

# Online Feedback Optimization

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ICSTCC 2023

# Acknowledgements

 Schweizerische Eidgenossenschaft  
Confédération suisse  
Confederazione Svizzera  
Confederaziun svizra

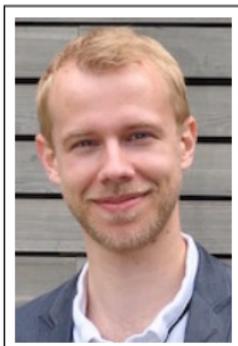
Bundesamt für Energie BFE  
Swiss Federal Office of Energy SFOE

 FNS-NF  
FONDS NATIONAL SUISSE  
SCHWEIZERISCHER NATIONALFONDS  
FONDO NAZIONALE SVIZZERO  
SWISS NATIONAL SCIENCE FOUNDATION

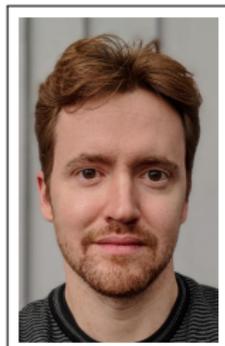
 **ETH**  
Eidgenössische Technische Hochschule Zürich  
Swiss Federal Institute of Technology Zurich



 **ABB Rte**  
Réseau de transport d'électricité



Adrian  
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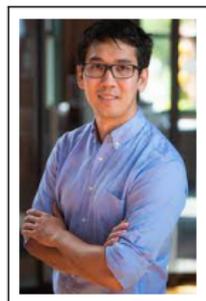
Saverio  
Bolognani



Lukas  
Ortmann



Zhiyu He



Dominic Liao  
McPherson



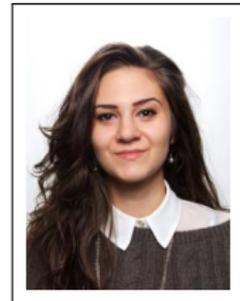
Giuseppe  
Belgioioso



Miguel Picallo



Verena Häberle

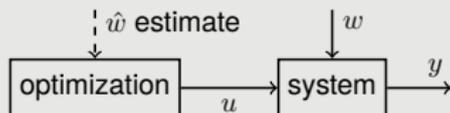


Irina Subotić

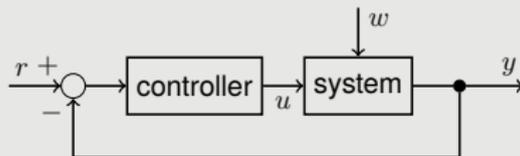
# feedforward optimization

vs.

# feedback control

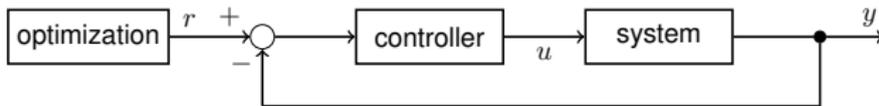


- **complex specifications & decision**  
optimal, constrained, & multivariable
- **strong requirements**  
precise model, full state, disturbance estimate, & computationally intensive



- **simple feedback policies**  
suboptimal, unconstrained, & SISO
- **forgiving nature of feedback**  
measurement driven, robust to uncertainty, fast & agile response

→ typically **complementary** methods are combined via **time-scale separation**

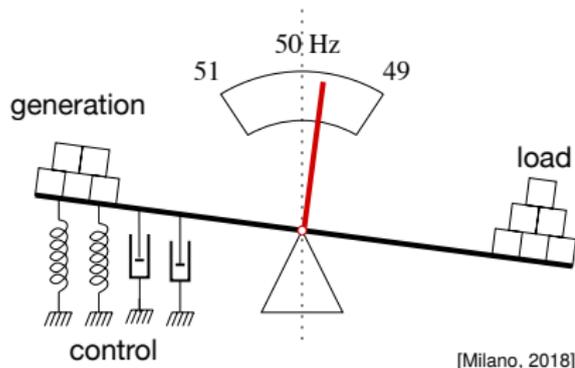
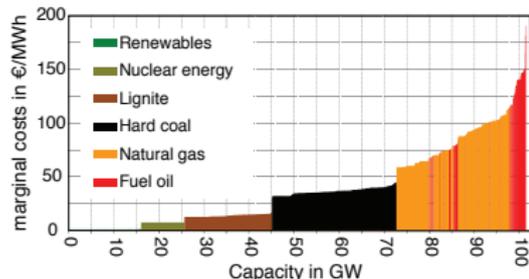


**offline & feedforward**

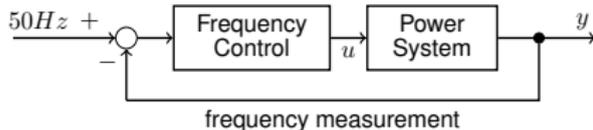
**real-time & feedback**

# Example: power system balancing

- **offline optimization**: dispatch based on forecasts of loads & renewables



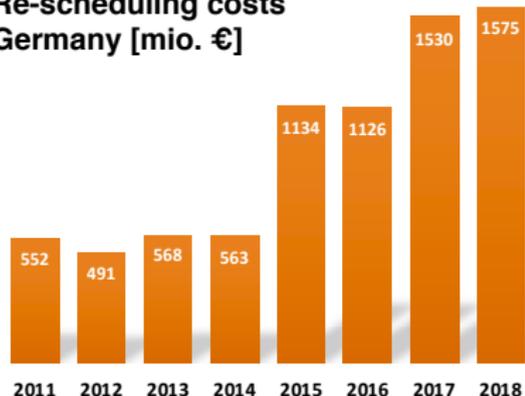
- **online control** based on frequency



- **re-schedule set-point** to mitigate severe forecasting errors (redispatch, reserve, etc.)

more uncertainty & fluctuations → **infeasible & inefficient** to separate optimization & control

## Re-scheduling costs Germany [mio. €]



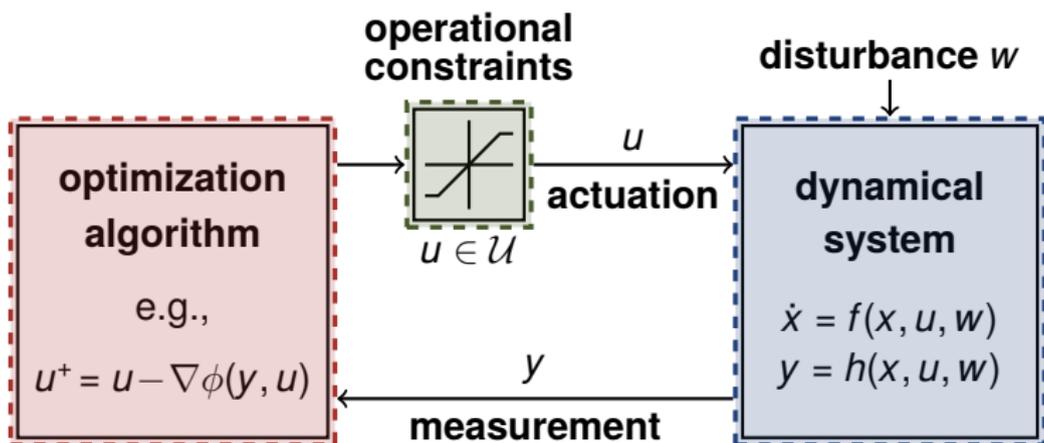
[Bundesnetzagentur, Monitoringbericht 2012-2019]

# Synopsis & proposal for control architecture

- **power grid**: separate decision layers hit limits under increasing uncertainty
- similar observations in other **large-scale & uncertain control systems** : process control systems & queuing/routing/infrastructure networks

proposal: **open** and **online optimization algorithm** as **feedback** control

with inputs & outputs      running & non-batch      real-time interconnected

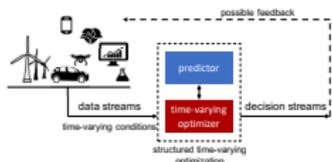


# Historical roots & conceptually related work

- **process control**: reducing the effect of uncertainty in successive optimization  
*Optimizing Control* [Garcia & Morari, 1981/84], *Self-Optimizing Control* [Skogestad, 2000], *Modifier Adaptation* [Marchetti et. al, 2009], *Real-Time Optimization* [Bonvin, ed., 2017], . . .
- **extremum-seeking**: derivative-free but hard for high dimensions & constraints  
[Leblanc, 1922], . . . [Wittenmark & Urquhart, 1995], . . . [Krstić & Wang, 2000], . . . , [Feiling et al., 2018]
- **MPC** with *anytime* guarantees (though for dynamic optimization): real-time MPC  
[Zeilinger et al. 2009], real-time iteration [Diel et al. 2005], [Feller & Ebenbauer 2017], etc.
- optimal routing, queuing, & congestion control in **communication networks**:  
e.g., TCP/IP [Kelly et al., 1998/2001], [Low, Paganini, & Doyle 2002], [Srikant 2012], [Low 2017], . . .
- **optimization algorithms as dynamic systems**: much early work [Arrow et al., 1958],  
[Brockett, 1991], [Bloch et al., 1992], [Helmke & Moore, 1994], . . . & recent revival [Holding & Lestas,  
2014], [Cherukuri et al., 2017], [Lessard et al., 2016], [Wilson et al., 2016], [Wibisono et al, 2016], . . .
- recent **system theory** approaches inspired by output regulation [Lawrence et al. 2018]  
& robust control methods [Nelson et al. 2017], [Colombino et al. 2018], [Simpson-Porco 2020], . . .

# Feedback optimization literature

- lots of recent theory development stimulated by **power systems** problems



## A Survey of Distributed Optimization and Control Algorithms for Electric Power Systems

Daniel K. Molzahn,<sup>\*</sup> *Member, IEEE*, Florian Dörfler,<sup>1</sup> *Member, IEEE*, Henrik Sandberg,<sup>2</sup> *Member, IEEE*, Steven H. Low,<sup>3</sup> *Fellow, IEEE*, Sambuddha Chakrabarti,<sup>4</sup> *Student Member, IEEE*, Ross Baldick,<sup>5</sup> *Fellow, IEEE*, and Javad Lavaei,<sup>\*\*</sup> *Member, IEEE*

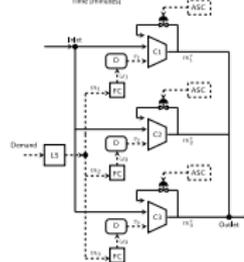
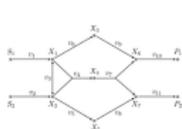
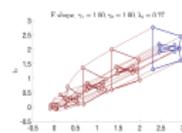
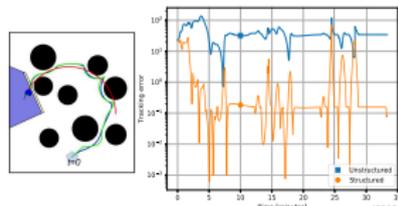
## Time-Varying Convex Optimization: Time-Structured Algorithms and Applications

Andrea Simonetto, Emiliano Dall'Anese, Santiago Paternain, Geert Leus, and Georgios B. Giannakis

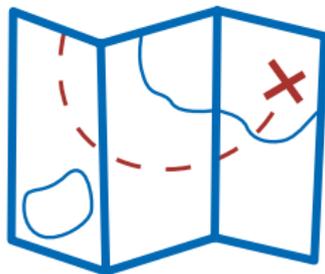
## Optimization Algorithms as Robust Feedback Controllers

Adrian Hauswirth, Saverio Bolognani, Gabriela Hug, and Florian Dörfler  
*Department of Information Technology and Electrical Engineering, ETH Zürich, Switzerland*

- theory** ↔ **power literature**: KKT control [Jokic et al, 2009] → really kick-started ~ 2013 by EU & US groups
- implemented** in microgrids (DTU, EPFL, Aachen ...), demo projects (PNNL, NREL), & commercially (AEW)
- feedback optimization increasingly **adopted** in robotics & process control domain + parallel work in comms
- recent theory**: distributed, games, nonlinear, data, ...



# Overview



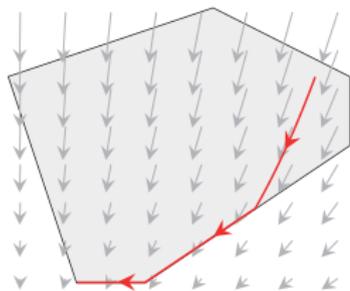
- **theory**: optimization algorithms in closed loop
  - stylized warm-up example & academic analysis
  - practical, robust, & performant extensions
- **power systems** case studies
  - device-level control & system-level operation
  - numerics, experiments, & industrial deployments

**ACADEMIC WARM-UP PROBLEM: STYLIZED  
ALGORITHM DESIGN & CLOSED-LOOP ANALYSIS**

# Stylized optimization problem & algorithm

## simple optimization problem

$$\begin{aligned} & \underset{y, u}{\text{minimize}} && \phi(y, u) \\ & \text{subject to} && y = h(u) \\ & && u \in \mathcal{U} \end{aligned}$$



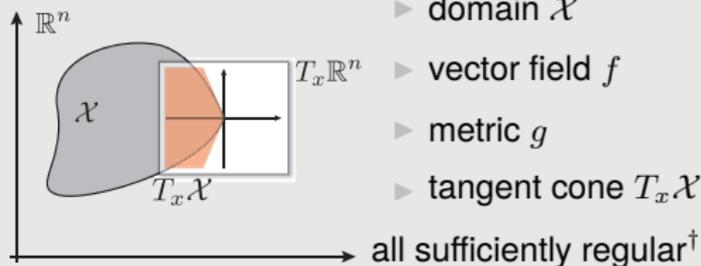
## cont.-time projected gradient flow

$$\begin{aligned} \dot{u} &= \Pi_{\mathcal{U}}^g \left( -\nabla \phi(h(u), u) \right) \\ &= \Pi_{\mathcal{U}}^g \left( - \left[ \frac{\partial h}{\partial u} \quad \mathbf{I} \right] \cdot \nabla \phi(y, u) \right) \Big|_{y=h(u)} \end{aligned}$$

**Fact:** a regular<sup>†</sup> solution  $u: [0, \infty] \rightarrow \mathcal{U}$  **converges** to critical points if  $\phi$  has Lipschitz gradient & compact sublevel sets.

## projected dynamical system

$$\dot{x} \in \Pi_{\mathcal{X}}^g [f](x) \triangleq \arg \min_{v \in T_x \mathcal{X}} \|v - f(x)\|_{g(x)}$$



# Algorithm in closed loop with LTI dynamics

## optimization problem

$$\underset{y,u}{\text{minimize}} \quad \phi(y,u)$$

$$\text{subject to} \quad y = H_{io}u + R_{do}w$$

$$u \in \mathcal{U}$$

→ open & scaled projected gradient flow

$$\dot{u} = \Pi_{\mathcal{U}} \left( -\epsilon [H_{io}^T \quad \mathbb{I}] \cdot \nabla \phi(y,u) \right)$$

## LTI dynamics

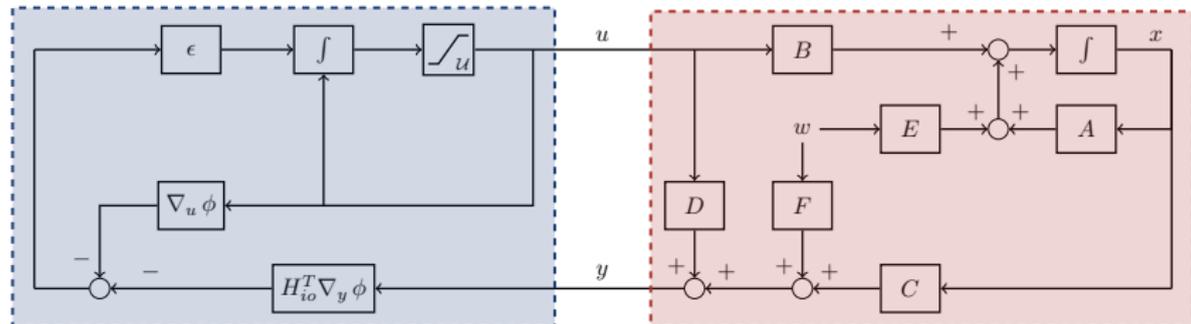
$$\dot{x} = Ax + Bu + Ew$$

$$y = Cx + Du + Fw$$

const. disturbance  $w$  & steady-state maps

$$x = \underbrace{-A^{-1}B}_{H_{is}} u \quad \underbrace{-A^{-1}E}_{R_{ds}} w$$

$$y = \underbrace{(D - CA^{-1}B)}_{H_{io}} u + \underbrace{(F - CA^{-1}E)}_{R_{do}} w$$



# Stability, feasibility, & asymptotic optimality

**Theorem:** Assume that

- **regularity** of cost function  $\phi$ : compact sublevel sets &  $\ell$ -Lipschitz gradient
- LTI system asymptotically **stable**:  $\exists \tau > 0, \exists P \succ 0 : PA + A^T P \preceq -2\tau P$
- sufficient **time-scale separation** (small gain):  $0 < \epsilon < \epsilon^* \triangleq \frac{2\tau}{\text{cond}(P)} \cdot \frac{1}{\ell \|H_{i\circ}\|}$

$$\iff \boxed{\text{system gain} \cdot \text{algorithm gain} < 1}$$

Then the closed-loop system is **stable** and **globally converges** to the critical points of the **optimization problem** while remaining **feasible** at all times.

**Proof:** LaSalle/Lyapunov analysis via **singular perturbation** [Saber & Khalil '84]

$$\Psi_\delta(u, e) = \delta \cdot \underbrace{e^T P e}_{\text{LTI Lyapunov function}} + (1 - \delta) \cdot \underbrace{\phi(h(u), u)}_{\text{merit function}}$$

with **parameter**  $\delta \in (0, 1)$  & steady-state **error coordinate**  $e = x - H_{is}u - R_{ds}w$

→ derivative  $\dot{\Psi}_\delta(u, e)$  is non-increasing if  $\epsilon \leq \epsilon^*$  & for judicious choice of  $\delta$

# Example: optimal frequency control

- **dynamic LTI power system model**
  - ▶ linearized swing dynamics
  - ▶ 1st-order turbine-governor
  - ▶ primary frequency droop
  - ▶ DC power flow approximation
- economic balancing **objective**
- **control** generation set-points
- unmeasured load **disturbances**
- **measurements**: frequency + constraint variables (injections & flows)

## ■ optimization problem

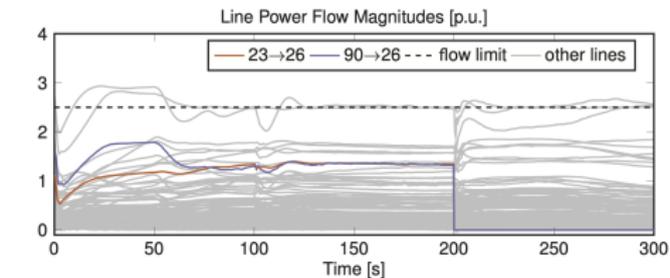
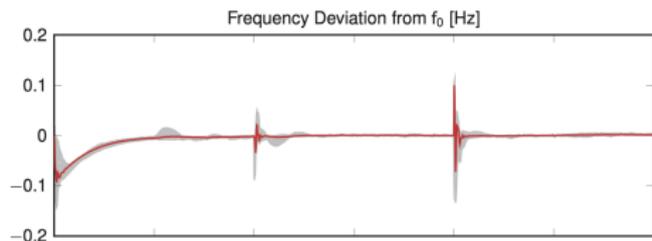
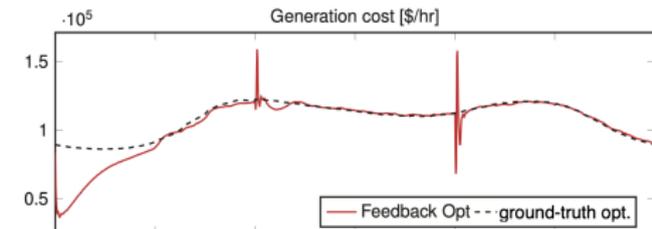
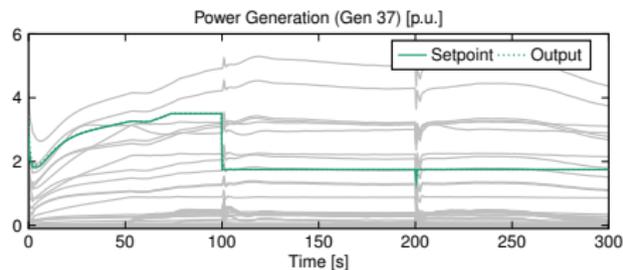
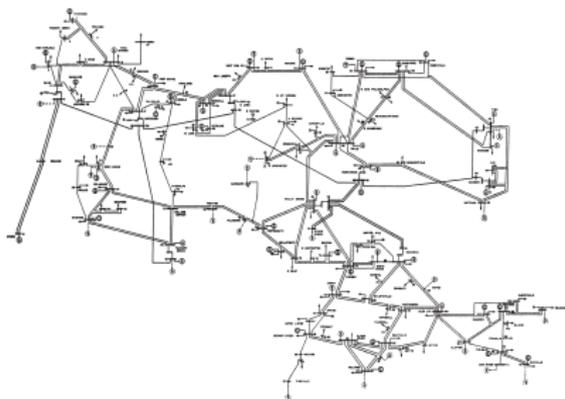
→ **objective**:  $\phi(y, u) = \underbrace{\text{cost}(u)}_{\text{economic generation}} + \underbrace{\frac{1}{2} \|\max\{0, \underline{y} - y\}\|_{\Xi}^2 + \frac{1}{2} \|\max\{0, y - \bar{y}\}\|_{\Xi}^2}_{\text{operational limits (line flows, frequency, \dots)}}$

→ **hard constraints**:  $\underbrace{\text{actuation } u \in \mathcal{U}}_{\text{enforced by projection}} \ \& \ \underbrace{\text{steady-state map } y = H_{io}u + R_{do}w}_{\text{enforced by physics}}$

→ **control**  $\dot{u} = \Pi_{\mathcal{U}}(\dots \nabla \phi) \equiv$  super-charged Automatic Generation Control

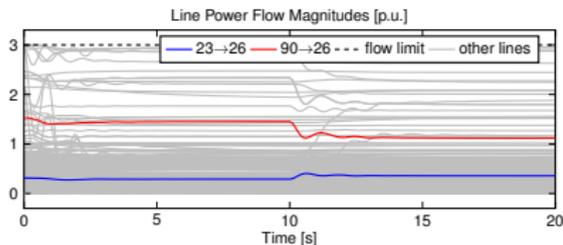
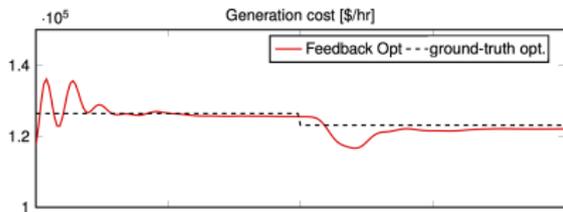
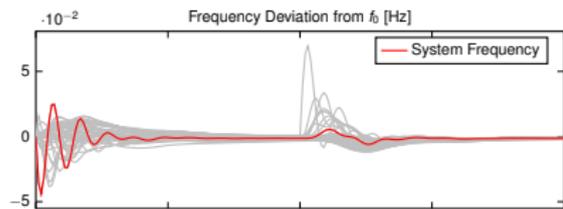
# Test case: contingencies in IEEE 118 system

events: generator outage at 100 s & double line tripping at 200 s

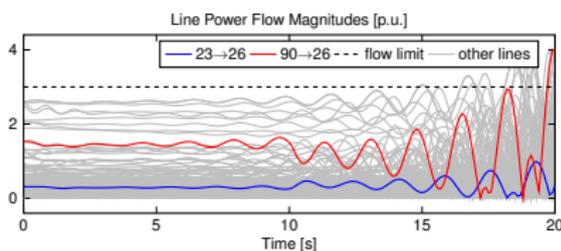
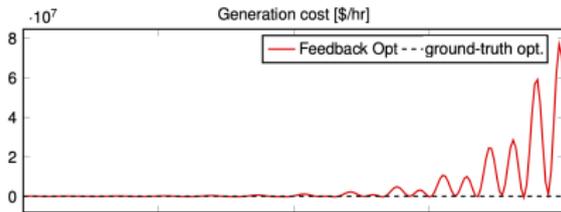
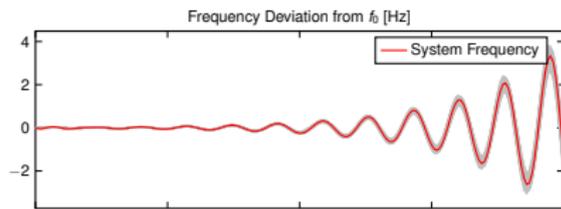


# How conservative is $\epsilon < \epsilon^*$ ?

still stable for  $\epsilon = 2\epsilon^*$



unstable for  $\epsilon = 10\epsilon^*$



**Note:** conservativeness depends on the problem, e.g., on soft penalty scalings

# Highlights & comparison of our approach

## Weak assumptions on plant

- internal stability
- no observability / controllability
- no passivity or primal-dual structure
- measurements & steady-state sensitivity
- no knowledge of model or disturbances
- no full state measurement
- steady-state constraint enforced by plant

## Weak assumptions on cost

- Lipschitz gradient + properness
- no (strict / strong) convexity required

→ **all of these insights** extend to much more general problem setups!

## Parsimonious but powerful setup

- potentially conservative bound on time-scale separation — but
- **minimal assumptions** on control system & optimization problem
- **robust & extendable methodology**
- nonlinear & sampled-data dynamics
- general equilibrium seeking algorithms
- time-varying disturbances, noise, ...

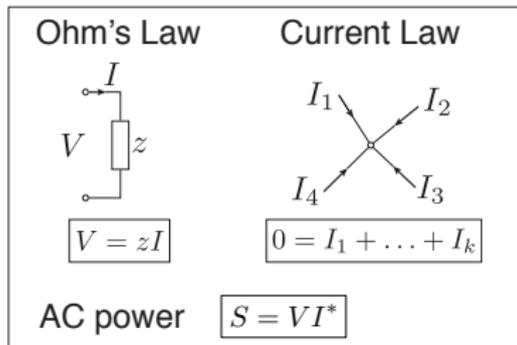
**take-away:** open online optimization algorithms can be applied in feedback

- Hauswirth et al. (2020)  
“Timescale Separation in Autonomous Optimization”
- Menta et al. (2018)  
“Stability of Dynamic Feedback Optimization ...”

# **GENERAL NONLINEAR SYSTEMS & DISTURBANCES**

# Motivation: steady-state AC power flow

## stationary model

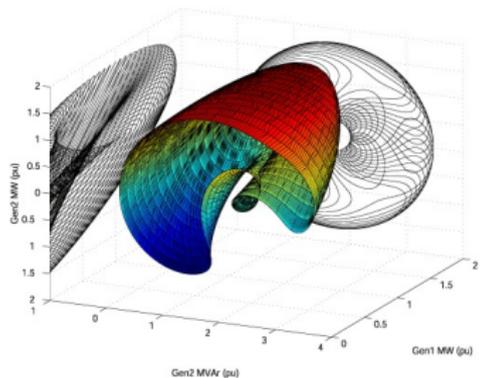


## AC power flow equations

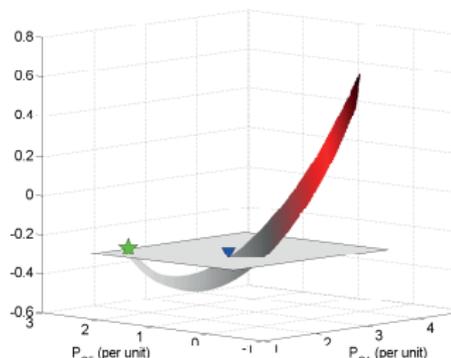
$$S_k = \sum_{l \in N(k)} \frac{1}{z_{kl}^*} V_k (V_k^* - V_l^*) \quad \forall k \in \mathcal{N}$$

- imagine **constraints slicing** this set  
 $\Rightarrow$  nonlinear, non-convex, disconnected
- additionally the parameters are  $\pm 20\%$  **uncertain** ... this is only the steady state!

## graphical illustration of AC power flow

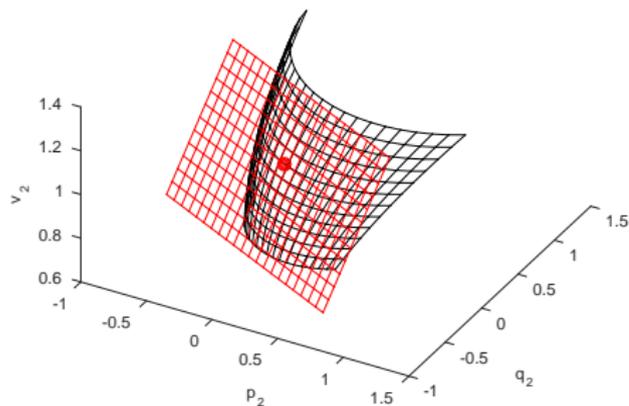


[Hiskens, 2001]



[Molzahn, 2016]

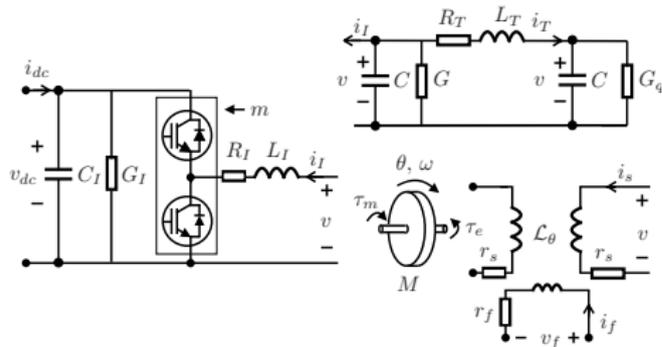
# Key insights on physical equality constraint



- AC power flow is complex but takes the form of a **smooth manifold**
- local tangent plane approximations, local invertibility, & generic LICQ
- **regularity** (algorithmic flexibility)

→ Bolognani et al. (2015)  
“Fast power system analysis via implicit linearization of the power flow manifold”

→ Hauswirth et al. (2018)  
“Generic Existence of Unique Lagrange Multipliers in AC Optimal Power Flow”



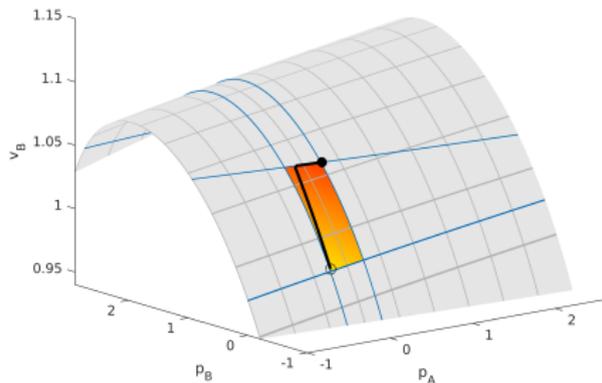
- AC power flow is **attractive steady state** for ambient physical dynamics
- physics enforce feasibility even for non-exact (e.g., discrete) updates
- **robustness** (algorithm & model)

→ Gross et al. (2018)  
“On the steady-state behavior of a nonlinear power system model”

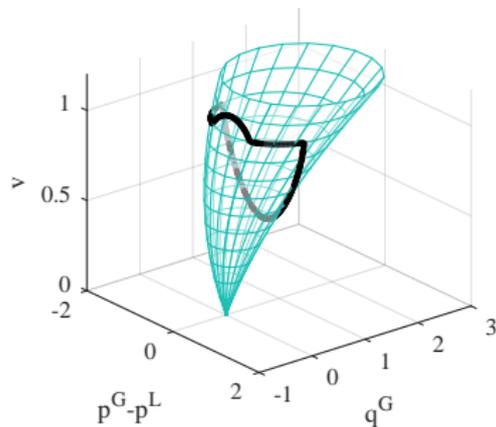
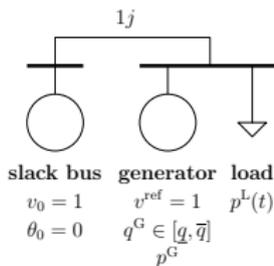
# Simple low-dimensional case studies ...

... can have **simple** feasible sets

output variables	$p_1, q_1$	$v_2, \theta_2$	$v_3, \theta_3$	$v_4, \theta_4$
control variables	$v_1 = 1$ $\theta_1 = 0$	$p_2$ $q_2 = 0$	$p_3 = P_L$ $q_3 = 0$	$p_4$ $q_4 = 0$
	slack bus	generator A	load	generator B
generation cost	$a = 0.1$ $b = 4$	$a = 0.1$ $b = 2$		$a = 0.1$ $b = 0.1$



... or can have **really complex** sets



application demands **sophisticated level of generality**!

# General nonlinear systems & disturbances

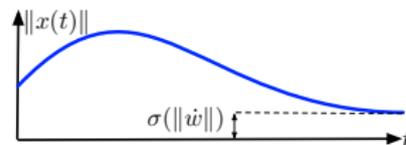
- Lipschitz continuous **nonlinear system**  $\dot{x} = f(x, u)$  (output setup also possible)
- explicit & differentiable **steady-state map**  $x = h(u)$  so that  $f(h(u), u) = 0$
- open-loop stable: **Lyapunov function**  $W(x, u) \approx W(x - h(u))$  w.r.t **steady-state error** satisfying  $\underbrace{\dot{W}(x, u) \leq -\gamma \|x - h(u)\|^2}_{\text{dissipation rate } \gamma}, \quad \underbrace{\|\nabla_u W(x, u)\| \leq \zeta \|x - h(u)\|}_{\zeta\text{-Lipschitz in steady-state error}}$

⇒ local / global closed-loop **stability, convergence to critical points, & feasibility** if

$$\boxed{\text{system gain} \cdot \text{algorithm gain} < 1}$$

where the system gain is  $\zeta/\gamma = \text{Lipschitz constant} / \text{dissipation rate}$

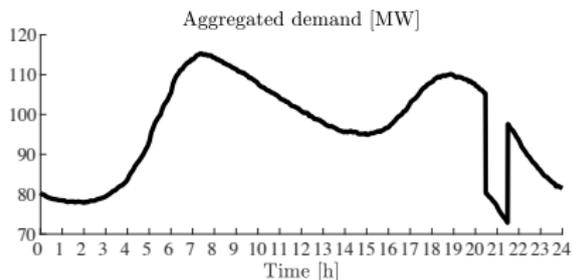
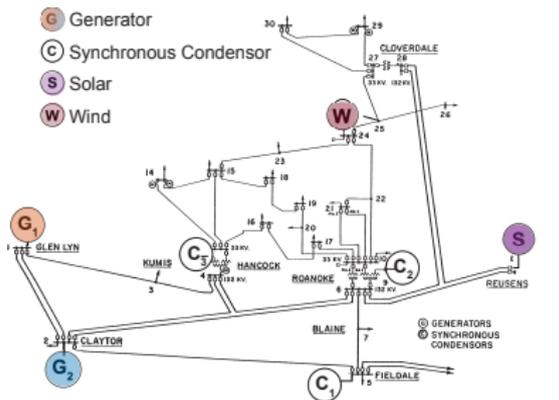
- **time-varying disturbances**:  $\dot{x} = f(x, u, w(t))$
- assume  $\|\dot{w}(t)\|$  **bounded** & system is **input-to-state stable** (ISS) w.r.t.  $\dot{w}$ :  $\dot{W} \leq -\gamma \|x - h(u, w)\|^2 + \sigma(\|\dot{w}\|)$



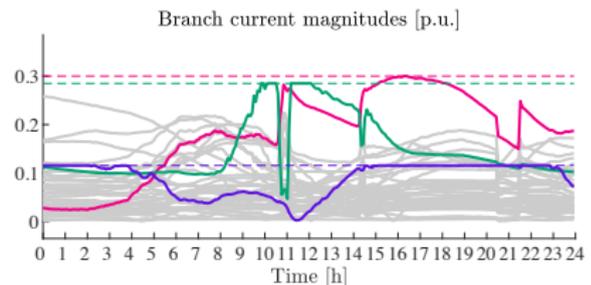
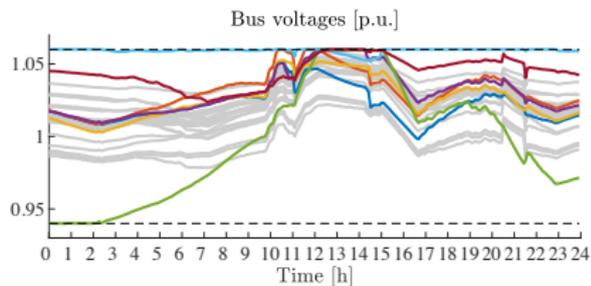
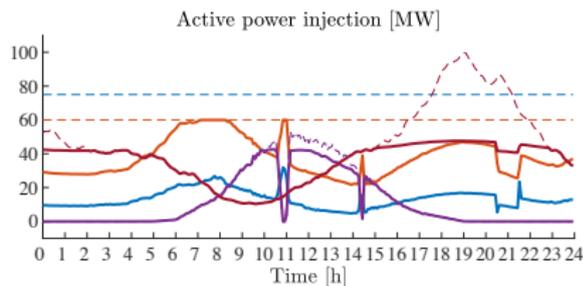
→ Hauswirth et al. (2020) “Timescale Separation in Autonomous Optimization”

→ Belgioioso et al. (2022) “Online Feedback Equilibrium Seeking”

# Tracking performance under disturbances

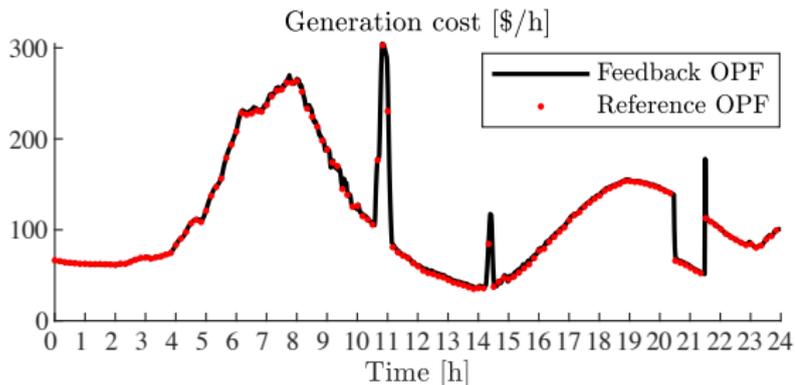


disturbance = net demand: load - (wind + solar)



# Optimality despite disturbances & uncertainty

- transient trajectory **feasibility**
- practically **exact tracking** of ideal optimal power flow (OPF)  
(omniscient & no computation delay)
- **robustness** to model mismatch  
(asymptotic optimality under wrong model)



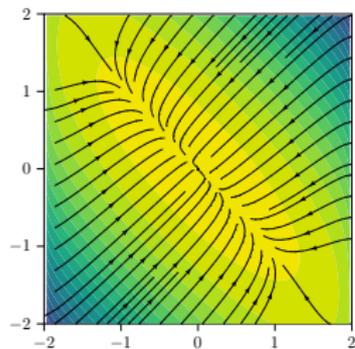
model uncertainty	offline optimization			feedback optimization		
	feasible ?	$\phi - \phi^*$	$\ v - v^*\ $	feasible ?	$\phi - \phi^*$	$\ v - v^*\ $
loads $\pm 40\%$	no	94.6	0.03	yes	0.0	0.0
line params $\pm 20\%$	yes	0.19	0.01	yes	0.01	0.003
2 line failures	no	-0.12	0.06	yes	0.19	0.007

**conclusion:** simple algorithm performs extremely well in challenging environment

**WITH DYNAMICS & DISTURBANCES TAKEN CARE OF,  
WE NOW FOCUS ON OPTIMIZATION**

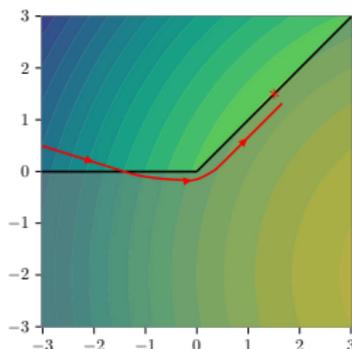
# More general optimization flows

variable metrics

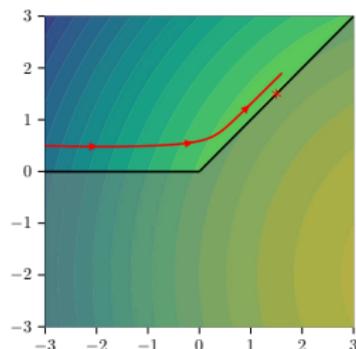


gradient:  $\dot{u} = -\nabla\phi(u)$

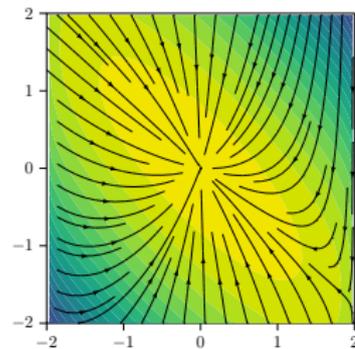
different ways of enforcing constraints



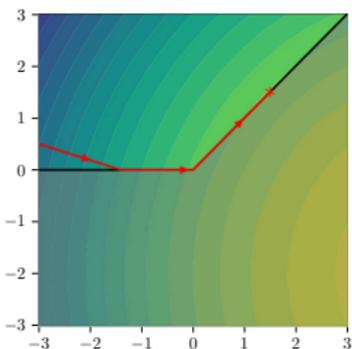
penalty function



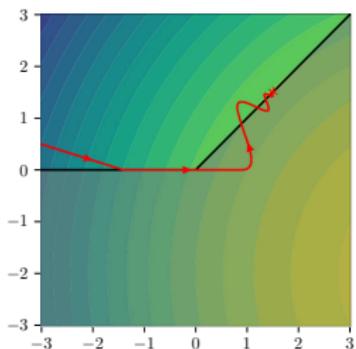
barrier function



Newton:  $\dot{u} = -\nabla^2\phi(u)^{-1} \cdot \nabla\phi(u)$



projected gradient flow



primal-dual saddle flow

# Certificates for general optimization flows

- **variable-metric**  $Q(u) \in \mathbb{S}_+^n$  gradient flow

$$\dot{u} = -Q(u)^{-1} \cdot \nabla \phi(u)$$

- examples: Newton method  $Q(u) = \nabla^2 \phi(u)$   
or mirror descent  $Q(u) = \nabla^2 \psi (\nabla \psi(u)^{-1})$
- **stability, convergence, & feasibility** if

$$\boxed{\text{system gain} \cdot \text{algorithm gain} < 1}$$

with algorithm gain  $\ell \cdot \nabla h(u) \cdot \sup_u \|Q(u)^{-1}\|$

Similar results for algorithms with memory:

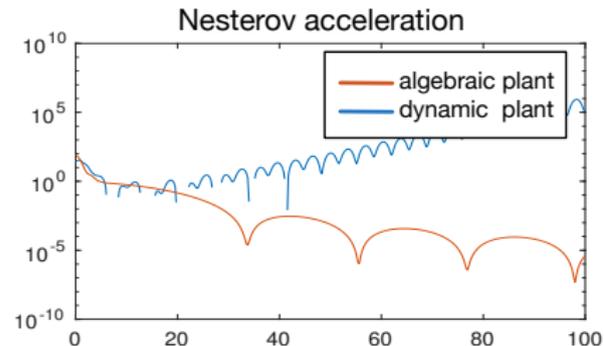
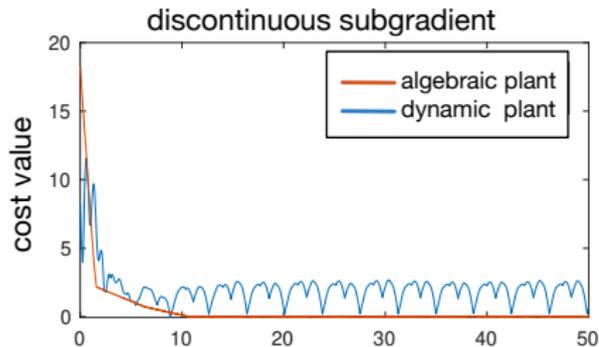
- **momentum methods** (e.g., heavy-ball)

$$\ddot{u} + D(u) \cdot \dot{u} = -Q(u)^{-1} \cdot \nabla \phi(u)$$

- (exp. stable) **primal-dual saddle flows**

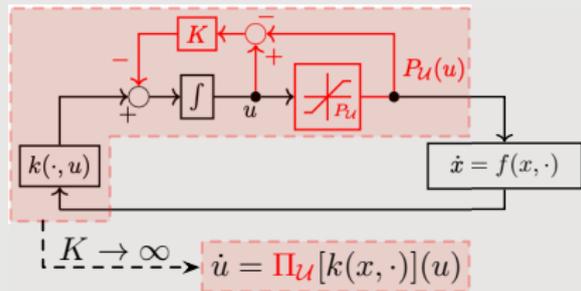
as long as the algorithm gain is bounded

a few **non-examples** for unbounded gain:

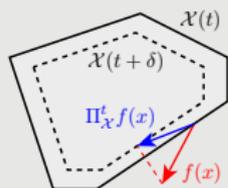


# Robust implementation of projections

- projection & integrator → **windup**
- **robust anti-windup** approximation
- saturation often “for free” by physics



- disturbance → **time-varying domain**



- ▶ **temporal tangent cone & vector field**
- ▶ ensure suff. regularity & tracking certificates

- handling **uncertainty** when enforcing **non-input constraints**:  $x \in \mathcal{X}$  or  $y \in \mathcal{Y}$

- ▶ **cannot measure** state  $x$  directly
- Kalman filtering: estimation & separation
- ▶ **cannot enforce constraints** on  $y = h(u)$  by projection (not actuated &  $h(\cdot)$  unknown)
- soft penalty or dualization + grad flows (inaccurate, violations, & strong assumptions)
- project on **1<sup>st</sup> order prediction** of  $y = h(u)$

$$y^+ \approx \underbrace{h(u)}_{\text{measured}} + \epsilon \underbrace{\frac{\partial h}{\partial u}}_{\text{steady-state I/O sensitivity}} \underbrace{w}_{\text{feasible descent direction}}$$

⇒ global convergence to critical points

→ Häberle et al. (2020)

“Enforcing Output Constraints in Feedback-based Optimization”

→ Hauswirth et al. (2018)

“Time-varying Projected Dynamical Systems with Applications”

→ Hauswirth et al. (2020)  
 “Anti-Windup Approximations of Oblique Projected Dynamical Systems for Feedback-based Optimization”

**COMPUTING HAPPENS IN DISCRETE TIME**

→ **SAMPLED-DATA SETTING**

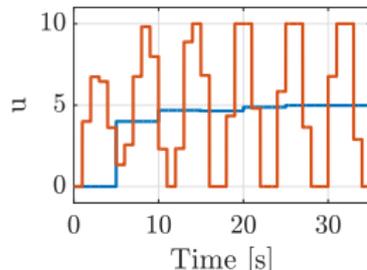
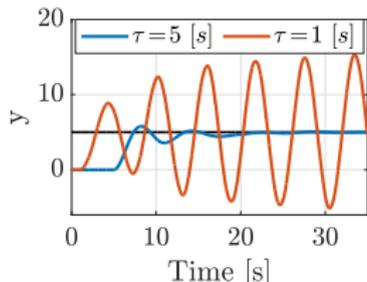
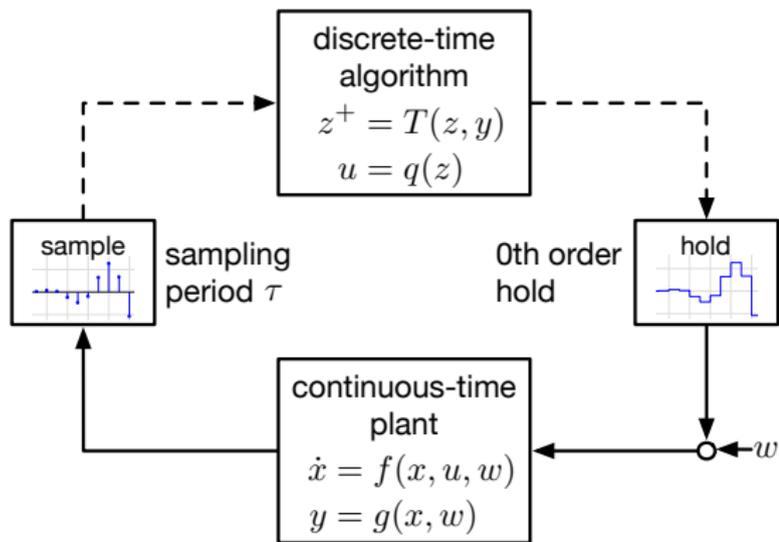
# Sampled-data setting

- **continuous-time plant**: same assumptions as before
- **sampling** rate  $\tau$  & 0th order hold
- **discrete-time algorithm** with strictly decreasing merit function & bounded gain
- examples: strongly quasi-non-expansive operator (ADMM, DR, prox, alternating projection, ...)

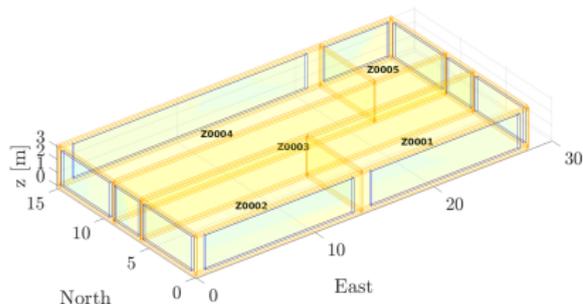
⇒ local / global closed-loop **ISS** if

$$\text{system gain} \cdot \text{algorithm gain} < 1$$

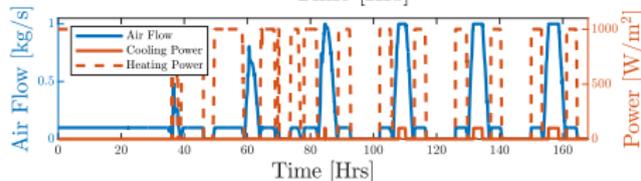
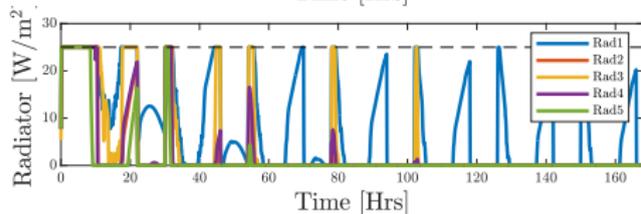
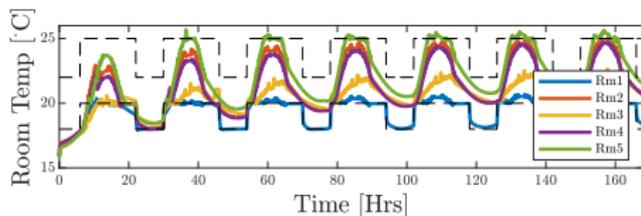
⇒ system gain decreases in  $\tau$   
i.e. **sufficiently slow sampling**



# Example: building temperature control



- **building model** from BRCM toolbox with 118 states (bilinear dynamics), 10 disturbances, 8 inputs, & 7 outputs
- **objective**: minimize energy cost & keep temperatures in comfort range
- **online SQP** (sequential quadratic programming) for feedback optimization
- note: algorithm is **not predictive** & doesn't use any forecast or reference



**comparison** to hysteresis (threshold-based) control: 32% cost reduction & 28% reduction in constraint violations

**ALL ALGORITHMS REQUIRE THE GRADIENT**

$$\frac{\partial}{\partial u} \phi(h(u), u) = \left[ \frac{\partial h}{\partial u} \quad \mathbb{I} \right] \cdot \nabla \phi(y, u) \Big|_{y=h(u)}$$

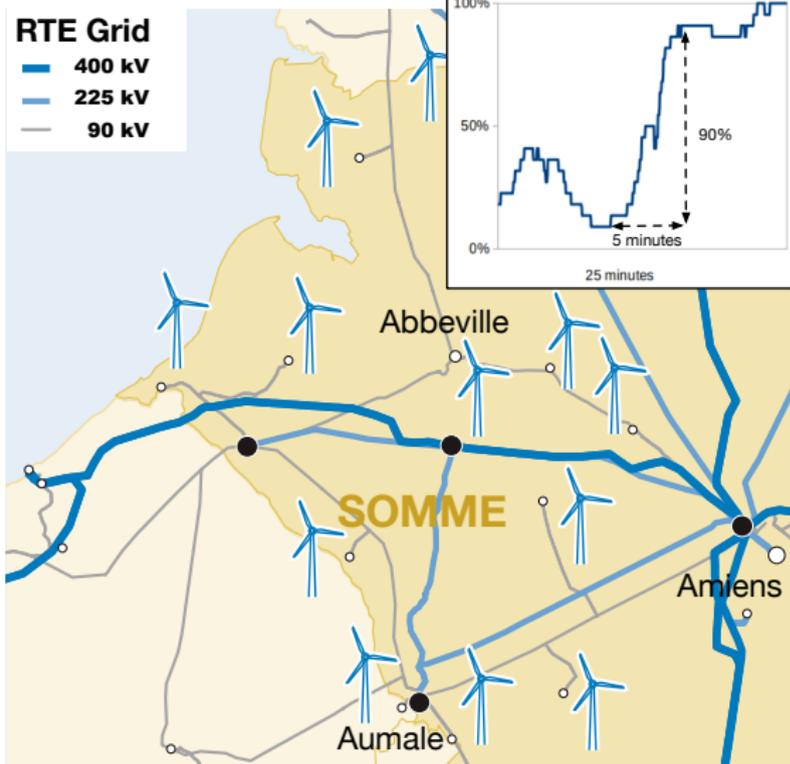
**& THUS THE MODEL SENSITIVITY  $\frac{\partial h}{\partial u}$  !**

**MODEL-FREE IMPLEMENTATIONS WITHOUT SENSITIVITY ?**

# Example: power grid operation

## RTE Grid

- 400 kV
- 225 kV
- 90 kV



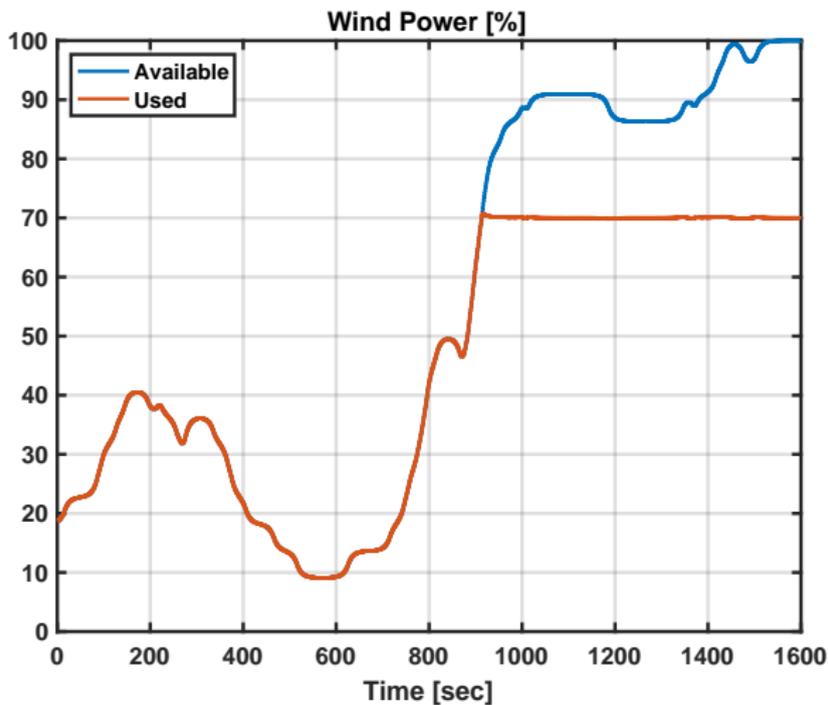
## UNICORN project with RTE

- automation of *Blocaux* zone
- **rapid change** in generation
  - **line / voltage limits** violations
  - resolve most **economically** & under severe **uncertainty** & time-varying **disturbances**

## Technical problem setup

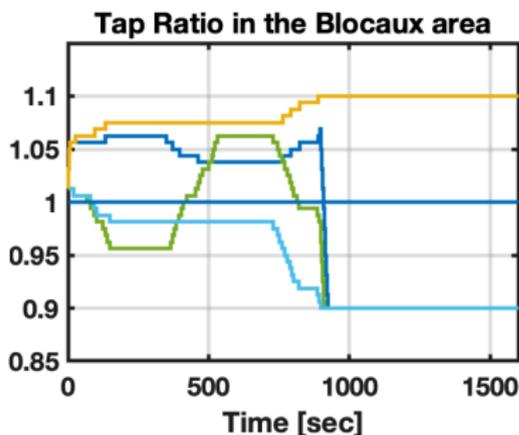
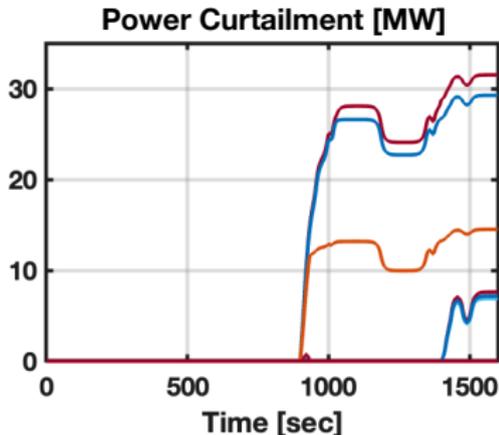
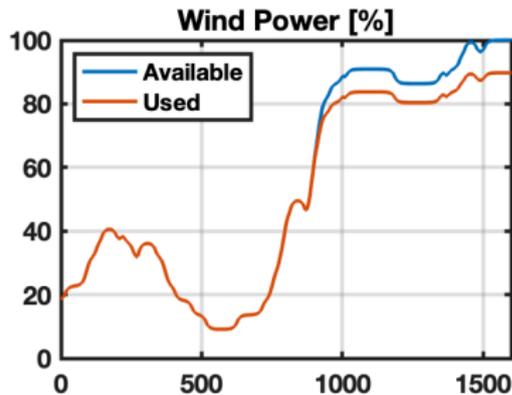
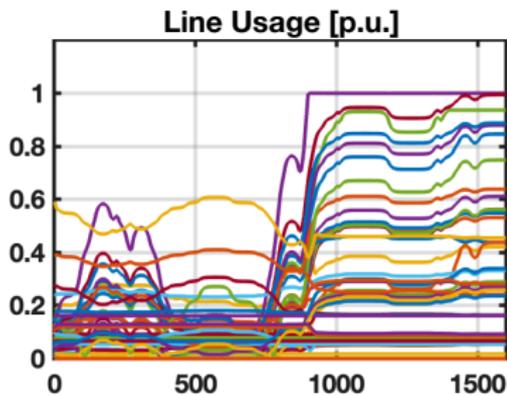
- simulation of entire French grid
  - power flow + tap changer
- actuation & sensing in Blocaux
  - tap, reactive & active power
  - voltage & current magnitudes
- realistic constraints & cost
  - curtailment + losses

# Current mode of operation



offline optimization & **curtailment at 70%** to not violate line / voltage limits

# Feedback optimization using constant (wrong) sensitivity



# Model-free feedback optimization

- **feedback optimization** is **robust** to inaccurate sensitivity, though the performance might be inferior

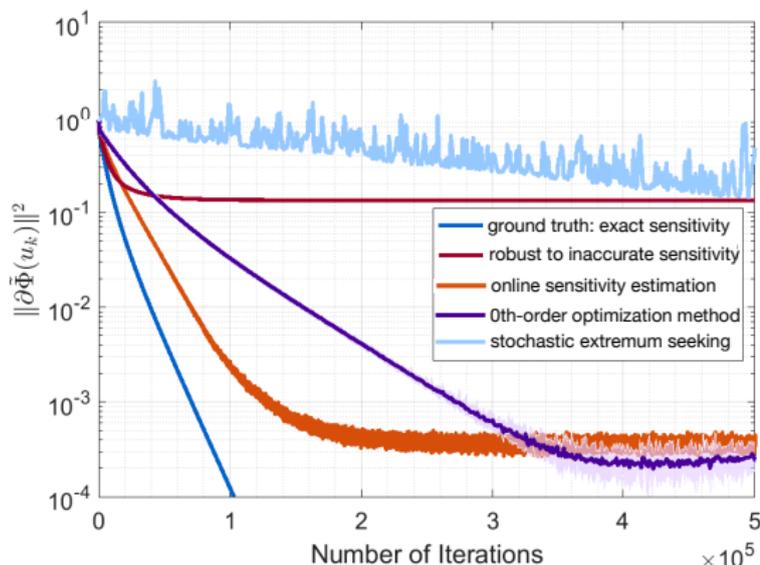
- **online sensitivity estimation** of  $\frac{\partial h}{\partial u} \approx \frac{y_{t+1} - y_t}{u_{t+1} - u_t}$  via Kalman filter

- **0<sup>th</sup>-order optimization** building one-point gradient estimates

$$\frac{\partial}{\partial u} \phi(u) = \lim_{\delta \searrow 0} \mathbb{E} \left[ \frac{\eta}{\delta} \phi(u + \delta \cdot \eta) \right]$$

where  $\eta$  is random probing direction &  $\delta$  is (small) smoothing parameter  
→ constructed via single actuation

- **others**: stochastic approximation, extremum seeking, ... do poorly



→ Colombino et al. (2020) "Towards robustness guarantees for feedback-based optimization"

→ Picallo et al. (2022) "Adaptive Real-Time Grid Operation via Online Feedback Optimization with Sensitivity Estimation"

→ He et al. (2022) "Model-Free Nonlinear Feedback Optimization"

**THE WORLD IS NOT AN OPTIMIZATION PROBLEM**

→ **EXTENSIONS TO THE GAME-THEORETIC SETUP**

# Feedback equilibrium seeking

**motivation:** multi-area power system

- ▶ different system operators whose cost functions are not aligned
- ▶ physical & operational coupling

■ **game theory** as lingua franca:

$$\min_{u_i} \phi_i(y_i, u_i)$$

subject to constraints coupling  $(u_i, y_i)$

■ opt. solution = **Nash equilibrium**

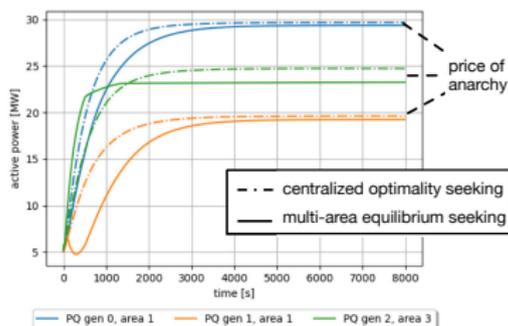
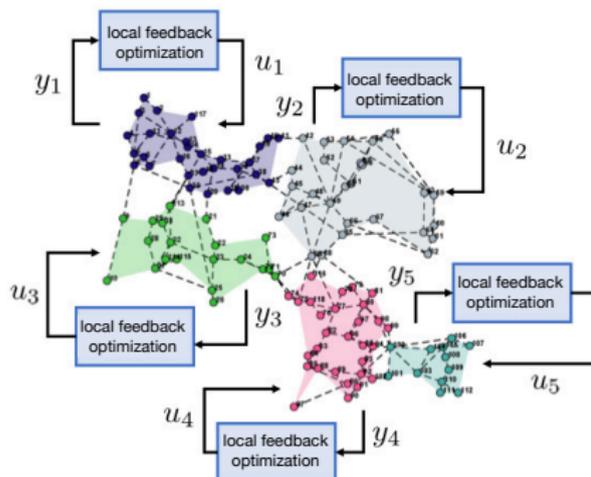
■ **equilibrium-seeking algorithms**

using local gradients  $\frac{\partial}{\partial u_i} \phi_i(y_i, u_i)$

■ similar assumptions as before &

$$\text{system gain} \cdot \text{algorithm gain} < 1$$

⇒ **all results extend** analogously!



**FROM LAB DEMONSTRATIONS  
TO COMMERCIAL DEPLOYMENT**

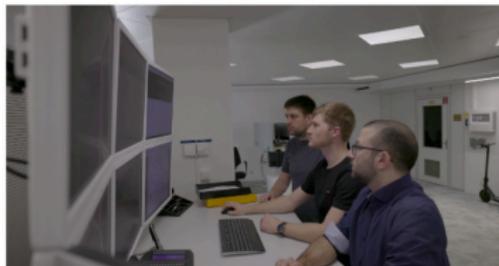
# Deployment at Swiss utility (AEW)



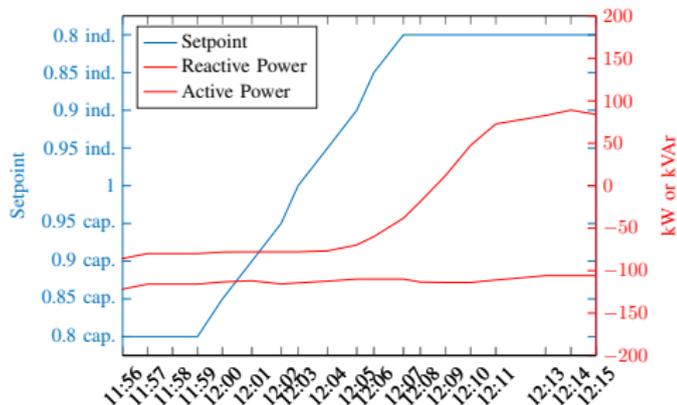
**AEW**



Communication channel



- **virtual grid reinforcement** through reactive power/voltage support & power flow control
- **strong economic incentives** (rewards & penalties) from higher-level system operator
- **feedback optimization** on legacy hardware
- **runs robustly, 24/7, & makes money** in presence of time-varying incentives



## **CONCLUSIONS**

# Conclusions

## Summary

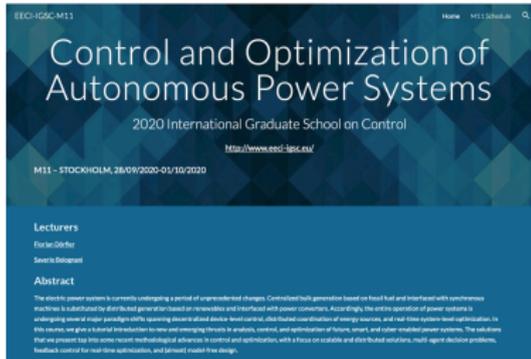
- open & online **feedback optimization** algorithms as controllers
- **unified framework** for broad class of systems, algorithms, decision-making problems, interconnection scenarios, & implementation aspects
- illustrated throughout with non-trivial **power systems** case studies
- **complete TRL scale** covered: theory → industrial deployment

## Ongoing work & open directions

- **theory**: get rid off time-scale separation & many other extensions
- **new application domains**: supply chains & recommender systems

*It works in theory and in practice !*

# Main resources for today



EECH-EGEC-M11 Home M11 COURSES

## Control and Optimization of Autonomous Power Systems

2020 International Graduate School on Control

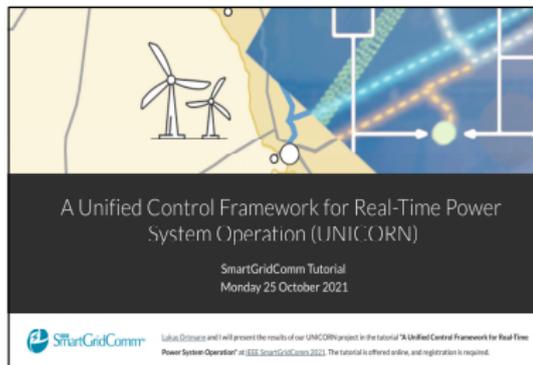
<https://www.eeci.se/>

M11 - STOCKHOLM, 28/09/2020-05/10/2020

**Lecturers**  
Florian Dörfler  
Saverio Bolognani

**Abstract**  
The electric power system is currently undergoing a period of unprecedented changes. Centralized bulk generation based on fossil-fuel and interfaced with synchronous machines is substituted by distributed generation based on renewable and distributed wind power generation. As a result, the entire operation of power networks is undergoing several major paradigm shifts, spanning distributed physical devices (renewable, distributed coordination of energy resources, and real-time system-level optimization). In this course, we give a tutorial introduction to some of the emerging trends in analysis, control, and optimization of future, smart, and cyber-enabled power systems. The solutions that are presented will be some of the most recent technological advances in control and optimization, with a focus on scalable and distributed solutions, multi-agent decision problems, feedback control for real-time applications, and advanced model-free design.

<https://sites.google.com/view/eeci-autonomous-power-systems>



## A Unified Control Framework for Real-Time Power System Operation (UNICORN)

SmartGridComm Tutorial  
Monday 25 October 2021

 Lukas Grigoriadis will present the results of our UNICORN project in the tutorial "A Unified Control Framework for Real-Time Power System Operation" at IEEE SmartGridComm 2021. The tutorial is offered online, and registration is required.

2021 SmartGridComm Tutorial [here](#)

## Optimization Algorithms as Robust Feedback Controllers

Adrian Hanswirth, Saverio Bolognani, Gabriela Hug, and Florian Dörfler  
*Department of Information Technology and Electrical Engineering, ETH Zurich, Switzerland*

### Abstract

Mathematical optimization is one of the cornerstones of modern engineering research and practice. Yet, throughout all application domains, mathematical optimization is, for the most part, considered to be a numerical discipline. Optimization problems are formulated to be solved numerically with specific algorithms running on microprocessors. An emerging alternative is to view optimization algorithms as dynamical systems. While this new perspective is insightful in itself, liberating optimization methods from specific numerical and algorithmic aspects opens up new possibilities to endow complex real-world systems with sophisticated self-optimizing behavior. Towards this goal, it is necessary to understand how numerical optimization algorithms can be converted into feedback controllers to enable robust "closed-loop optimization". In this article, we review several research streams that have been pursued in this direction, including extremum seeking and perturbed methods from model predictive and process control. However, our primary focus lies on recent methods under the name of "feedback-based optimization". This research stream studies control designs that directly implement optimization algorithms in closed loop with physical systems. Such ideas are finding widespread application in the design and retrofit of control protocols for communication networks and electricity grids. In addition to an overview over continuous-time dynamical systems for optimization, our particular emphasis in this survey lies on closed-loop stability as well as the enforcement of physical and operational constraints in closed-loop implementations. We further illustrate these methods in the context of classical problems, namely congestion control in communication networks and optimal frequency control in electricity grids, and we highlight one potential future application in the form of autonomous reserve dispatch in power systems.

2021 Survey paper <https://arxiv.org/abs/2103.11329>

## Publications about 'Online Optimization'

### Articles in journal, book chapters

1. V. Häberle, A. Hanswirth, L. Ortmann, S. Bolognani, and F. Dörfler. **Non-convex Feedback Optimization with Input and Output Constraints**. *IEEE Control Systems Letters*, 5(1):343-348, 2021.    **Keyword(s)**: Online Optimization, Nonlinear Optimization, Nonlinear Control Design. [[bibtex-entry](#)]
2. A. Hanswirth, S. Bolognani, and F. Dörfler. **Projected Dynamical Systems on Irregular Non-Euclidean Domains for Nonlinear Optimization**. *SIAM Journal on Control and Optimization*, 59(1):635-668, 2021.    **Keyword(s)**: Online Optimization, Nonlinear Optimization. [[bibtex-entry](#)]
3. A. Hanswirth, S. Bolognani, G. Hug, and F. Dörfler. **Optimization Algorithms as Robust Feedback Controllers**. January 2021. Note: Submitted. Available at <http://arxiv.org/abs/2103.11329>.   **Keyword(s)**: Power Networks, Power Flow Optimization, Online Optimization, Nonlinear Optimization. [[bibtex-entry](#)]

Publications <http://people.ee.ethz.ch/~floriand/>

# Thanks !

**Florian Dörfler**

<http://control.ee.ethz.ch/~floriand>

[link] to related publications