Plug-and-Play Operation of Microgrids

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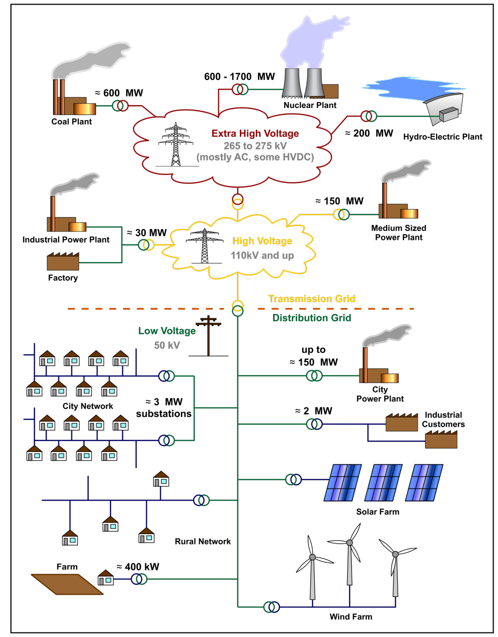
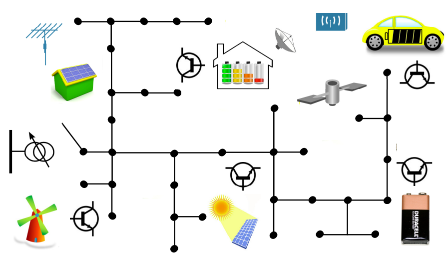
Electric power networks & their conventional operation



- **electric energy** is our lifeblood
- purpose of electric **power grid**: generate/transmit/distribute
- **constraints**: op, econ, & stab

Paradigm shifts & new problem scenarios . . . in a nutshell



1. controllable fossil fuel sources ⇒ stochastic renewable sources
2. centralized bulk generation ⇒ distributed low-voltage generation
3. synchronous generators ⇒ low/no inertia power electronics
4. generation follows load ⇒ controllable load follows generation
5. monopolistic energy markets ⇒ deregulated energy markets
6. centralized top-to-bottom control ⇒ distributed non-hierarchical control
7. human in the loop & heuristics ⇒ "smart" real-time decision making

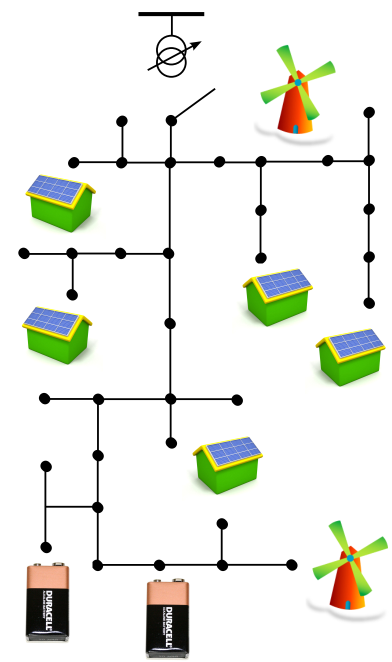
Microgrids

Structure
- low-voltage distribution networks
- grid-connected or islanded
- autonomously managed

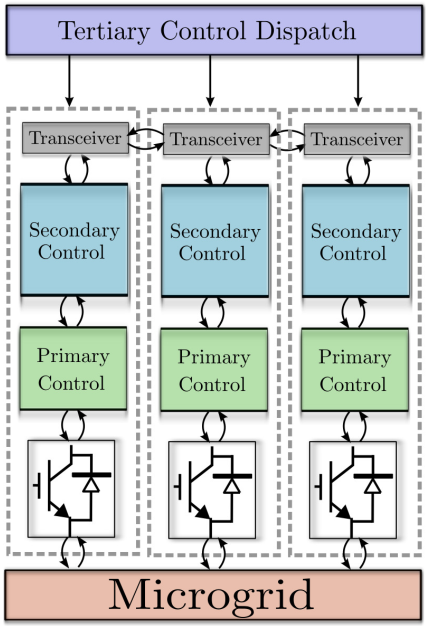
Applications
- hospitals, military, campuses, large vehicles, & isolated communities

Benefits
- naturally distributed for renewables
- flexible, efficient, & reliable

Operational challenges
- volatile dynamics & low inertia
- plug'n'play & no central authority



Conventional control architecture from bulk power ntwks



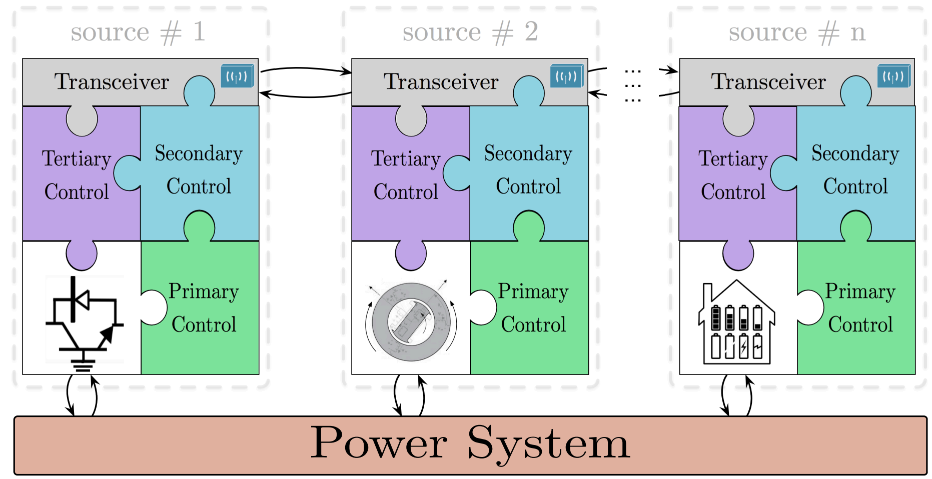
3. **Tertiary control** (offline)
   - Goal: optimize operation
   - Strategy: centralized & forecast
2. **Secondary control** (slower)
   - Goal: maintain operating point
   - Strategy: centralized
1. **Primary control** (fast)
   - Goal: stabilization & load sharing
   - Strategy: decentralized

Microgrids: distributed, model-free, online & without time-scale separation

⇒ **break** vertical & horizontal **hierarchy**

A preview – plug-and-play operation architecture

flat hierarchy, distributed, no time-scale separations, & model-free . . .



Outline

Introduction

Modeling

Primary Control

Tertiary Control

Secondary Control

Virtual Oscillator Control

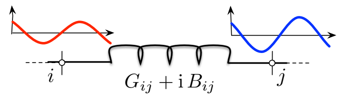
Conclusions

we will illustrate all theorems with experiments

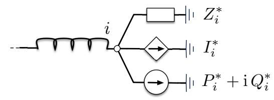
modeling & assumptions

Modeling: a power system is a circuit

1. synchronous **AC circuit** with harmonic waveforms $E_i e^{i(\theta_i + \omega^* t)}$



2. **ZIP loads**: constant impedance, current, & power $P_i^* + iQ_i^*$ (today)



3. **coupling** via Kirchhoff & Ohm

   injection $=$ $\sum$ power flows

4. identical lines $G/B = const.$ (equivalent to lossless case $G/B = 0$)

5. decoupling: $P_i \approx P_i(\theta)$ & $Q_i \approx Q_i(E)$ (near operating point)

- active power: $P_i = \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j)$
- reactive power: $Q_i = -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j)$

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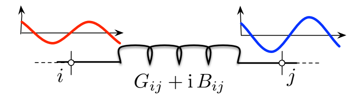

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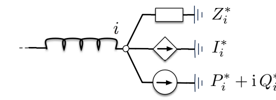

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4. identical lines $G/B = const.$ (equivalent to lossless case $G/B = 0$)

5. decoupling: $P_i \approx P_i(\theta)$ & $Q_i \approx Q_i(E)$ (near operating point)

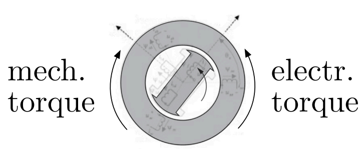
- trigonometric active power flow: $P_i(\theta) = \sum_j B_{ij} \sin(\theta_i - \theta_j)$
- polynomial reactive power flow: $Q_i(E) = -\sum_j B_{ij} E_i E_j$ (not today)

Modeling the "essential" network dynamics & controls

(models can be arbitrarily detailed)

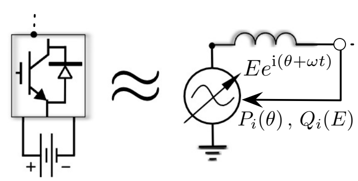
1. synchronous **machines** (swing dynamics)

$$M_i \ddot{\theta}_i = P_i^* + \mathbf{P_i^c} - P_i(\theta)$$



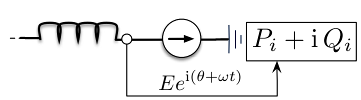
2. DC & variable AC sources interfaced with voltage-source **converters**

$$P_i^* + \mathbf{P_i^c} = P_i(\theta)$$



3. controllable **loads** (voltage- and frequency-responsive)

$$P_i^* + \mathbf{P_i^c} = P_i(\theta)$$

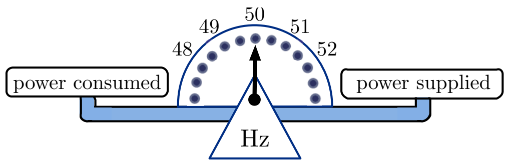


primary control (droop characteristic)

Decentralized primary control of active power

Emulate physics of dissipative coupled **synchronous machines**:

$$M_i\ddot{\theta} + D_i\dot{\theta}_i = P_i^* - \sum\nolimits_j B_{ij}\sin(\theta_i - \theta_j)$$



$\Rightarrow$ sum equations & set $\dot{\theta}_i = \omega_{\text{sync}}$:

$$\omega_{\text{sync}} = \sum\nolimits_i P_i^* / \sum\nolimits_i D_i$$

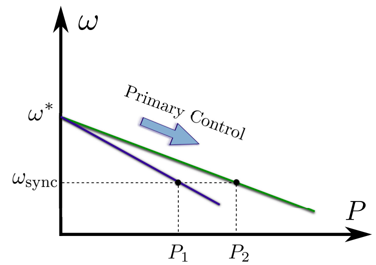
Conventional wisdom: physics are naturally stable & sync frequency reveals power imbalance

$P/\dot{\theta}$ droop control:

$$(\omega_i - \omega^*) \propto (P_i^* - P_i(\theta))$$

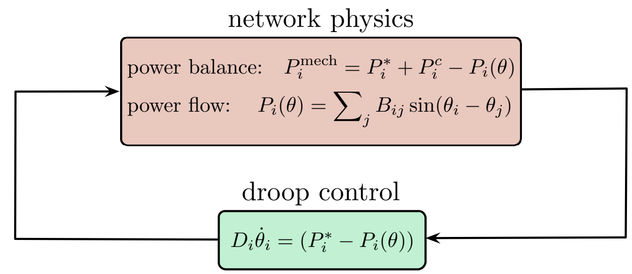
$$\Updownarrow$$

$$D_i\dot{\theta}_i = P_i^* - P_i(\theta)$$



Putting the pieces together...

differential-algebraic, nonlinear, large-scale closed loop



passive loads: $0 = P_i^* - \sum\nolimits_j B_{ij}\sin(\theta_i - \theta_j)$

synchronous machines: $M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i^* - \sum\nolimits_j B_{ij}\sin(\theta_i - \theta_j)$

inverter sources: $D_i\dot{\theta}_i = P_i^* - \sum\nolimits_j B_{ij}\sin(\theta_i - \theta_j)$

controllable loads: $D_i\dot{\theta}_i = P_i^* - \sum\nolimits_j B_{ij}\sin(\theta_i - \theta_j)$

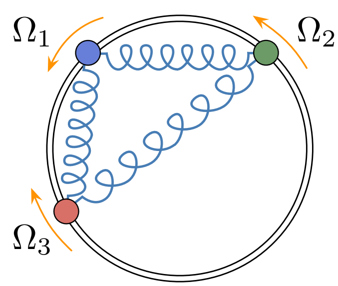
A perspective from coupled oscillators

Mechanical oscillator network

Angles $(\theta_1, \dots, \theta_n)$ evolve on $\mathbb{T}^n$ as

$$M_i\ddot{\theta}_i + D_i\dot{\theta}_i = \Omega_i - \sum\nolimits_j K_{ij}\sin(\theta_i - \theta_j)$$

- inertia constants $M_i > 0$
- viscous damping $D_i > 0$
- external torques $\Omega_i \in \mathbb{R}$
- spring constants $K_{ij} \geq 0$

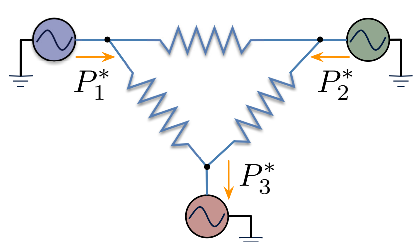


Droop-controlled power system

$$0 = P_i^* - \sum\nolimits_j B_{ij}\sin(\theta_i - \theta_j)$$

$$D_i\dot{\theta}_i = P_i^* - \sum\nolimits_j B_{ij}\sin(\theta_i - \theta_j)$$

$$M_i\ddot{\theta} + D_i\dot{\theta}_i = P_i^* - \sum\nolimits_j B_{ij}\sin(\theta_i - \theta_j)$$



Closed-loop stability under droop control

Theorem: stability of droop control [J. Simpson-Porco, FD, & F. Bullo, '12]

$\exists$ unique & exp. stable frequency sync $\iff$ active power flow is feasible

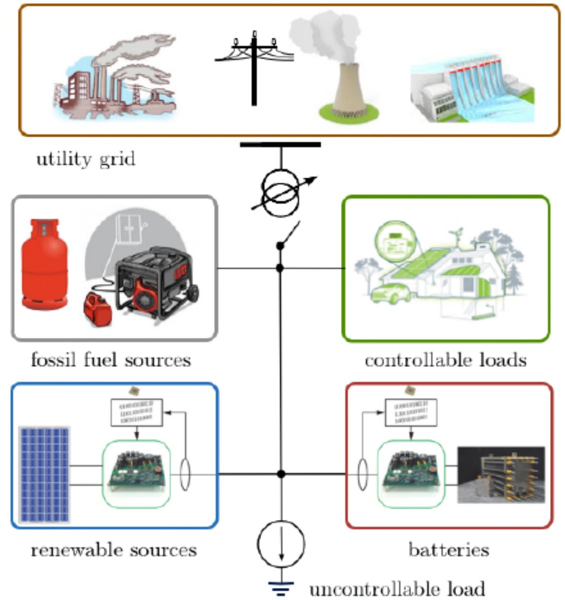
Main **proof ideas** and some **further results**:

- synchronization frequency: $\omega_{\text{sync}} = \omega^* + \dfrac{\sum_{\text{sources}} P_i^* + \sum_{\text{loads}} P_i^*}{\sum_{\text{sources}} D_i}$
  ($\propto$ power balance)
- steady-state power injections: $\mathcal{P}_i = \begin{cases} P_i^* & (\text{load } \#i) \\ P_i^* - D_i(\omega_{\text{sync}} - \omega^*) & (\text{source } \#i) \end{cases}$
  (depend on D_i & P_i^*)
- stability via incremental Chetaev energy function [C. Zhao, E. Mallada, & FD '14]

tertiary control (energy management)

Tertiary control and energy management

an offline resource allocation & scheduling problem



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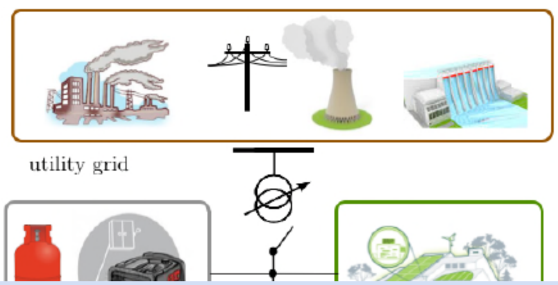


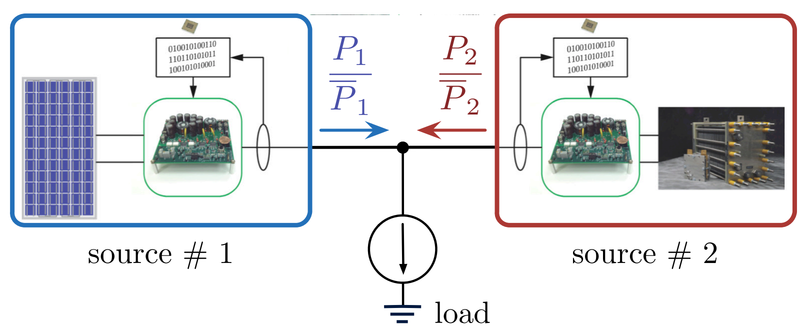
minimize	{cost of generation, losses, . . . }
subject to	
equality constraints:	power balance equations
inequality constraints:	flow/injection/voltage constraints
logic constraints:	commit generators yes/no
	⋮

Objective I: decentralized proportional load sharing

1) Sources have **injection constraints**: $P_i(\theta) \in [0, \overline{P}_i]$

2) Load must be **serviceable**: $0 \leq \left|\sum_{\text{loads}} P_j^*\right| \leq \sum_{\text{sources}} \overline{P}_j$

3) **Fairness:** load should be shared proportionally: $P_i(\theta)\,/\,\overline{P}_i = P_j(\theta)\,/\,\overline{P}_j$



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A little calculation reveals in steady state:

$$\frac{P_i(\theta)}{\overline{P}_i} \overset{!}{=} \frac{P_j(\theta)}{\overline{P}_j} \quad \Rightarrow \quad \frac{P_i^* - (D_i\omega_{\text{sync}} - \omega^*)}{\overline{P}_i} \overset{!}{=} \frac{P_j^* - (D_j\omega_{\text{sync}} - \omega^*)}{\overline{P}_i}$$

. . . so choose

$$\frac{P_i^*}{\overline{P}_i} = \frac{P_j^*}{\overline{P}_j} \quad \text{and} \quad \frac{D_i}{\overline{P}_i} = \frac{D_j}{\overline{P}_j}$$

Objective I: decentralized proportional load sharing

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3) **Fairness:** load should be shared proportionally: $P_i(\theta)\,/\,\overline{P}_i = P_j(\theta)\,/\,\overline{P}_j$

Theorem: fair proportional load sharing [J. Simpson-Porco, FD, & F. Bullo, '12]

Let the droop coefficients be selected **proportionally**:

$$\boxed{D_i/\overline{P}_i = D_j/\overline{P}_j \;\;\&\;\; P_i^*/\overline{P}_i = P_j^*/\overline{P}_j}$$

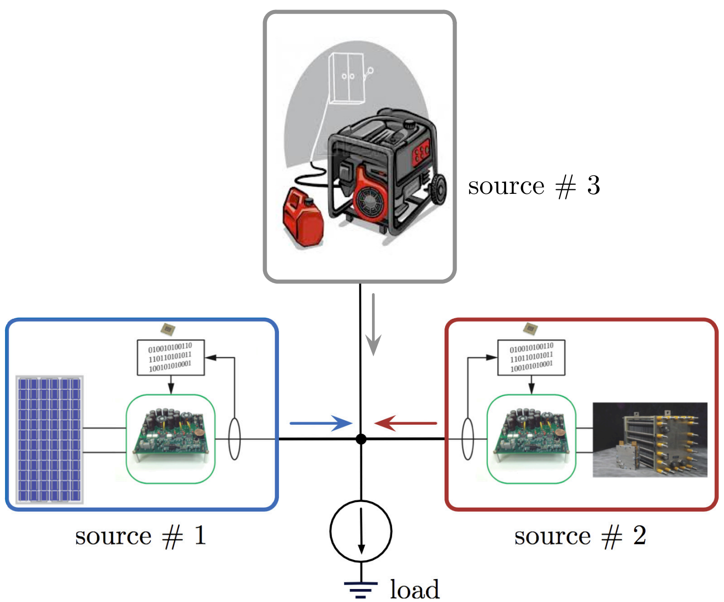
The the following statements hold:

(i) Proportional load sharing: $P_i(\theta)\,/\,\overline{P}_i = P_j(\theta)\,/\,\overline{P}_j$

(ii) Constraints met: $0 \leq \left|\sum_{\text{loads}} P_j^*\right| \leq \sum_{\text{sources}} \overline{P}_j \;\Leftrightarrow\; P_i(\theta) \in [0, \overline{P}_i]$

Objective I: fair proportional load sharing

proportional load sharing is not always the right objective



Objective II: economic generation dispatch

minimize the total accumulated generation (many variations possible)

$$\text{minimize}_{\,\theta \in \mathbb{T}^n,\, u \in \mathbb{R}^{n_I}} \quad f(u) = \sum\nolimits_{\text{sources}} \alpha_i u_i^2$$

subject to

source power balance: $P_i^* + u_i = P_i(\theta)$

load power balance: $P_i^* = P_i(\theta)$

branch flow constraints: $|\theta_i - \theta_j| \leq \gamma_{ij} < \pi/2$

Unconstrained case: identical marginal costs $\alpha_i u_i^* = \alpha_j u_j^*$ at optimality

In conventional power system operation, the economic dispatch is

- solved **offline**, in a **centralized** way, & with a **model & load forecast**

In a grid with distributed energy resources, the economic dispatch should be

- solved **online**, in a **decentralized** way, & **without knowing a model**

Objective II: decentralized dispatch optimization

Insight: droop-controlled system = decentralized primal/dual algorithm

Theorem: optimal droop [FD, Simpson-Porco, & Bullo '13, Zhao, Mallada, & FD '14]

The following statements are equivalent:

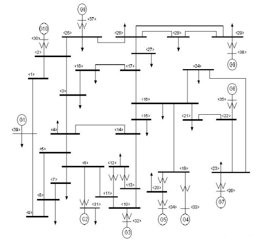
(i) the economic dispatch with cost coefficients α_i is **strictly** feasible with global minimizer (θ^*, u^*).

(ii) $\exists$ droop coefficients D_i such that the power system possesses a unique & locally exp. stable sync'd solution θ.

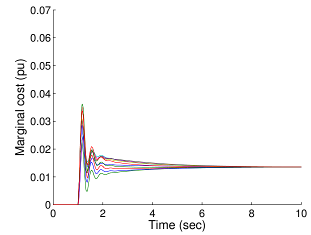
If (i) & (ii) are true, then $\theta_i \sim \theta_i^*$, $u_i^* = -D_i(\omega_{\text{sync}} - \omega^*)$, & $\boxed{D_i\alpha_i = D_j\alpha_j}$.

- recover load sharing for $\alpha_i \propto 1/\overline{P}_i$ & similar results in **constrained** case
- similar results in transmission ntwks with DC flow [E. Mallada & S. Low, '13] & [N. Li, L. Chen, C. Zhao, & S. Low '13] & [X. Zhang & A. Papachristodoulou, '13] & [M. Andreasson, D. V. Dimarogonas, K. H. Johansson, & H. Sandberg, '13] & ...

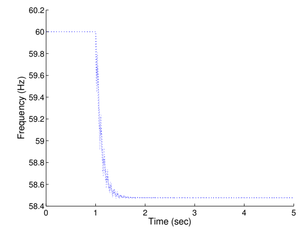
Some quick simulations & extensions



IEEE 39 New England with load step at 1s



$t \to \infty$: convergence to identical marginal costs

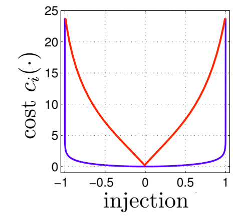


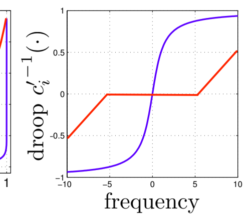
$t \to \infty$: frequency $\propto$ power imbalance

$\Rightarrow$ strictly convex & differentiable cost $f(u) = \sum_{\text{sources}} c_i(u_i)$

$\Rightarrow$ non-linear frequency droop curve $c_i'^{-1}(\dot{\theta}_i) = P_i^* - P_i(\theta)$

$\Rightarrow$ include dead-bands, saturation, etc.

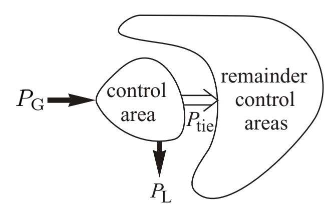




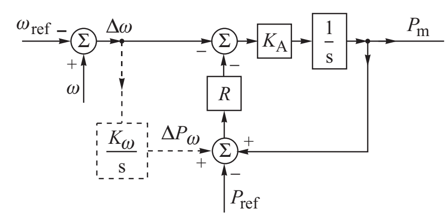
secondary control (frequency regulation)

Conventional secondary frequency control in power systems

Interconnected Systems	Isolated Systems
• **Centralized** automatic generation control (AGC)	• **Decentralized** PI control



compatible with econ. dispatch [N. Li, L. Chen, C. Zhao, & S. Low '13]

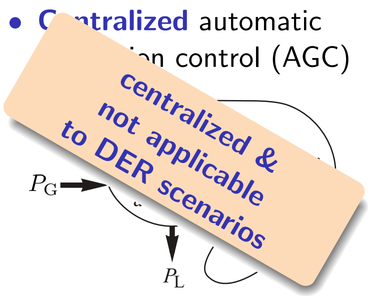


is *globally* stabilizing [C. Zhao, E. Mallada, & FD, '14]

Conventional secondary frequency control in power systems

Interconnected Systems

- **Centralized** automatic generation control (AGC)

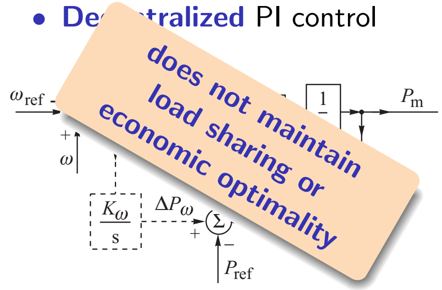


compatible with econ. dispatch

[N. Li, L. Chen, C. Zhao, & S. Low '13]

Isolated Systems

- **Decentralized** PI control



is *globally* stabilizing

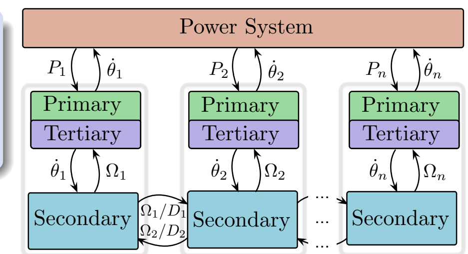
[C. Zhao, E. Mallada, & FD, '14]

Distributed energy ressources require **distributed** (**!**) secondary control.

Distributed **A**veraging **PI** (DAPI) control

$$D_i\dot{\theta}_i = P_i^* - P_i(\theta) - \Omega_i$$

$$k_i\dot{\Omega}_i = D_i\dot{\theta}_i - \sum_{j \subseteq \text{sources}} a_{ij} \cdot (\alpha_i\Omega_i - \alpha_j\Omega_j)$$



- no tuning & no time-scale separation: $k_i, D_i > 0$
- distributed & modular: connected comm. $\subseteq$ sources
- recovers primary op. cond. (load sharing & opt. dispatch)

$\Rightarrow$ plug'n'play implementation

Theorem: stability of DAPI

[J. Simpson-Porco, FD, & F. Bullo, '12]

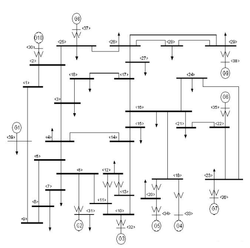
[C. Zhao, E. Mallada, & FD '14]

primary droop controller works

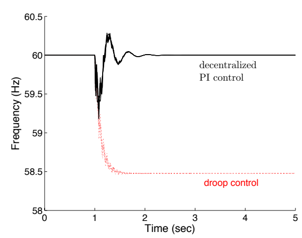
$\Longleftrightarrow$

secondary DAPI controller works

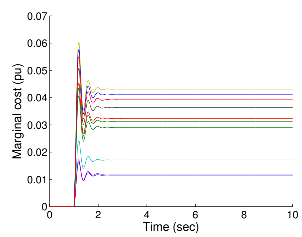
Simulations cont'd



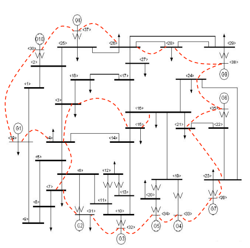
IEEE 39 New England with decentralized PI control



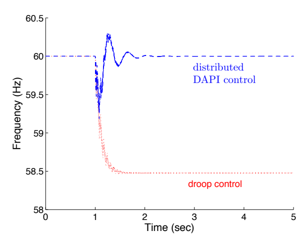
$t \to \infty$: decentralized PI control regulates frequency



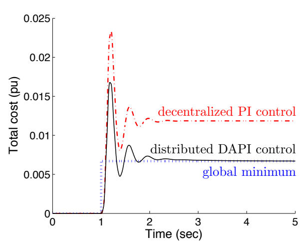
$t \to \infty$: decentralized PI control is not optimal



IEEE 39 New England with distributed DAPI control



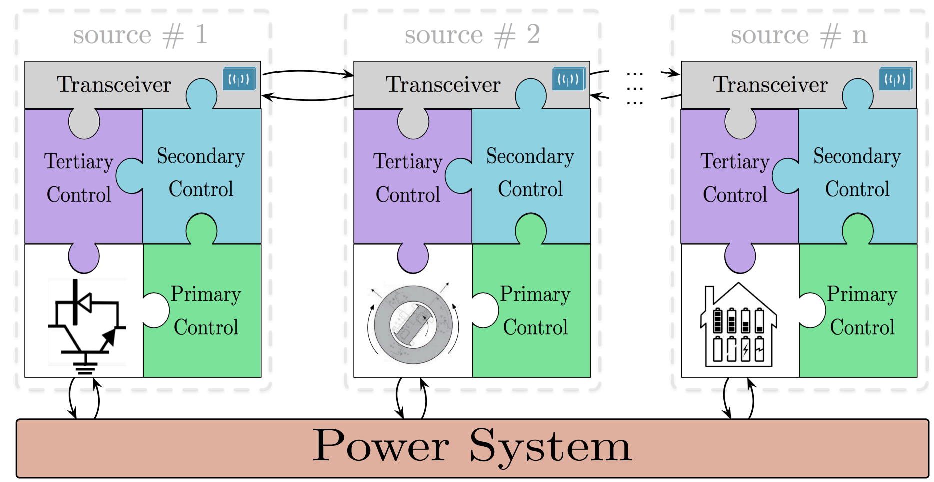
$t \to \infty$: DAPI control regulates frequency



DAPI control minimizes cost with little effort

Plug'n'play architecture

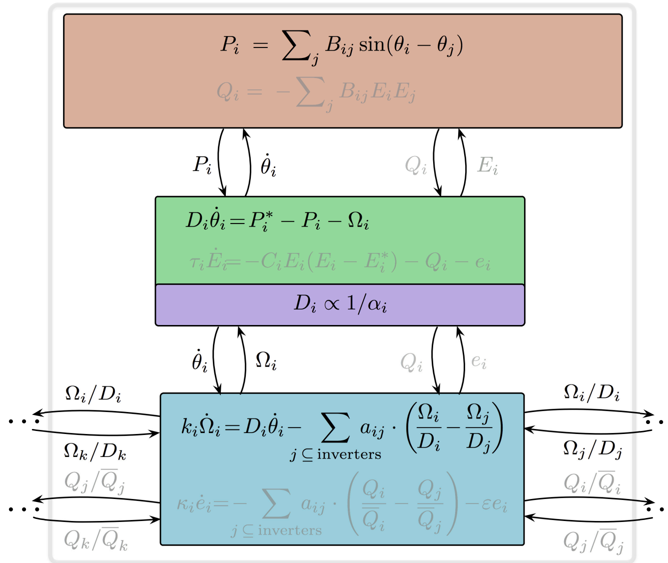
flat hierarchy, distributed, no time-scale separations, & model-free



plug-and-play experiments

Plug'n'play architecture

recap of detailed signal flow (active power only)



Power system: physics & loadflow

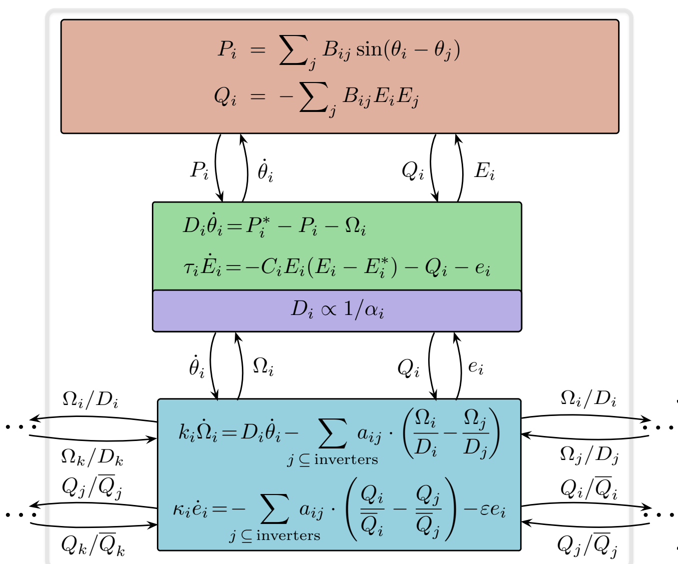
Primary control: mimic oscillators & polyn. symmetry

Tertiary control: marginal costs $\propto$ 1 /control gains

Secondary control: diffusive averaging of injection ratios

Plug'n'play architecture

recap of detailed signal flow (with reactive power)



Power system: physics & loadflow

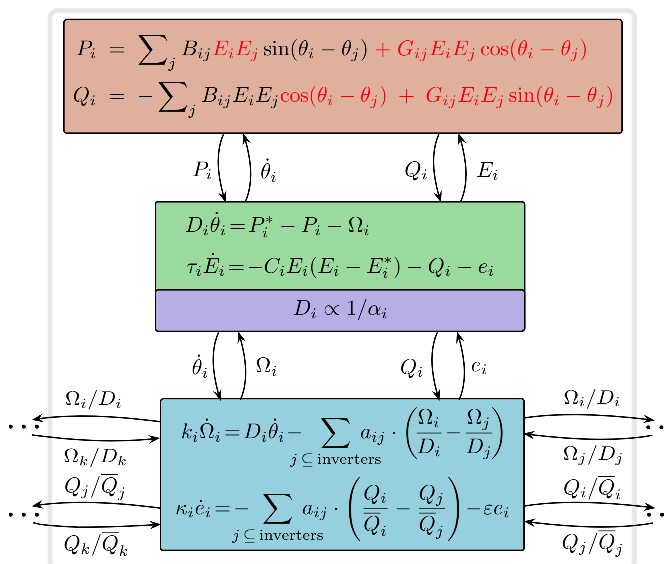
Primary control: mimic oscillators & polyn. symmetry

Tertiary control: marginal costs $\propto$ 1 /control gains

Secondary control: diffusive averaging of injection ratios

Plug'n'play architecture

experiments also work well in the coupled & lossy case



Power system: physics & loadflow

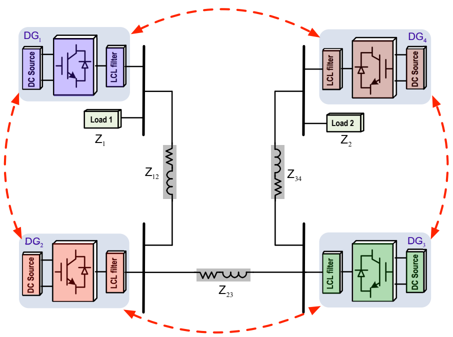
Primary control: mimic oscillators & polyn. symmetry

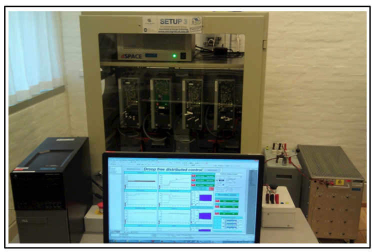
Tertiary control: marginal costs $\propto$ 1 /control gains

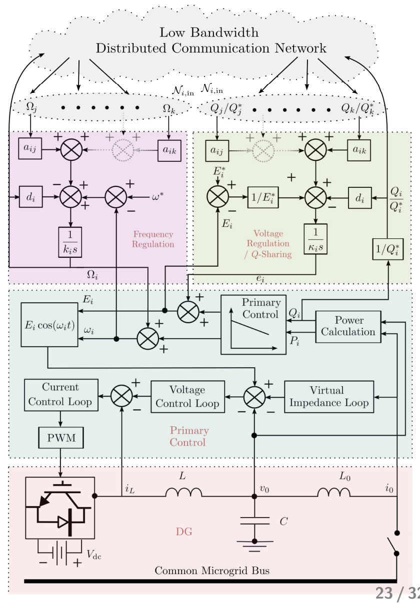
Secondary control: diffusive averaging of injection ratios

Experimental validation of control & opt. algorithms

in collaboration with Q. Shafiee & J.M. Guerrero @ Aalborg University

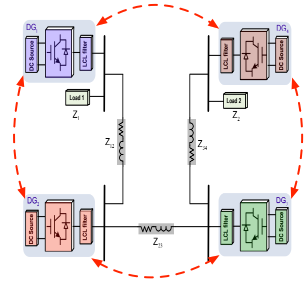






Experimental validation of control & opt. algorithms

frequency/voltage regulation & active/reactive load sharing

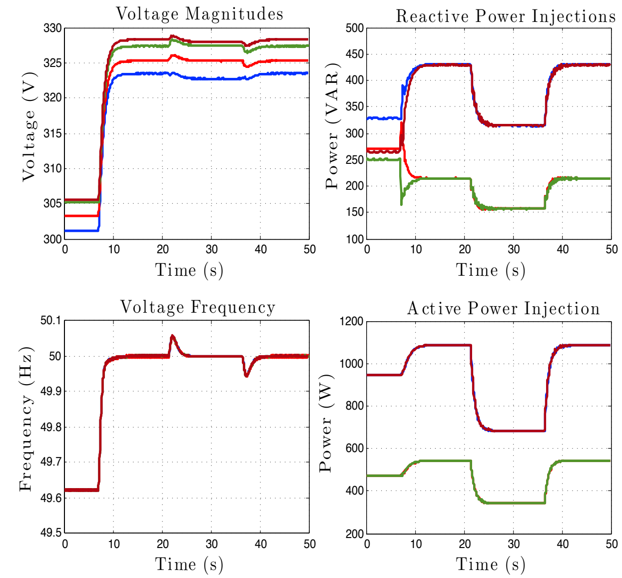


$t \in [0s, 7s]$: primary & tertiary control

$t = 7s$: secondary control activated

$t = 22s$: load # 2 unplugged

$t = 36s$: load # 2 plugged back



what can we do better?

algorithms, detailed models, cyber-physical aspects, . . .

today: **virtual oscillator control**

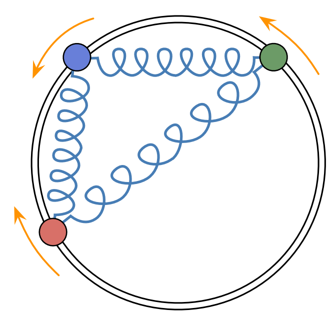
Removing the assumptions of droop control

- **idealistic assumptions:** quasi-stationary operation & phasor coordinates

⇒ **future grids:** more power electronics & renewables and fewer machines

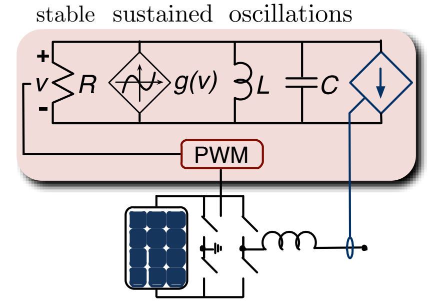
- **droop control** = coupled phase oscillators constrained to limit-cycle

⇒ **Virtual Oscillator Control:** control inverters as limit cycle oscillators
[Torres, Moehlis, & Hespanha, '12, Johnson, Dhople, Hamadeh, & Krein, '13]



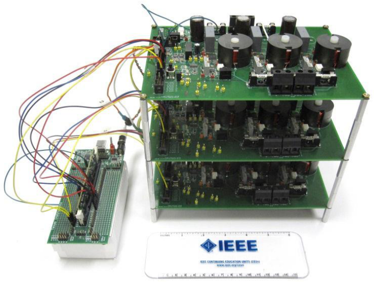
dynamic behavior of droop control

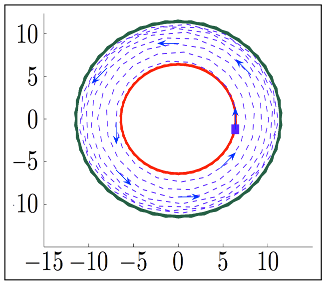
stable sustained oscillations



digitally implemented VOC

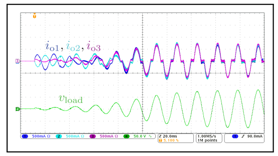
Plug'n'play **V**irtual **O**scillator **C**ontrol (VOC)



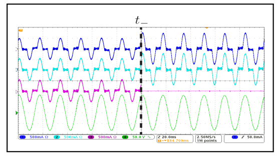


change of setpoint

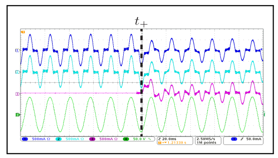
Oscilloscope plots:



emergence of synchrony



removal of inverter



addition of inverter

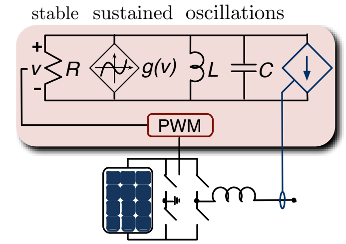
Crash course on planar limit cycle oscillators

$$L\frac{d}{dt}i = v$$

$$C\frac{d}{dt}v = -Rv - g(v) - i - i_{\text{grid}}$$

$\Rightarrow$ normalized coordinates ($\varepsilon = \sqrt{L/C}$):

$$\ddot{v} + \varepsilon k_1 g(\dot{v}) + v = \varepsilon k_2 u$$

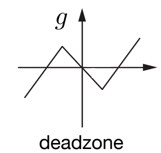


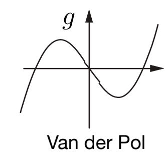
Liénard's oscillation condition

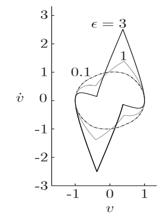
for our VOC oscillator (in a nutshell):

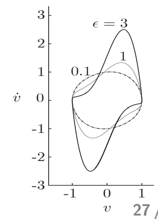
1. 2nd order harmonic oscillator without forcing & state-dependent damping
2. damping: negative in neighborhood of the origin & positive elsewhere

$\Rightarrow$ unique & stable limit cycle

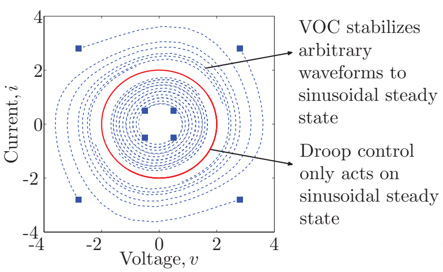


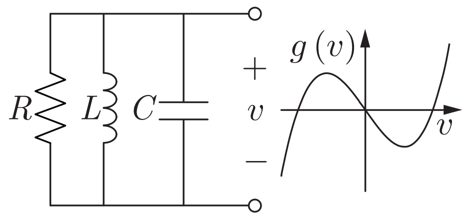






Backward compatibility to droop [M. Sinha, FD, B. Johnson, & S. Dhople, '14]





Van der Pol nonlinearity: $g(v) \propto v^3 - v$

in normalized coordinates: $\ddot{v} + \varepsilon k_1 g(\dot{v}) + v = \varepsilon k_2 u$

$\Rightarrow$ transf. to polar coordinates, averaging, & generalized power definitions

Thm: in vicinity of the limit cycle:

VOC $\supset$ droop:

$$\frac{d}{dt}\theta_{\text{avg}} = \textit{constant} \cdot (\text{reactive power})$$

$$r_{\text{avg}} - r^* = \textit{constant} \cdot (P^* - \text{active power})$$

Experimental validation of backward compatibility

[B. Johnson, M. Sinha, N. Ainsworth, FD, & S. Dhople, '15]

VOC model: $\ddot{v} + \varepsilon k_1 g(\dot{v}) + v = \varepsilon k_2 u$

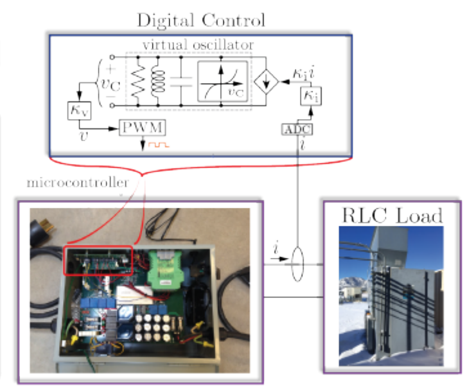
1. **VOC $\supset$ droop:**

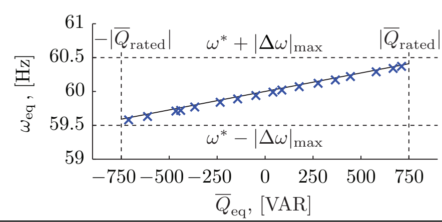
   $\frac{d}{dt}\theta_{\text{avg}} = \textit{constant} \cdot (\text{reactive power})$

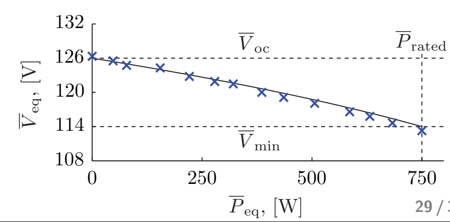
   $r_{\text{avg}} - r^* = \textit{constant} \cdot (P^* - \text{active power})$

2. **VOC $\xrightarrow{\varepsilon \to 0}$ harmonic oscillator**

   with 1/3 harmonics ratio $\propto \varepsilon/8$



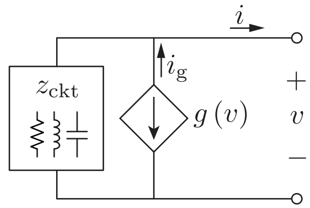




Co-evolution: "dynamic process over dynamic network"

Nonlinear oscillators:

- passive circuit impedance $z_{\mathrm{ckt}}(s)$
- active current source $g(v)$



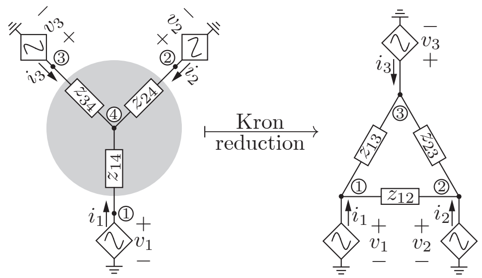
Co-evolving network:

- RLC network is LTI
- Kron reduction: eliminate loads

Homogeneity assumptions:

- identical oscillators & local loads after Kron reduction

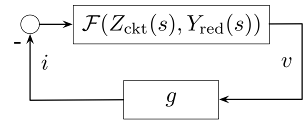
⇝ perfect sync of waveforms



Time-domain analysis

[S. Dhople, B. Johnson, FD, & A. Hamadeh, '13]

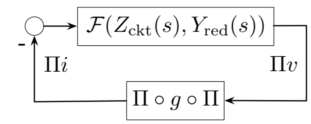
❶ **Compartmentalization** of linear and nonlinear systems



Linear fractional transformation:
$\mathcal{F}(G, H) = \left(I + GH^{-1}\right)^{-1} G$

❷ **Projection** $\Pi = \left(I_n - \frac{1}{n}\mathbb{1}_n\mathbb{1}_n^T\right)$

⇒ sync problem ⇝ stability problem



❸ apply **Lure system** analysis: passivity, $\mathcal{L}_2$ small-gain, IQC,...

frequency domain **sync** criterion: "stability of $\mathcal{F}$" > "instability of g"

❹ **Liénard limit-cycle condition:** sync'd & decoupled system oscillates if

"instability of g" > "local dissipation"

for heterogeneous systems?

many open questions:

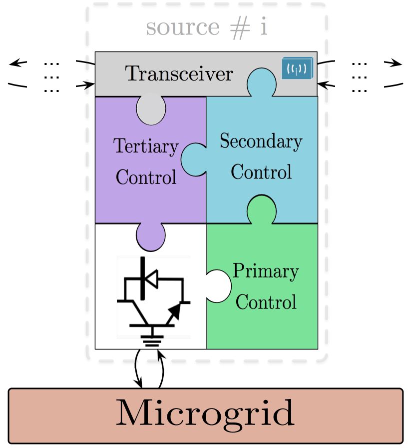
- some IQCs work only for some networks
- sync analysis of heterogeneous VOCs
- nonlinear constant power load models
- secondary amplitude & frequency control
- ...

conclusions

Conclusions

Summary

- primary $P/\dot{\theta}$ droop control
- new quadratic droop control
- fair proportional load sharing & economic dispatch optimization
- distributed secondary control strategies based on averaging
- virtual oscillator control
- experimental validation



Ongoing work & next steps

- better models & sharper analysis
- other energy management tasks
- solve these problems without comm
- many open problems for VOC inverters

addendum: proof of optimality of droop control

Key ingredients of the proof

1. **convexification** via flow bijection:

   AC flow: $P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j)$ DC flow: $P_i = \sum_j B_{ij}(\delta_i - \delta_j)$

   The flow map $\sin(\theta_i - \theta_j) = (\delta_i - \delta_j)$ is bijective in acyclic networks.

   Argument can be extended to **cyclic** networks [C. Zhao, E. Mallada, & FD, '14]

2. droop control is **surjective & 1-to-1**: $\exists$ droop coefficients to uniquely reach every feasible steady-state (with flow & injection constraints)

3. **KKT** conditions = steady state & identical marginal costs (= frequs)

$$\frac{\partial \mathcal{L}}{\partial \theta_i} = 0 : \; 0 = \sum_j \lambda_j \cdot \frac{\partial P_j(\theta)}{\partial \theta_i} \qquad \frac{\partial \mathcal{L}}{\partial \lambda_i} = 0 : \; -u_i = P_i^* - P_i(\theta) \text{ (controllable)}$$

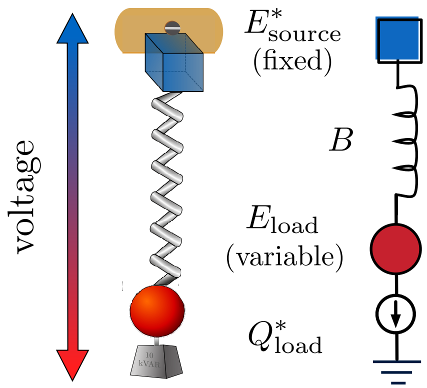
$$\frac{\partial \mathcal{L}}{\partial u_i} = 0 : \; 2\alpha_i u_i = -\lambda_i \qquad \frac{\partial \mathcal{L}}{\partial \lambda_i} = 0 : \; 0 = P_i^* - P_i(\theta) \text{ (passive)}$$

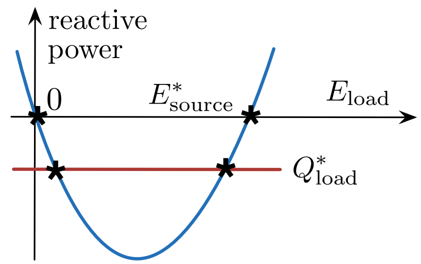
4. droop-controlled **dynamics** converge to stable KKT steady state

addendum: reactive power

Back of the envelope calculations

reactive power balance at load: $Q^*_{\text{load}} = B\, E_{\text{load}}(E_{\text{load}} - E^*_{\text{source}})$



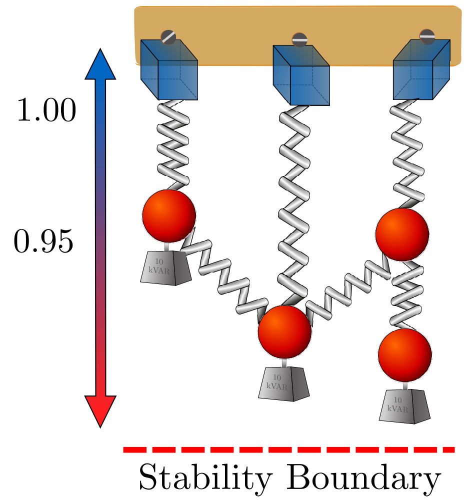


$E_{\text{load}} \in \mathbb{R} \;\Leftrightarrow\; Q^*_{\text{load}} \geq -B\,(E^*_{\text{source}})^2/4$

$\exists$ high load voltage solution $\Leftrightarrow$ (load) < (network)(source voltage)$^2/4$

Intuition extends to complex networks – essential insights

Reactive power balance:
$Q_i = -\sum_j B_{ij} E_i E_j$



Suff. & tight cond' for general case [J. Simpson-Porco, FD, & F. Bullo, '14]:

$\exists$ unique high-voltage solution E^*_{load}

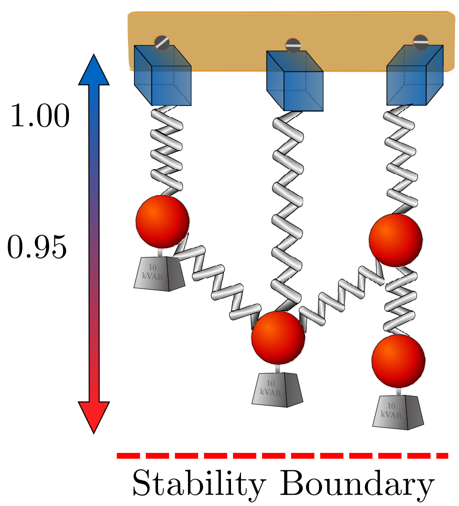
$\Leftrightarrow$

$$\frac{4 \cdot \text{load}}{(\text{admittance})(\text{nominal voltage})^2} < 1$$

1. via nominal (zero load) voltage E_{nom}
   $0 = -\sum_j B_{ij}\, E_{i,\text{nom}}\, E_{j,\text{nom}}$
2. coord-trafo to solution guess:
   $x_i = E_i/E_{i,\text{nom}} - 1$
3. Picard fixed point iteration:
   $x(k+1) = f(x(k))$

Intuition extends to complex networks – essential insights

Reactive power balance:
$Q_i = -\sum_j B_{ij} E_i E_j$



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$\exists$ unique high-voltage solution E^*_{load}

$\Leftrightarrow$

$$\frac{4 \cdot \text{load}}{(\text{admittance})(\text{nominal voltage})^2} < 1$$

Moreover . . . [B. Gentile, J. Simpson-Porco, FD, S. Zampieri, & F. Bullo, '14]

1. load flow Jacobian at E^*_{load} is Hurwitz $\Rightarrow$ voltage stability
2. linear $\mathcal{O}(1/{E^*_{\text{source}}}^3)$ approx:
   $E^*_{\text{load}} \approx E_{\text{nom}} - B^\dagger Q^*_{\text{load}}/E_{\text{nom}}$

primary control of reactive power

Decentralized primary control of reactive power

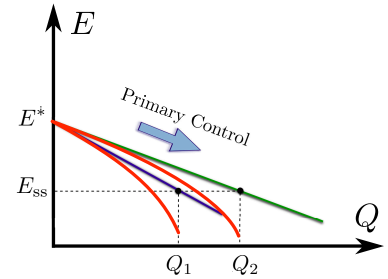
Recall: $Q_i(E) = -\sum_j B_{ij} E_i E_j$

Heuristic linear Q/E droop:

$$(E_i - E_i^*) \propto (Q_i^* - Q_i(E))$$

Implemented with integrator:

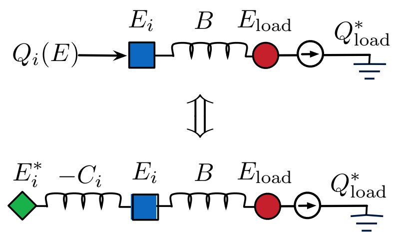
$$\tau_i \dot{E}_i = -C_i (E_i - E_i^*) - Q_i(E)$$



Mostly works but hardly tractable & conflicts with network (a)symmetries

Circuit theory suggests **quadratic & asymmetric droop control** [J. Simpson-Porco, FD, & F. Bullo, '13]:

$$\tau_i \dot{E}_i = -C_i E_i (E_i - E_i^*) - Q_i(E)$$

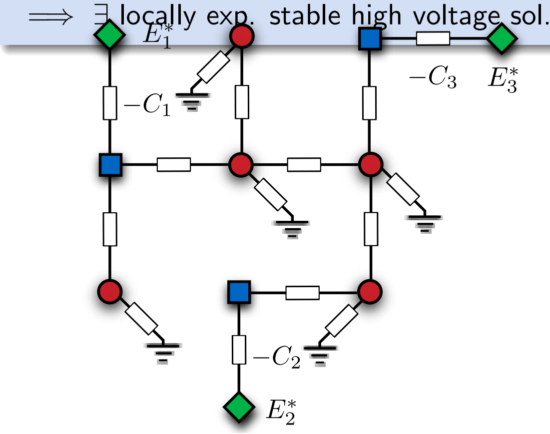


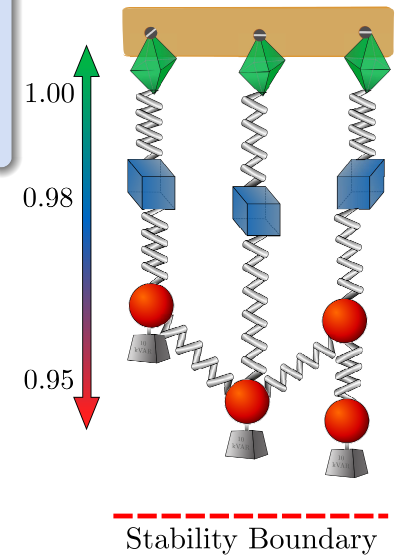
Closed-loop stability under quadratic droop control

Corollary combining previous results

$$\frac{4 \cdot \text{load}}{(\text{nominal voltage})^2 \times (\text{admittance})} < 1$$

$\Longrightarrow \exists$ locally exp. stable high voltage sol.





secondary control of reactive power

Active & reactive power DAPI control

DAPI control for **reactive power sharing** [J. Simpson-Porco, FD, & F. Bullo, '12]

$$D_i \dot{\theta}_i = P_i^* - P_i(\theta) - \Omega_i$$

$$k_i \dot{\Omega}_i = D_i \dot{\theta}_i - \sum_{j \subseteq \text{sources}} a_{ij} \cdot \left(\frac{\Omega_i}{D_i} - \frac{\Omega_j}{D_j}\right)$$

$$\tau_i \dot{E}_i = -C_i E_i (E_i - E_i^*) - Q_i(E) - e_i$$

$$\kappa_i \dot{e}_i = -\sum_{j \subseteq \text{sources}} a_{ij} \cdot \left(\frac{Q_i}{\overline{Q}_i} - \frac{Q_j}{\overline{Q}_j}\right) - \varepsilon e_i$$

Reactive DAPI control =

(quadratic droop) $\cap$ ((injection ratio averaging) $\cup$ $\varepsilon\cdot$ (voltage regulation))

- **Case** $\varepsilon \to \infty \Rightarrow$ steady-state voltage regulation
- **Case** $\varepsilon \to 0 \Rightarrow$ reactive load sharing (with non-unique voltages)

  [J. Schiffer, T. Seel, J. Raisch, & T. Sezi, '14] & [L.Y. Yu & C.C. Chu '14]

Active & reactive power DAPI control

DAPI control for **reactive power sharing** [J. Simpson-Porco, FD, & F. Bullo, '12]

$$D_i\dot{\theta}_i = P_i^* - P_i(\theta) - \Omega_i$$

$$k_i\dot{\Omega}_i = D_i\dot{\theta}_i - \sum_{j \subseteq \text{sources}} a_{ij} \cdot \left(\frac{\Omega_i}{D_i} - \frac{\Omega_j}{D_j}\right)$$

$$\tau_i\dot{E}_i = -C_iE_i(E_i - E_i^*) - Q_i(E) - e_i$$

$$\kappa_i\dot{e}_i = -\sum_{j \subseteq \text{sources}} a_{ij} \cdot \left(\frac{Q_i}{\overline{Q}_i} - \frac{Q_j}{\overline{Q}_j}\right) - \varepsilon e_i$$