ETH zürich

Feedback Optimization on the Power Flow Manifold

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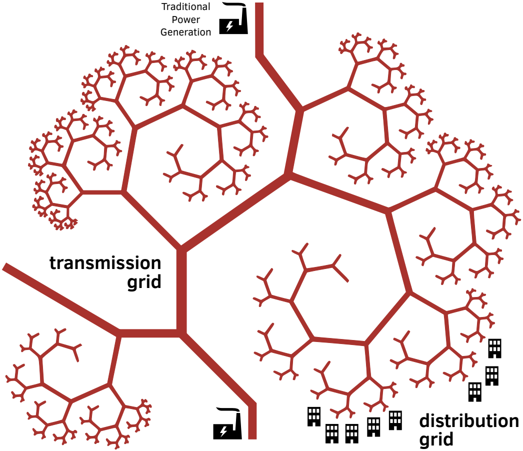
HORIZON 2020

ABB

Rte
Réseau de transport d'électricité

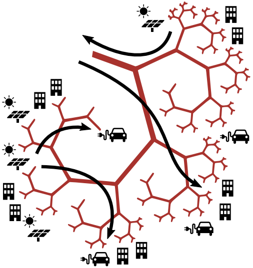
Power system operation: supply chain without storage

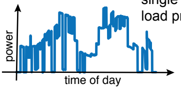


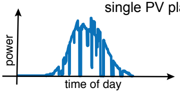
- **principle:** deliver power from generators to loads
- **physical constraints:** Kirchhoff's and Ohm's laws
- **operational constraints:** thermal and voltage limits
- **performance objectives:** running costs, reliability, quality of service
- **fit-and-forget design:** historically designed according to worst-case possible demand

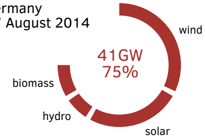
New challenges and opportunities

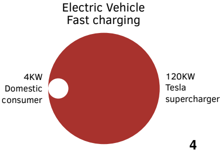
- **variable renewable energy sources**
  - poor short-range prediction & correlations
  - fluctuations on all time scales (low inertia)
- **distributed microgeneration**
  - conventional and renewable sources
  - congestion and under-/over-voltage
- **electric mobility**
  - large peak (power) & total (energy) demand
  - flexible but spatio-temporal patterns
- **inverter-interfaced storage/generation**
  - extremely fast actuation
  - modular & flexible control
- **information & comm technology**
  - inexpensive reliable communication
  - increasingly ubiquitous sensing





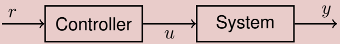






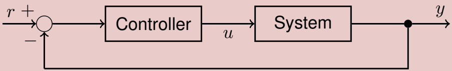
Recall: feedforward vs. feedback or optimization vs. control

feedforward optimization



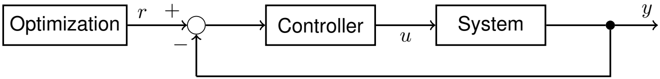
- highly model based
- computationally intensive
- **optimal decision**
- **operational constraints**

feedback control



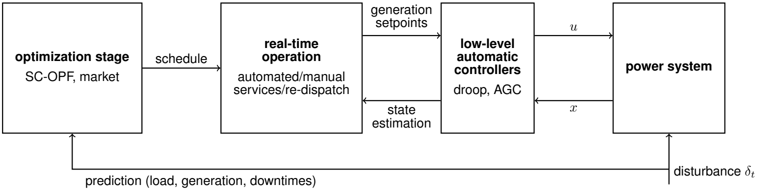
- **model-free (robust) design**
- **fast response**
- suboptimal operation
- unconstrained operation

$\Rightarrow$ typically **complementary** methods are combined via **time-scale separation**

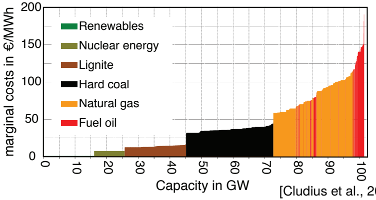


offline & feedforward | **real-time & feedback**

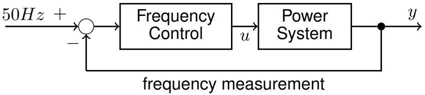
Example: power systems load / generation balancing



- **optimization stage**
  economic dispatch based
  on load/renewable prediction
- **real-time interface**
  manual re-dispatch,
  area balancing services
- **low-level automatic control**
  frequency regulation at
  the individual generators



[Cludius et al., 2014]



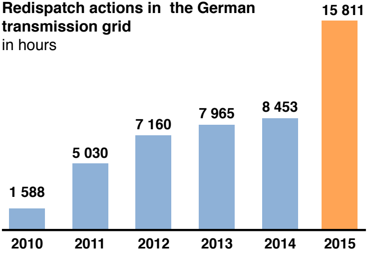
The price for time-scale separation: sky-rocketing re-dispatch

- **re-dispatch** to deal with unforeseen load, congestion, & renewables

⇒ ever more **uncertainty** & **fluctuations** on all time scales

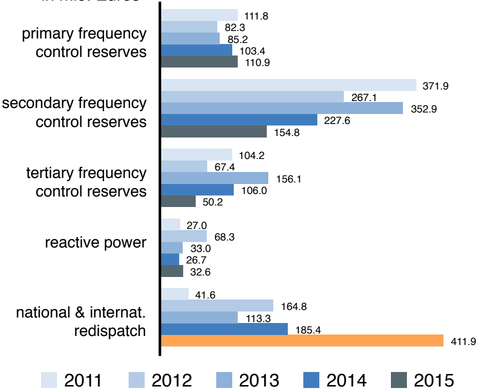
⇒ operation architecture becomes **infeasible & inefficient**

Redispatch actions in the German transmission grid
in hours



[Bundesnetzagentur, Monitoringbericht 2016]

Cost of ancillary services of German TSOs
in mio. Euros



[Bundesnetzagentur, Monitoringbericht 2016]

There must be a better way of operation.

Synopsis . . . for essentially all ancillary services

- real-time balancing
- frequency control
- economic re-dispatch
- voltage regulation
- voltage collapse prevention
- line congestion relief
- reactive power compensation
- losses minimization

recall new challenges:

- increased variability
- poor short-term prediction
- correlated uncertainties

recall new opportunities:

- fast actuation
- ubiquitous sensing
- reliable communication

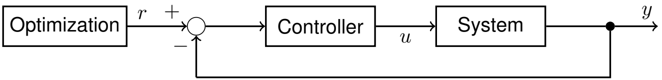
Today: these services are partially automated, implemented independently, online or offline, based on forecasts (or not), and operating on different time/spatial scales.

One central paradigm of "smart(er) grids" : real-time operation

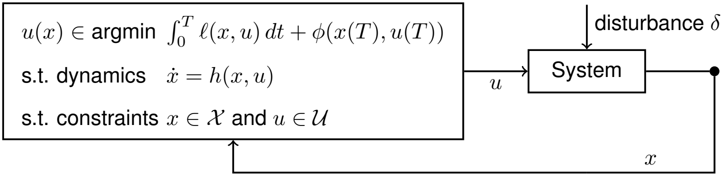
Future power systems will require faster operation, based on online control and monitoring, in order to meet the operational specifications in real time.

Control-theoretic core of the problem

- **time-scale separation** of complementary feedback/feedforward architectures



- ideal approach: **optimal feedback policies** (from HJB, Pontryagin, etc.)

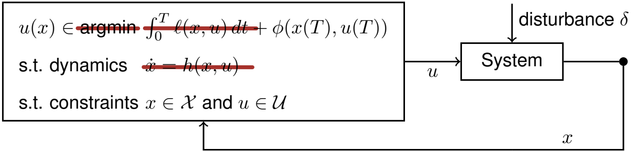


→ explicit ($T = \infty$) feedback policies are **not tractable** analytically or computationally

→ usually a decent trade-off: receding horizon model predictive control **MPC**

  ⇒ not suited for power systems (due to dimension, robustness, uncertainty, etc.)

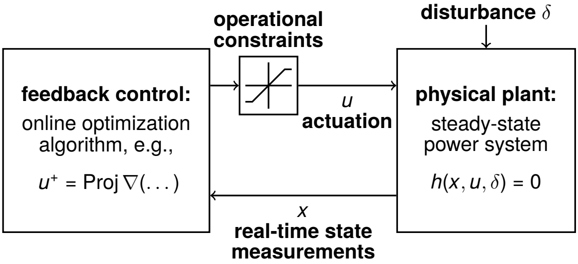
Today we will follow a different approach



- drop exact argmin
- drop integral/stage costs
- let physics solve equality constraints (dynamics)

Instead we apply **online optimization in closed loop** with fast/stationary physics:

- robust (feedback)
- fast response
- operational constraints
- steady-state optimal



Very brief review on related online optimization in closed loop

- **historical roots**: optimal routing and queuing in communication networks, e.g., in the internet (TCP/IP) [Kelly et al. 1998/2001, Low, Paganini, and Doyle 2002, Srikant 2012, . . .]

- lots of recent theory development in **power systems** & other infrastructures

  lots of related work: [Bolognani et. al, 2015], [Dall'Anese and Simmonetto, 2016/2017], [Gan and Low, 2016], [Tang and Low, 2017], . . .

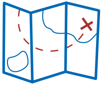
  A Survey of Distributed Optimization and Control Algorithms for Electric Power Systems

  Daniel K. Molzahn,* *Member, IEEE*, Florian Dörfler,† *Member, IEEE*, Henrik Sandberg,‡ *Member, IEEE*, Steven H. Low,§ *Fellow, IEEE*, Sambuddha Chakrabarti,¶ *Student Member, IEEE*, Ross Baldick,¶ *Fellow, IEEE*, and Javad Lavaei,** *Member, IEEE*

- **MPC version** of "dropping argmin": real-time iteration [Diel et al. 2005], real-time MPC [Zeilinger et al. 2009], . . . and related papers with *anytime* guarantees

- independent literature in **process control** [Bonvin et al. 2009/2010] or **extremum seeking** [Krstic and Wang 2000], . . . and probably much more

- plenty of interesting recent **system theory** coming out [Nelson and Mallada 2017]

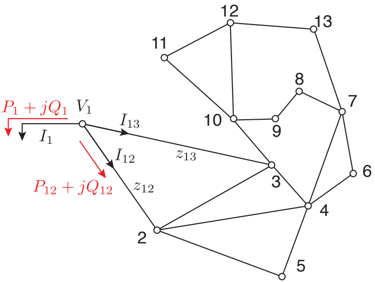


OVERVIEW

1. Problem setup & preview of a solution
2. Technical ingredient I: the power flow manifold
3. Technical ingredient II: manifold optimization
4. Case studies: tracking, feasibility, & dynamics

AC power flow model, constraints, and objectives

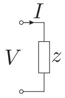
- **quasi-stationary** (for now) dynamics



V_k nodal voltage
I_k current injection
P_k, Q_k power injections

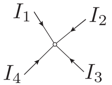
z_{kl} line impedance
I_{kl} line current
P_{kl}, Q_{kl} power flow

Ohm's Law



$V = zI$

Current Law



$0 = I_1 + \ldots + I_k$

AC power $S = P + jQ = VI^*$

AC power flow equations

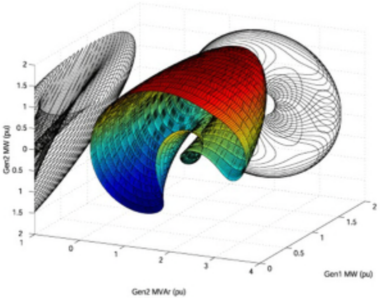
$$S_k = \sum_{l \in N(k)} \frac{1}{z^*_{kl}} V_k (V_k^* - V_l^*) \quad \forall k \in \mathcal{N}$$

(all variables and parameters are $\mathbb{C}$-valued)

- **objective:** economic dispatch, minimize losses, distance to collapse, etc.
- **operational constraints:** generation capacity, voltage bands, congestion
- **control:** state measurements and actuation via generation set-points

What makes power flow optimization interesting?

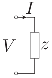
graphical illustration of AC power flow



[Hiskens, 2001]

- imagine **constraints slicing** this set
  $\Rightarrow$ nonlinear, non-convex, disconnected
- additionally the parameters are $\pm 20\%$ **uncertain** …this is only the steady state!

Ohm's Law



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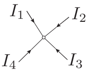
Current Law



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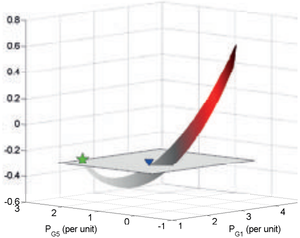


[Molzahn, 2016]

Ancillary services as a real-time optimal power flow

Offline optimal power flow (OPF)

minimize	$\phi(x,u)$	e.g., losses, generation
subject to	$h(x,u,\delta) = \mathbb{0}$	AC power flow
	$(x,u) \in \mathcal{X} \times \mathcal{U}$	operational constraints

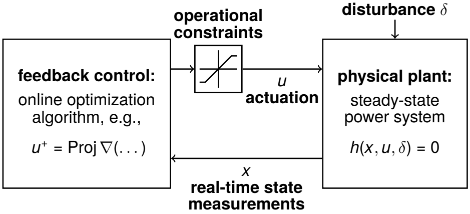
- exogenous variables
  - $\rightarrow$ u controllable generation
  - $\rightarrow$ δ exogenous disturbances (e.g., loads & renewables)
- x endogenous variables (voltages)

Idea for an online algorithm

- goal: **closed-loop gradient flow**

$$\begin{bmatrix} \dot{x} \\ \dot{u} \end{bmatrix} = -\text{Proj}_{\mathcal{U}\cap\mathcal{X}\cap\{\text{linearization of } h\}} \nabla\phi(x,u)$$

- implement **control** $\dot{u}$ (as above)
- **consistency** of x ensured by non-singular physics $h(x,u,\delta) = \mathbb{0}$
- **discrete-time** implementation

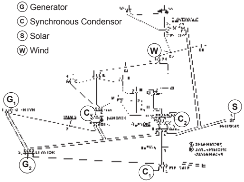


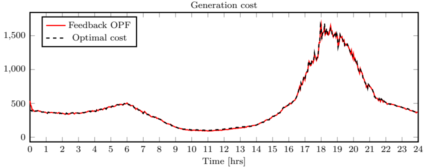
Pretty hand-waivy . . . I know.

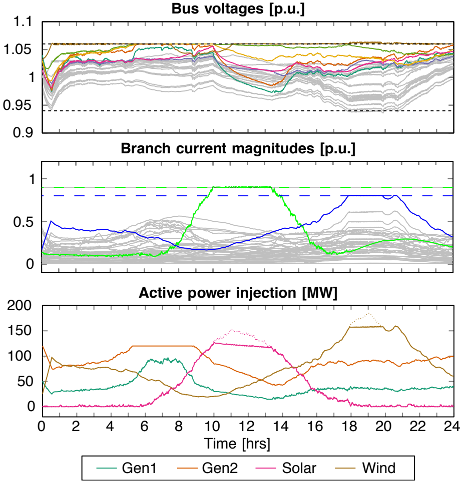
I will make it more precise later.

Let's see if it works!

Preview: simple algorithm solves many problems





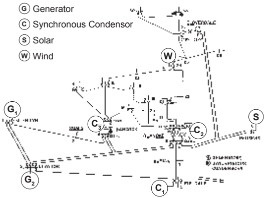


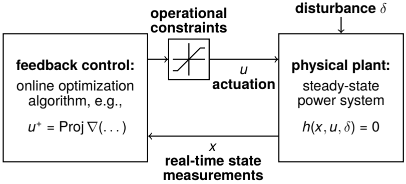
controller: gradient + saturation

 ∇(generation + voltage violation)

- time-variant disturbances/constraints ✓
- robustness to noise & uncertainty ✓
- dynamics of physical system ✓
- crude discretization/linearization ✓

Preview cont'd: robustness to model mismatch





gradient controller:

- saturation of generation constraints
- soft penalty for operational constraints

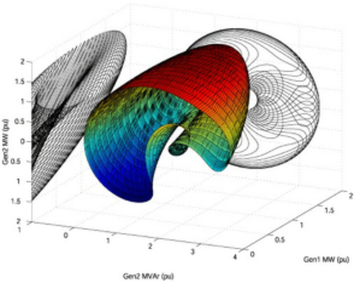
	no automatic re-dispatch			feedback optimization		
model uncertainty	feasible ?	$f - f^*$	$\|v - v^*\|$	feasible ?	$f - f^*$	$\|v - v^*\|$
loads $\pm 40\%$	no	94.6	0.03	yes	0.0	0.0
line params $\pm 20\%$	yes	0.19	0.01	yes	0.01	0.003
2 line failures	no	-0.12	0.06	yes	0.19	0.007

conclusion: simple algorithm performs extremely well & robust → **closer look!**

TECHNICAL INGREDIENT I:

THE POWER FLOW MANIFOLD

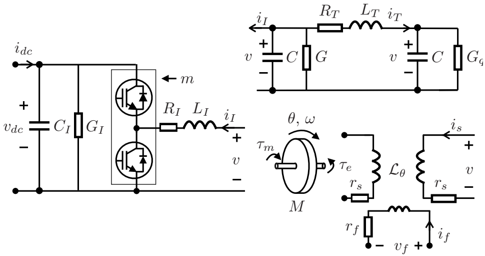
Key insights about our physical equality constraint



- AC power flow is complex but it defines a **smooth manifold**

→ local tangent plane approximations & $h(x, u, \delta) = \mathbb{0}$ locally solvable for x

→ Bolognani & Dörfler (2015)
"Fast power system analysis via implicit linearization of the power flow manifold"

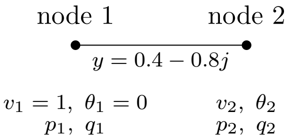


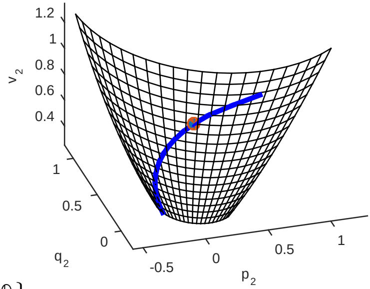
- AC power flow is **attractive*** **steady state** for ambient physical dynamics

→ physics enforce feasibility even for non-exact (e.g., discretized) updates

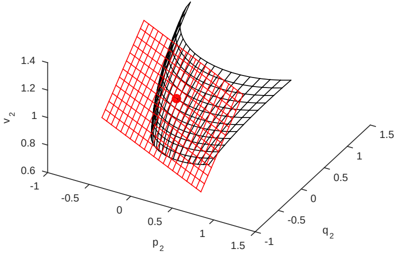
→ Gross, Arghir, & Dörfler (2018)
"On the steady-state behavior of a nonlinear power system model"

Geometric perspective: the power flow manifold

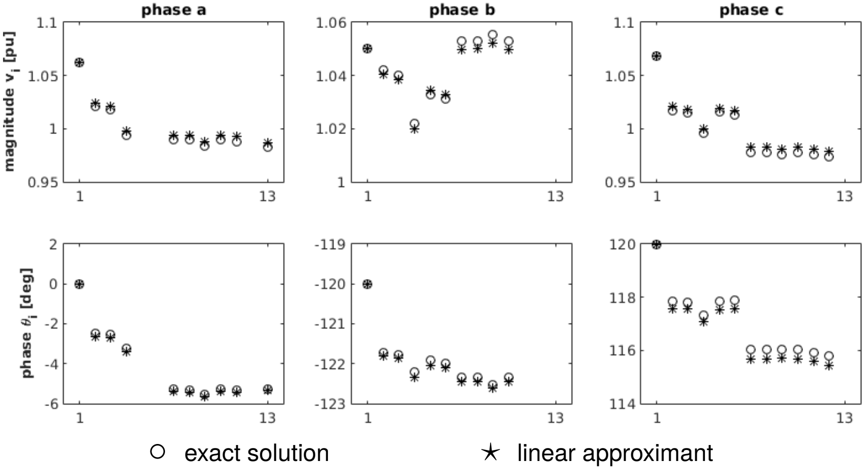




- **variables:** all of $x = (|V|, \theta, P, Q)$
- **power flow manifold:** $\mathcal{M} = \{x : h(x) = \mathbb{0}\}$
  $\rightarrow$ submanifold in $\mathbb{R}^{2n}$ or $\mathbb{R}^{6n}$ (3-phase)
- **tangent space** $\frac{\partial h(x)}{\partial x}\Big|_{x^*}^{\top} (x - x^*) = \mathbb{0}$
  $\rightarrow$ best linear approximant at x^*
- **accuracy** depends on curvature $\frac{\partial^2 h(x)}{\partial x^2}$
  $\rightarrow$ constant in rectangular coordinates



Accuracy illustrated with unbalanced three-phase IEEE13



dirty secret: power flow manifold is very flat (linear) near usual operating points

→ **Matlab/Octave code** @ `https://github.com/saveriob/1ACPF`

Coordinate-dependent linearizations reveal old friends

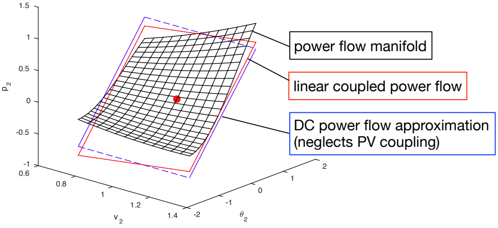
- **flat-voltage/0-injection point:** $x^* = (|V|^*, \theta^*, P^*, Q^*) = (\mathbb{1}, \mathbb{0}, \mathbb{0}, \mathbb{0})$

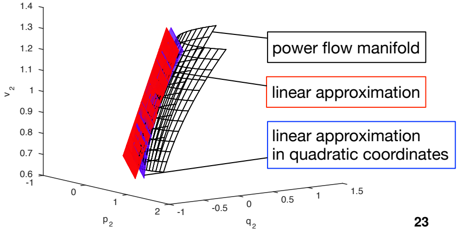
$\Rightarrow$ tangent space parameterization $\begin{bmatrix} \Re(Y) & -\Im(Y) \\ -\Im(Y) & \Re(Y) \end{bmatrix} \begin{bmatrix} |V| \\ \theta \end{bmatrix} = \begin{bmatrix} P \\ Q \end{bmatrix}$

is **linear coupled power flow** and $\Re(Y) \approx \mathbb{0}$ gives **DC power flow** approximation

- nonlinear change to **quadratic coordinates** $|V| \to |V|^2$

$\Rightarrow$ linearization is (non-radial) **LinDistFlow** [M.E. Baran and F.F. Wu, '88] $\Rightarrow$ more exact in $|V|$





TECHNICAL INGREDIENT II:

MANIFOLD OPTIMIZATION

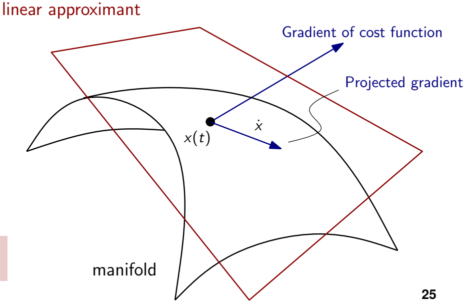
Unconstrained manifold optimization: the smooth case

- **geometric objects:**

manifold	$\mathcal{M} = \{x : h(x) = \mathbb{0}\}$	objective	$\phi : \mathcal{M} \to \mathbb{R}$
tangent space	$T_x\mathcal{M} = \ker \frac{\partial h(x)}{\partial x}^\top$	Riemann metric (degree of freedom)	$g : T_x\mathcal{M} \times T_x\mathcal{M} \to \mathbb{R}$

- **target state:** local minimizer on the manifold $x^\star \in \arg\min_{x \in \mathcal{M}} \phi(x)$
- **always feasible** $\leftrightarrow$ trajectory/sequence $x(t)$ remains on manifold $\mathcal{M}$
- continuous-time **gradient descent** on $\mathcal{M}$:

1. $\text{grad}\,\phi(x)$: **gradient** of cost function in ambient space
2. $\Pi_{\mathcal{M}}(x, -\text{grad}\phi(x))$: **projection** of gradient on tangent space $T_x\mathcal{M}$
3. **flow** on manifold: $\dot{x} = \Pi_{\mathcal{M}}(x, -\text{grad}\phi(x))$



Constrained manifold optimization: the wild west

dealing with **operational constraints** $g(x) \leq 0$

1. penalty in cost function ϕ

$\rightarrow$ barrier: not practical for online implementation

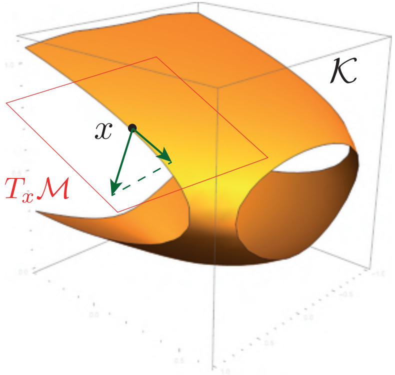
$\rightarrow$ soft penalty: practical but no real-time feasibility

2. dualization and gradient flow on Lagrangian

$\rightarrow$ poor performance & no real-time feasibility

$\rightarrow$ theory: close to none available on manifolds

$\rightarrow$ Hauswirth, Bolognani, Hug, & Dörfler (2018)
"Generic Existence of Unique Lagrange Multipliers in AC Optimal Power Flow"



3. projection of gradient flow trajectory $x(t)$ on feasible set $\mathcal{K} = \mathcal{M} \cap \{g(x) \leq 0\}$

$$\dot{x} = \Pi_{\mathcal{K}}(x, -\text{grad}\phi(x)) \in \arg \min_{v \in T_x^{>}\mathcal{K}} \| -\text{grad}\, \phi(x) - v\|_g$$

where $T_x^{>}\mathcal{K} \subset T_x\mathcal{M}$ is inward tangent cone

Projected gradient descent on manifolds

$\mathcal{K} = \left\{ x : \|x\|_2^2 = 1 \,,\ \|x\|_1 \leq \sqrt{2} \right\}$

Theorem (simplified)

Let $x : [0, \infty) \to \mathcal{K}$ be a Carathéodory solution of the initial value problem

$$\dot{x} = \Pi_{\mathcal{K}}\left(x, -\text{grad}\phi(x)\right) \,, \quad x(0) = x_0 \,.$$

If ϕ has compact level sets on $\mathcal{K}$, then $x(t)$ will converge to a critical point $x^\star$ of ϕ on $\mathcal{K}$.

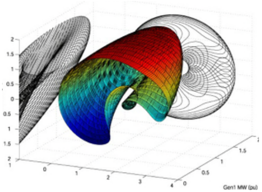
→ Hauswirth, Bolognani, Hug, & Dörfler (2016)
"Projected gradient descent on Riemanniann manifolds with applications to online power system optimization"

Hidden assumption: existence of a Carathéodory solution $x(t) \in \mathcal{K}$

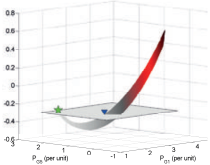
→ when does it exist, is forward complete, unique, and sufficiently regular ?

(in absence of convexity, Euclidean space, and other regularity properties)

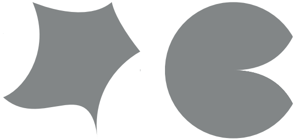
Analysis via projected systems hit mathematical bedrock



power flow manifold



disconnected regions



cusps & corners (convex and/or inward)

	constraint set	gradient field	metric	manifold
existence (Krasovski)	loc. compact	loc. bounded	-	C^1
Krasovski = Carathéodory	Clarke regular	C^0	C^0	C^1
uniqueness of solutions	prox regular	$C^{0,1}$	$C^{0,1}$	$C^{1,1}$

→ also forward-Lipschitz continuity of time-varying constraints

→ continuity with respect to initial conditions and parameters

→ Hauswirth, Bolognani, Hug, & Dörfler (2018) **"Projected Dynamical Systems on Irregular, Non-Euclidean Domain for Nonlinear Optimization"**

→ Hauswirth, Subotic, Bolognani, Hug, & Dörfler (2018) **"Time-varying Projected Dynamical Systems with Applications to Feedback Optimization of Power Systems"**

Implementation issue: how to **induce** the gradient flow?

Open-loop system

$\dot{x}_1 = u$ controlled generation

$\mathbb{0} = h(x_1, x_2, \delta)$ AC power flow manifold relating x_1 & other variables

Desired closed-loop system

$\dot{x}_1 = f_1(x_1, x_2)$ desired projected

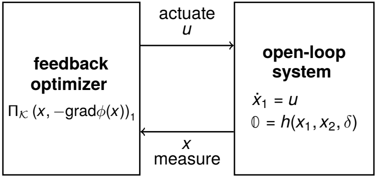
$\dot{x}_2 = f_2(x_1, x_2)$ gradient descent

where $f(x) = \Pi_{\mathcal{K}}(x, -\mathrm{grad}\phi(x))$

Solution use **non-singularity** of the physics: $\mathbb{0} = h(x_1, x_2, \delta)$ can be solved for x_2

Feedback equivalence

The trajectories of the desired closed loop **coincide** with those of the open loop under the feedback $u = f_1(x_1, x_2)$.



$\rightarrow$ closed-loop trajectory remains feasible at all times and converges to optimality

$\rightarrow$ no need to numerically solve the optimization problem or any power flow equation

Implementation issue: discrete-time manifold optimization

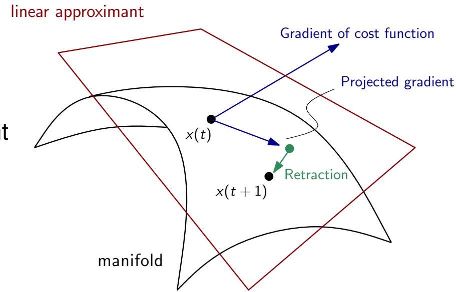
- **always feasible** $\leftrightarrow$ trajectory/sequence $x(t)$ remains on manifold $\mathcal{M}$

- **discrete-time** gradient descent on $\mathcal{M}$:

1. grad $\phi(x)$: **gradient** of cost function
2. $\Pi_{\mathcal{M}}(x, -\text{grad}\phi(x))$: **projection** of gradient
3. **Euler integration** of gradient flow:

$$\tilde{x}(t+1) = x(t) - \varepsilon\, \Pi_{\mathcal{M}}(x, -\text{grad}\phi(x))$$

4. **retraction step**: $x(t+1) = \mathcal{R}_{x(t)}\left(\tilde{x}(t+1)\right)$



Discrete-time control implementation:

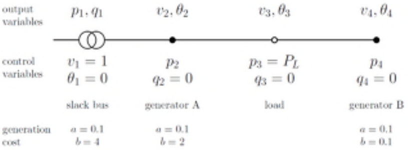
$\rightarrow$ manifold is attractive steady state for ambient dynamics

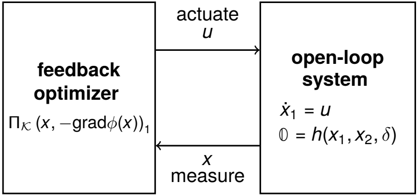
$\rightarrow$ retraction is taken care of by the physics: "nature enforces feasibility"

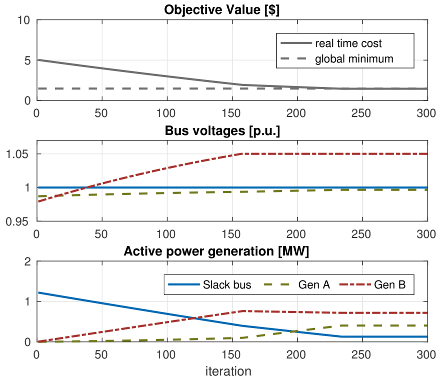
$\rightarrow$ can be made rigorous using singular perturbation theory (Tikhonov)

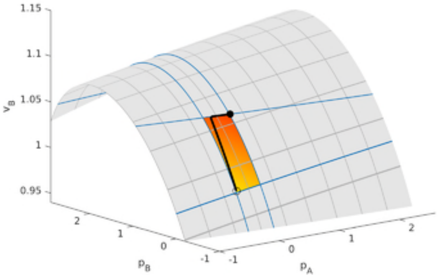
CASE STUDIES: TRACKING, FEASIBILITY, & DYNAMICS

Simple illustrative case study





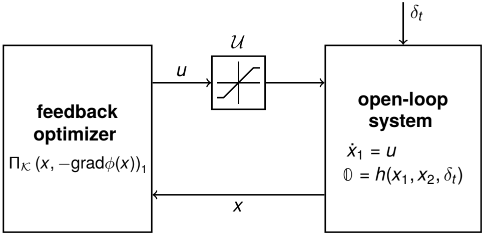




The tracking problem

- power system affected by **exogeneous time-varying inputs** δ_t

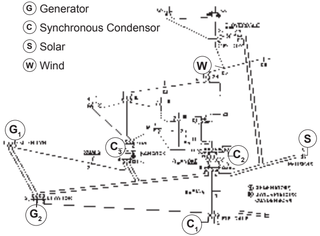
$\rightarrow$ under disturbances state could leave feasible region $\mathcal{K}$ (ill-defined)

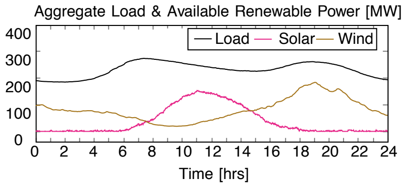


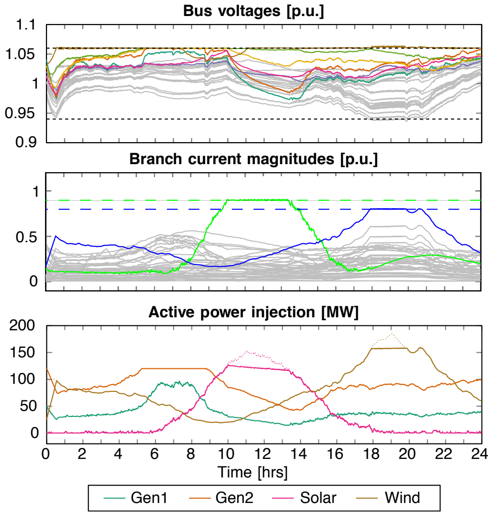
constraints satisfaction for non-controllable variables:

- $\mathcal{K}$ accounts only for **hard constraints** on controllable variables u (e.g., generation limits)
- gradient projection becomes **input saturation** (saturated proportional feedback control)
- **soft constraints** included via penalty functions in ϕ (e.g., thermal and voltage limits)

Tracking performance

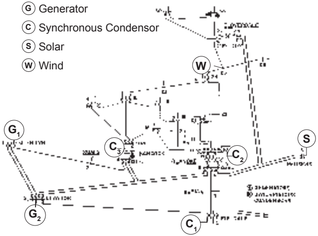






controller: penalty + saturation

Tracking performance



Comparison

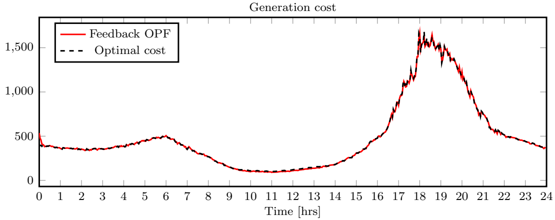
- closed-loop feedback trajectory
- benchmark: feedforward OPF

  (ground-truth solution of an ideal OPF with access to exact disturbance and without computation delay)

- practically **exact tracking**

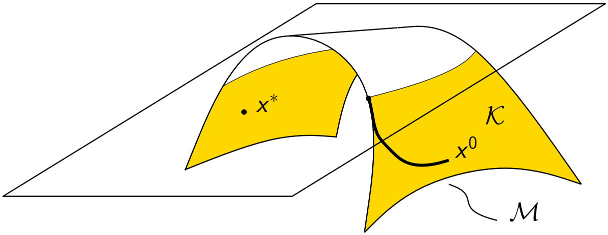
\+ trajectory **feasibility**

\+ **robustness** to model mismatch



Trajectory feasibility

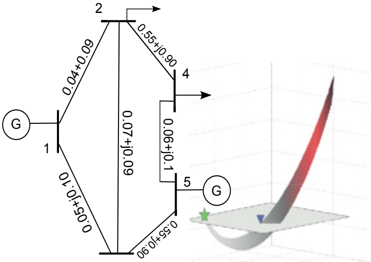
The feasible region $\mathcal{K} = \mathcal{M} \cap \mathcal{X}$ often has **disconnected components**.



- **feedforward (OPF)**
  - optimizer $x^\star = \arg\min_{x \in \mathcal{K}} \phi(x)$ can be in different **disconnected component**
  - $\rightarrow$ no feasible trajectory exists: $x_0 \rightarrow x^\star$ must **violate constraints**
- **feedback (gradient descent)**
  - $\rightarrow$ continuous closed-loop trajectory $x(t)$ guaranteed to be **feasible**
  - $\rightarrow$ convergence of $x(t)$ to a **local minimum** is guaranteed

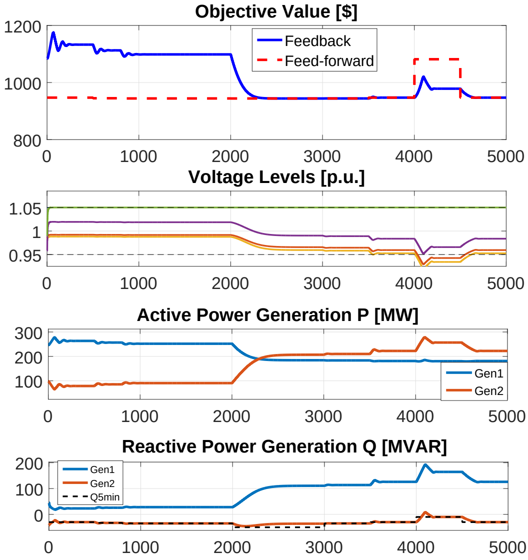
Illustration of continuous trajectories & reachability

5-bus example known to have two disconnected feasible regions:



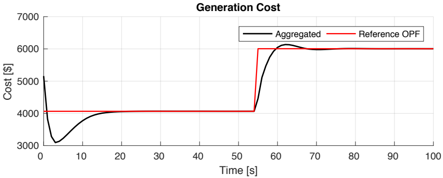
[Molzahn, 2016]

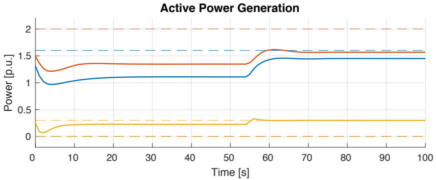
- [0s,2000s]: separate feasible regions
- [2000s,3000s]: loosen limits on reactive power $\underline{Q}_2$ → regions merge
- [4000s,5000s]: tighten limits on $\underline{Q}_2$ → vanishing feasible region

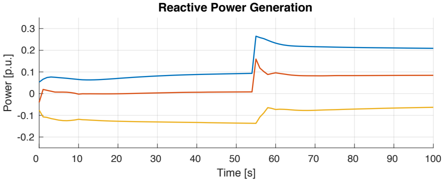


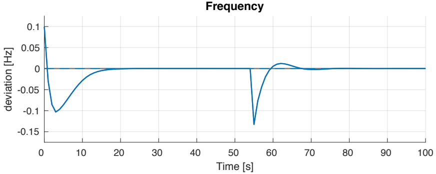
Feedback optimization with frequency

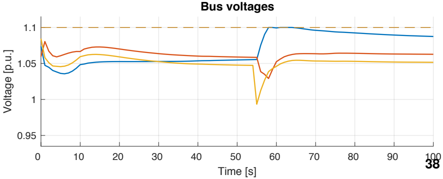
- **frequency** ω as global variable
- **primary control**: $P = P_G - K\omega$
- **secondary frequency control** incorporated via dual multiplier
- 20% step increase in load



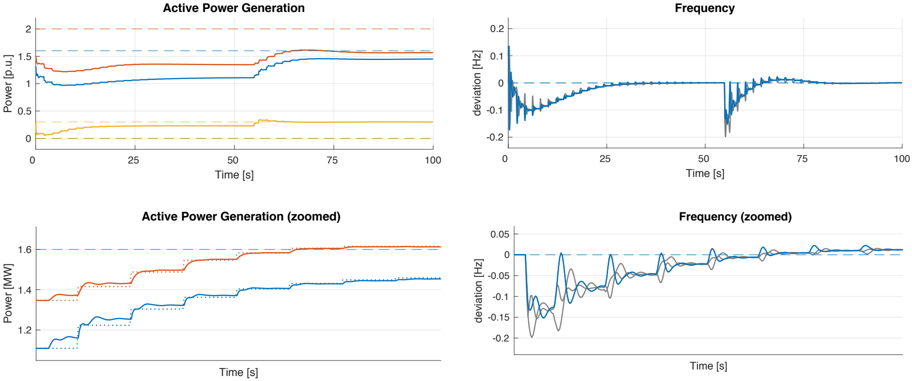








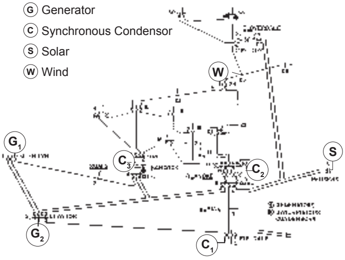
Same feedback optimization with grid dynamics

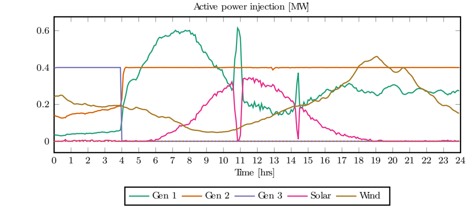


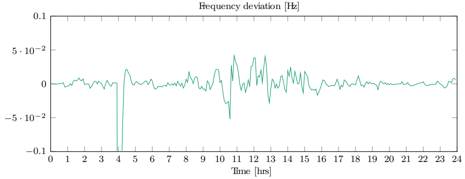
- **dynamic grid model:** swing equation & simple turbine governor
- **work in progress** based on singular perturbation methods

  $\Rightarrow$ dynamic and quasi-stationary dynamics are "close" and converge to the same optimal solutions under "sufficient" time-scale separation

Feedback optimization in dynamic IEEE 30-bus system





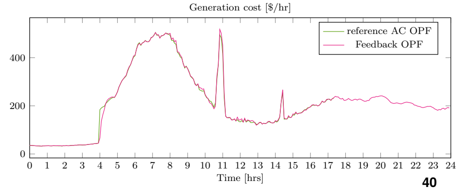


- **events:**

→ generator outage at 4:00

→ PV generation drops at 11:00 and 14:15

⇒ feedback optimization can provide **all ancillary services** + optimal + constraints + robust + scalable + . . .



Conclusions

Summary:

- necessity of **real-time power system operation**
- our starting point: **online optimization as feedback control**
- **technical approach:** manifold optimization & projected dyn. systems
- **unified framework** accommodating various constraints & objectives

Ongoing and future work:

- **fun:** questions on existence of trajectories, reachability, ...
- quantitative **guarantees for robustness**, tracking, etc.
- interaction of optimization algorithm with **low level grid dynamics**
- efficient **implementation**, discretization, experiments, RTE collaboration
- **extensions:** transient optimality à la MPC & model-free à la extremum seeking

Thanks !

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