - R(\theta_{jk}^*) v_k)}_{\text{synchronization with global knowledge}} + c_2 \cdot \underbrace{(v^{*2} - \|v_k\|^2) v_k}_{\text{local amplitude regulation}} \\
 &= \underbrace{\begin{bmatrix} 0 & -\omega_0 \\ \omega_0 & 0 \end{bmatrix} v_k}_{\text{rotation at } \omega_0} + c_1 \cdot \underbrace{R(\kappa) \left(\frac{1}{v^{*2}} \begin{bmatrix} q_k^* & p_k^* \\ -p_k^* & q_k^* \end{bmatrix} v_k - i_{o,k} \right)}_{\text{synchronization through physics}} + c_2 \cdot \underbrace{(v^{*2} - \|v_k\|^2) v_k}_{\text{local amplitude regulation}}
 \end{aligned}$$

2. **almost global stability** result:

If the ... condition holds, the system is **almost globally asymptotically stable** with respect to a **limit cycle** corresponding to a **pre-specified** solution of the **AC power-flow** equations at a **synchronous** frequency ω_0 .

Main results cont'd

3. certifiable, sharp, and intuitive **stability conditions**:

- ▶ **power transfer** “small enough” compared to **network connectivity**
- ▶ **amplitude control** slower than **synchronization control**

e.g., for resistive grid:

$$\frac{1}{2} \underbrace{\lambda_2}_{\text{algebraic connectivity}} > \max_k \sum_{j=1}^n \frac{1}{v^{*2}} \underbrace{|p_{jk}|}_{\text{power transfer}} + \frac{c_2}{c_1} v^*$$

4. connection to **droop control** revealed in polar coordinates (for inductive grid):

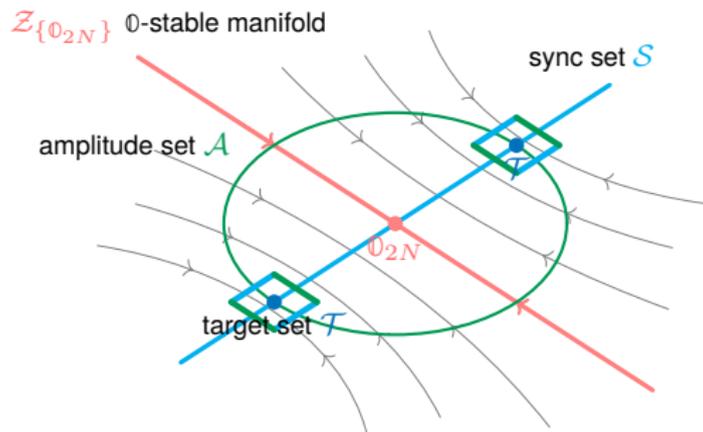
$$\frac{d}{dt} \theta_k = \omega_0 + c_1 \left(\frac{p_k^*}{v^{*2}} - \frac{p_k}{\|v_k\|^2} \right) \Big|_{\|v_k\| \approx 1} \approx \omega_0 + c_1 (p_k^* - p_k) \quad (p - \omega \text{ droop})$$

$$\frac{d}{dt} \|v_k\| \Big|_{\|v_k\| \approx 1} \approx c_1 (q_k^* - q_k) + c_2 (v^* - \|v_k\|) \quad (q - \|v\| \text{ droop})$$

Proof sketch for algebraic grid: Lyapunov & center manifold

Lyapunov function:

$$V(v) = \frac{1}{2} \text{dist}(v, \mathcal{S})^2 + \frac{c_2}{v^{*2}} \sum_k (v^{*2} - \|v_k\|^2)^2$$



$\mathcal{T} \cup 0_{2N}$ is globally attractive

$$\lim_{t \rightarrow \infty} \|v(t)\|_{\mathcal{T} \cup 0_{2N}} = 0$$

\mathcal{T} is stable

$$\|v(t)\|_{\mathcal{T}} \leq \chi(\|v_0\|_{\mathcal{T}})$$

\mathcal{T} is almost globally attractive

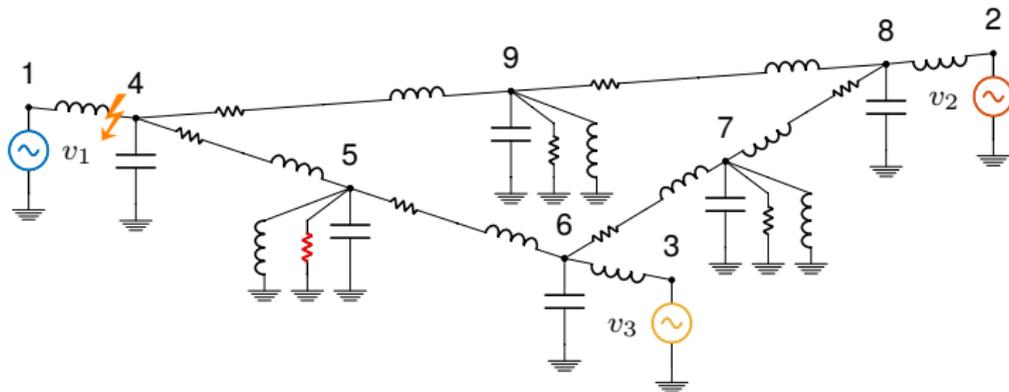
0_{2N} exponentially unstable

$\implies \mathcal{Z}_{\{0_{2N}\}}$ has measure zero

$$\forall v_0 \notin \mathcal{Z}_{\{0_{2N}\}} : \lim_{t \rightarrow \infty} \|v(t)\|_{\mathcal{T}} = 0$$

stability & almost global attractivity \implies **almost global asymptotic stability**

Case study: IEEE 9 Bus system



t = 0 s: black start of three inverters

- **initial state:** $\|v_k(0)\| \approx 10^{-3}$
- **convergence to set-point**

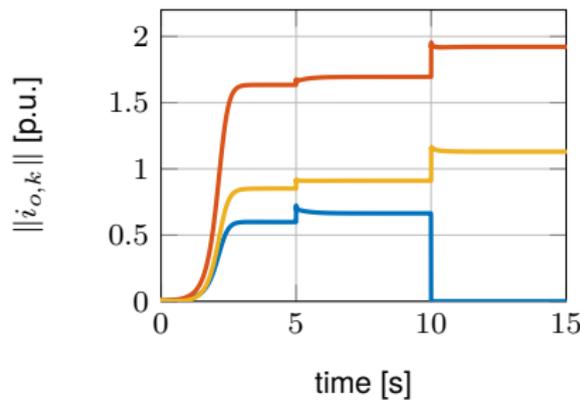
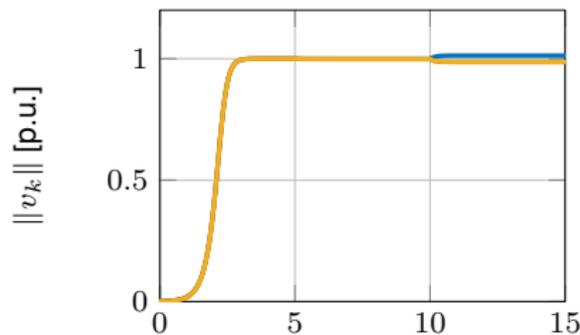
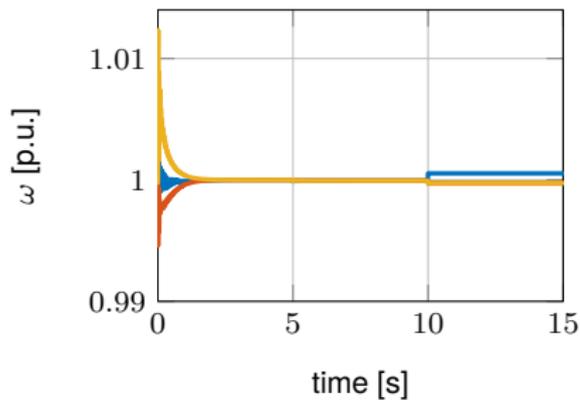
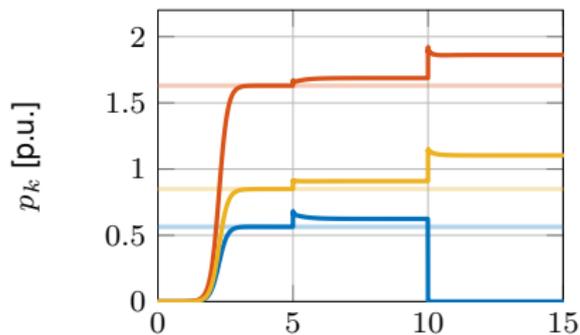
t = 5 s: load step-up

- **20% load increase** at bus 5
- consistent **power sharing**

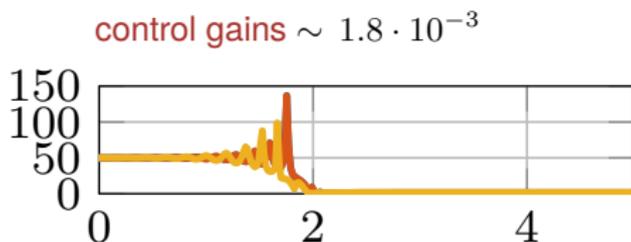
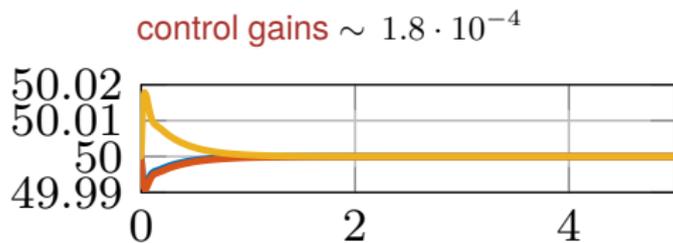
t = 10 s: loss of inverter 1

- the **remaining** inverters **synchronize**
- they **supply** the load **sharing** power

Simulation of IEEE 9 Bus system



Dropping assumptions: dynamic lines



re-do the math leading to updated condition:
amplitude control slower than **sync control**
slower than line dynamics

observations

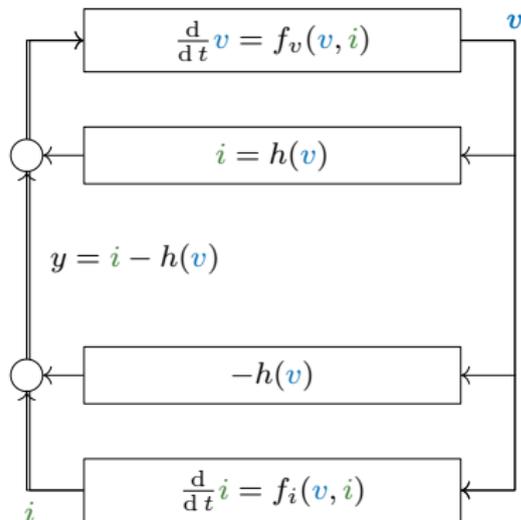
- ▶ inverter control **interferes** with the line dynamics
- ▶ controller needs to be artificially **slowed down**
- ▶ recognized problem

[Vorobev, Huang, Hosaini, & Turitsyn, '17]

“networked control” reason

- ▶ **communication through currents** to infer voltages
- ▶ very inductive lines **delay** the information transfer
- ▶ the controller must be **slow** in very inductive networks

Proof sketch for dynamic grid: perturbation-inspired Lyapunov



Individual Lyapunov functions

- ▶ slow system: $V(v)$ for $\frac{d}{dt}v = f_v(v, h(v))$
- ▶ fast system: $W(y)$ for $\frac{d}{dt}y = f_i(v, y + h(v))$
where $\frac{d}{dt}v = 0$ & coordinate $y = i - h(v)$

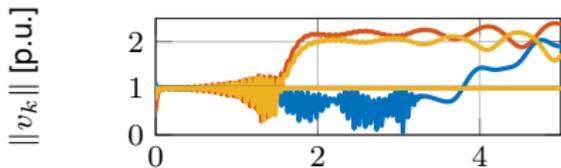
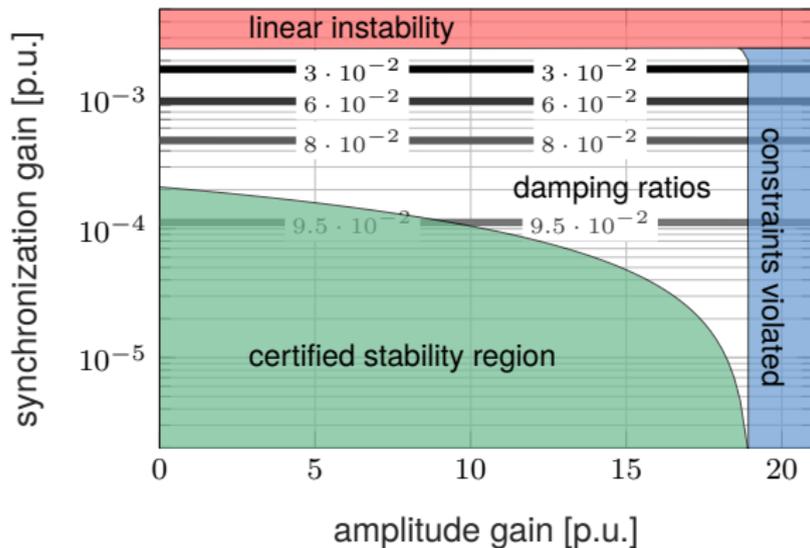
Lyapunov function for the full system

- ▶ $\nu(x) = dW(i - h(v)) + (1 - d)V(v)$
where $d \in [0, 1]$ is free convex coefficient
- ▶ $\frac{d}{dt}\nu(x)$ is decaying under stability condition

Almost global asymptotic stability

- ▶ $\mathcal{T}' \cup \{0_n\}$ globally attractive & \mathcal{T}' stable
- ▶ $\mathcal{Z}_{\{0_n\}}$ has measure zero

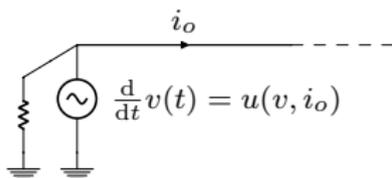
Evaluation of stability conditions



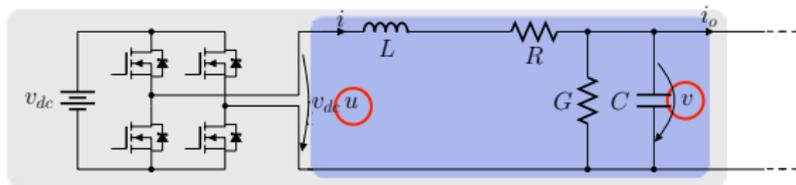
- increase of control gains by factor 10
- \Rightarrow oscillations, overshoots, & instability
- \Rightarrow ***conditions are highly accurate***

Dropping assumptions: detailed converter model

voltage source model:



detailed converter model with LC filter:



► **idea:** invert LC filter so that $v \approx v_{dc}u$

→ **control:** perform robust inversion of LC filter via **cascaded PI**

► **analysis:** repeat proof via **singular perturbation Lyapunov** functions

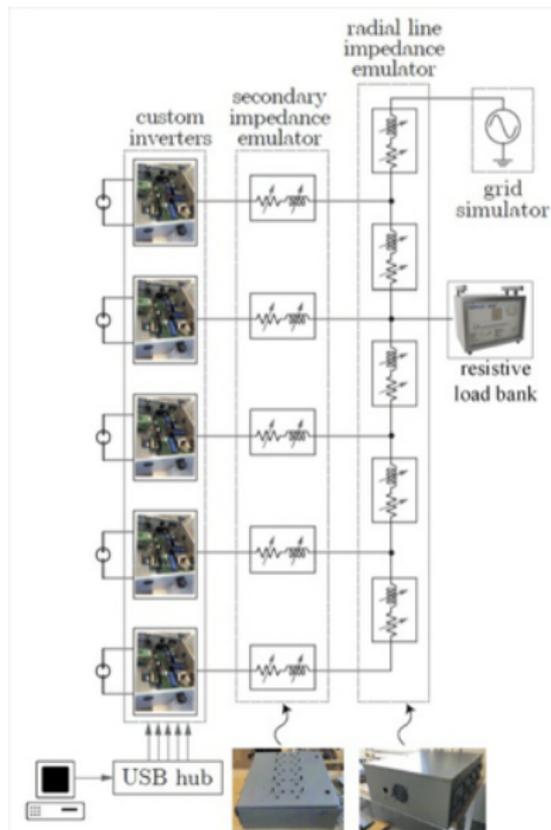
→ **almost global stability** for sufficient time scale separation (quantifiable)

VOC model < line dynamics < voltage PI < current PI

[Subotic, ETH Zürich Master thesis '18]

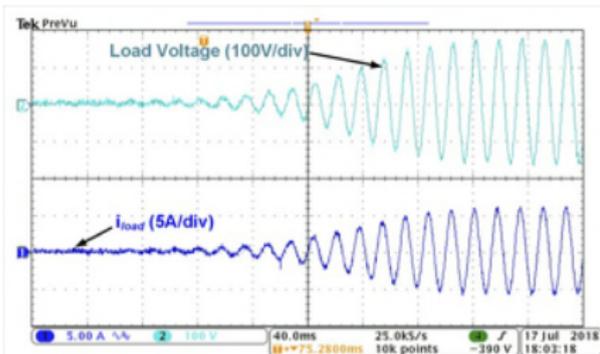
► ... similar steps for control of v_{dc} in a more detailed model

Experimental setup @ NREL

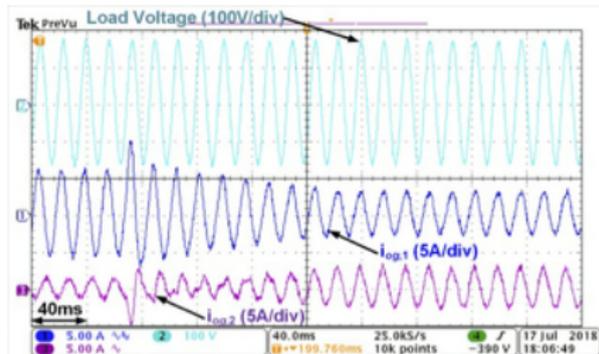


Experimental results

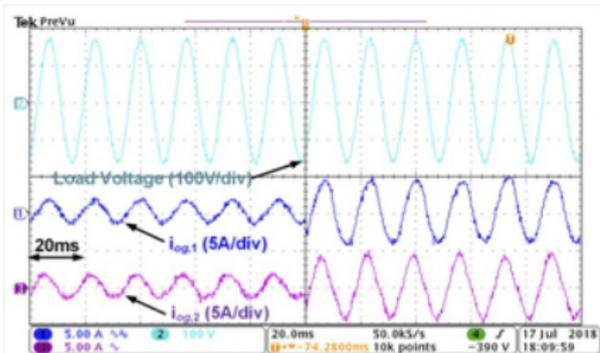
[Seo, Subotic, Johnson, Colombino, Groß, & Dörfler, APEC'18]



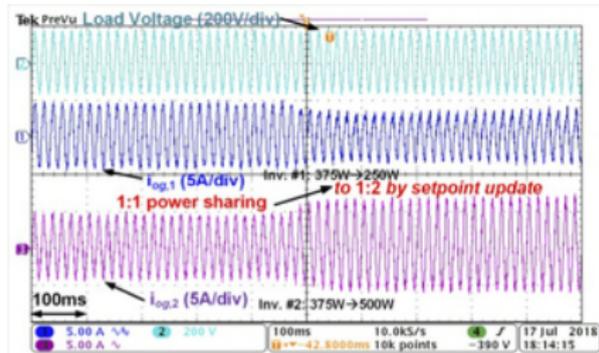
black start of inverter #1 under 500 W load
(making use of almost global stability)



connecting inverter #2 while inverter #1
is regulating the grid under 500 W load



250 W to 750 W load transient with two
inverters active

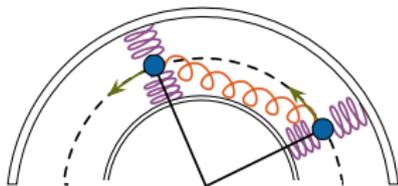


change of setpoint: p^* of inverter #2
updated from 250 W to 500 W

Conclusions

Summary

- challenges of low-inertia systems
- dispatchable virtual oscillator control
- theoretic analysis & experiments



Ongoing & future work

- theoretical questions: robustness & regulation
- practical issue: compatibility with legacy system
- experimental validations @ ETH, NREL, AIT



Main references (others on website)

D. Groß, M Colombino, J.S. Brouillon, & F. Dörfler. *The effect of transmission-line dynamics on grid-forming dispatchable virtual oscillator control.*

M. Colombino, D. Groß, J.S. Brouillon, & F. Dörfler. *Global phase and magnitude synchronization of coupled oscillators with application to the control of grid-forming power inverters.*

POWER IS NOTHING WITHOUT CONTROL

