Plug-and-Play Control and Optimization in Power Systems

Laboratoire d'Automatique Seminar
École Polytechnique Fédérale de Lausanne

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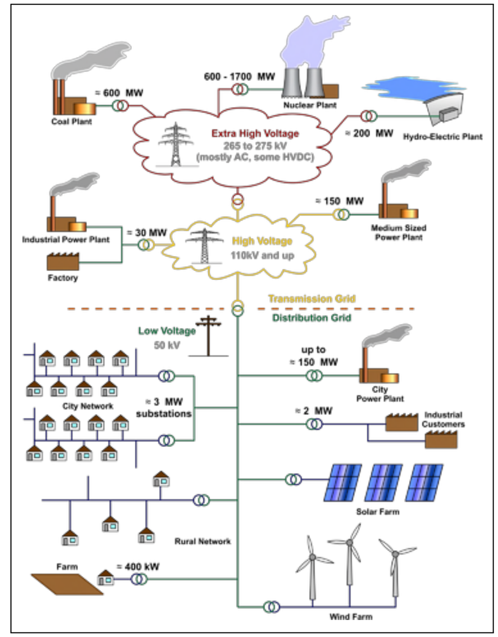
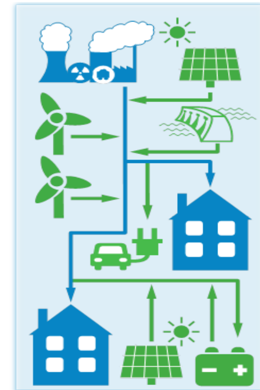
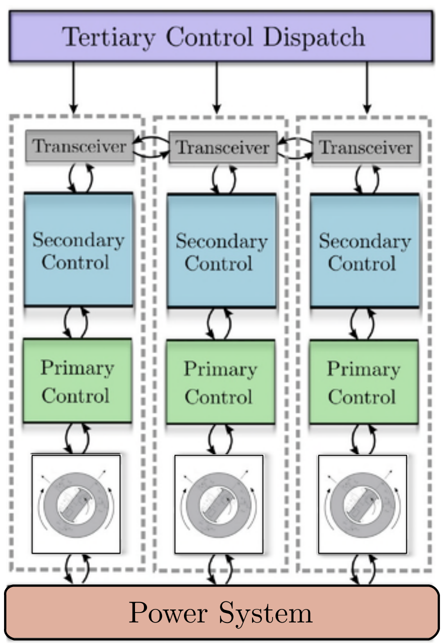
Operation of electric power networks



- purpose of electric **power grid**: generate/transmit/distribute
- **operation**: hierarchical & based on bulk generation
- things are changing . . .



Conventional hierarchical control architecture



3. **Tertiary control** (offline)
   - Goal: optimize operation
   - Strategy: centralized & forecast
2. **Secondary control** (slower)
   - Goal: maintain operating point
   - Strategy: centralized
1. **Primary control** (fast)
   - Goal: stabilization & load sharing
   - Strategy: decentralized

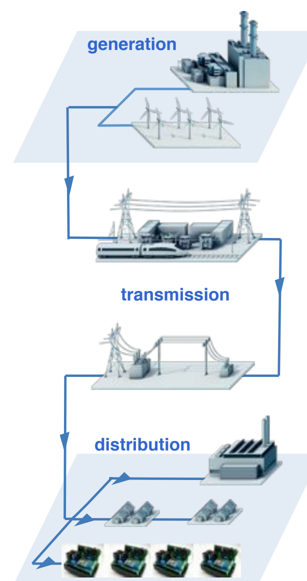
Is this **top-to-bottom architecture** based on **bulk generation control** still appropriate in tomorrow's grid?

A few (of many) game changers

synchronous generator
⇒ power electronics

scaling

distributed generation



other paradigm shifts

Challenges & opportunities in tomorrow's power grid

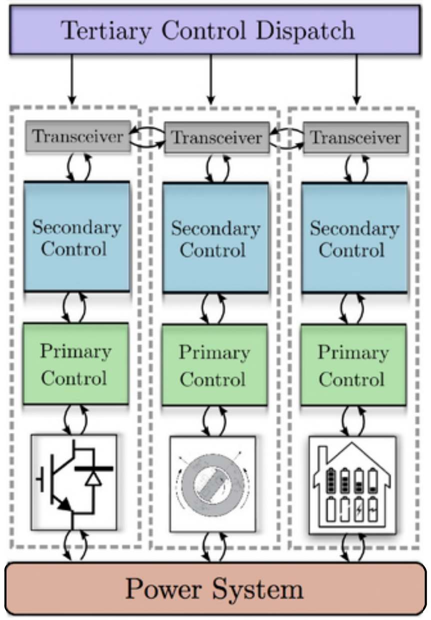
☹perational challenges

- more uncertainty & less inertia
- more volatile & faster fluctuations

☺pportunities

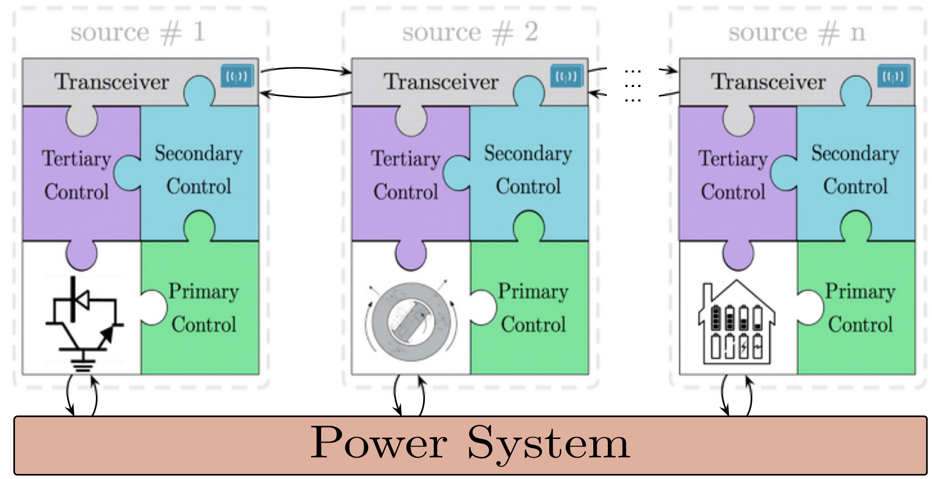
- re-instrumentation: comm & sensors and actuators throughout grid
- advances in control of cyber-physical & complex systems

- **break** vertical & horizontal **hierarchy**
- **plug'n'play** control: fast, model-free, & without central authority



A preview – plug-and-play operation architecture

flat hierarchy, distributed, no time-scale separations, & model-free ...



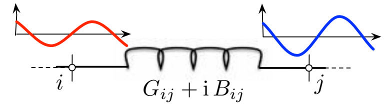
Outline

we will illustrate all theorems with experiments

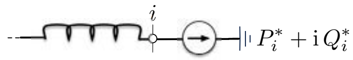
modeling & assumptions

Modeling: a power system is a circuit

1. synchronous **AC circuit** with harmonic waveforms $E_i e^{i(\theta_i + \omega^* t)}$



2. **loads** demand constant power



3. **coupling** via Kirchhoff & Ohm

injection $= \sum$ power flows

4. identical lines $G/B = const.$ (equivalent to lossless case $G/B = 0$)
5. decoupling: $P_i \approx P_i(\theta)$ & $Q_i \approx Q_i(E)$ (for simplicity of presentation)

- active power: $P_i = \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j)$
- reactive power: $Q_i = -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j)$

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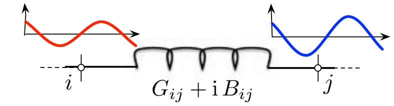

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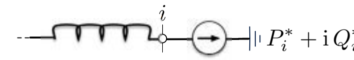

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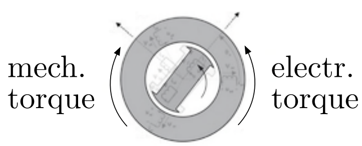
- trigonometric active power flow: $P_i(\theta) = \sum_j B_{ij} \sin(\theta_i - \theta_j)$
- polynomial reactive power flow: $Q_i(E) = -\sum_j B_{ij} E_i E_j$ (not today)

Modeling the "essential" network dynamics & controls

(models can be arbitrarily detailed)

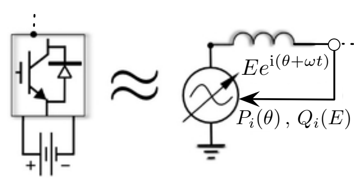
1. synchronous **machines** (swing dynamics)

$$M_i \ddot{\theta}_i = P_i^* + \mathbf{P_i^c} - P_i(\theta)$$



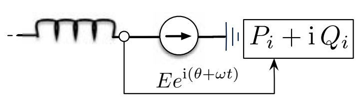
2. DC & variable AC sources interfaced with voltage-source **converters**

$$P_i^* + \mathbf{P_i^c} = P_i(\theta)$$



3. controllable **loads** (voltage- and frequency-responsive)

$$P_i^* + \mathbf{P_i^c} = P_i(\theta)$$

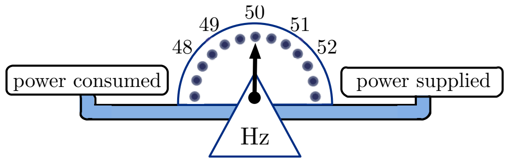


primary control (droop characteristic)

Decentralized primary control of active power

Emulate physics of dissipative coupled **synchronous machines**:

$$M_i\ddot{\theta} + D_i\dot{\theta}_i = P_i^* - \sum\nolimits_j B_{ij}\sin(\theta_i - \theta_j)$$



Conventional wisdom: physics are naturally stable & sync frequency reveals power imbalance

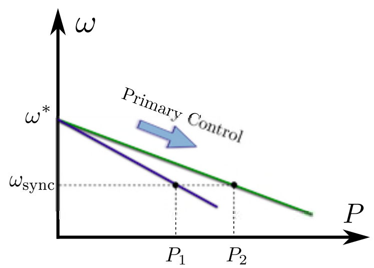
$$\omega_{\text{sync}} = \sum\nolimits_i P_i^* / \sum\nolimits_i D_i$$

$P/\dot{\theta}$ droop control:

$$(\omega_i - \omega^*) \propto (P_i^* - P_i(\theta))$$

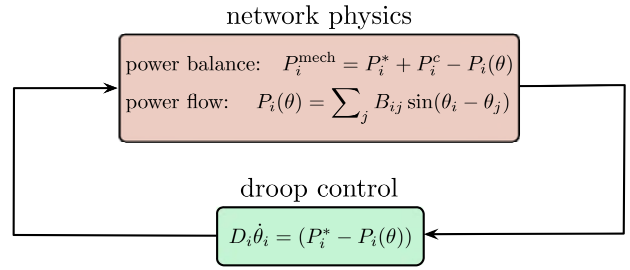
$$\Updownarrow$$

$$D_i\dot{\theta}_i = P_i^* - P_i(\theta)$$



Putting the pieces together...

differential-algebraic, nonlinear, large-scale closed loop



synchronous machines: $M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i^* - \sum_j B_{ij}\sin(\theta_i - \theta_j)$

inverter sources: $D_i\dot{\theta}_i = P_i^* - \sum_j B_{ij}\sin(\theta_i - \theta_j)$

controllable loads: $D_i\dot{\theta}_i = P_i^* - \sum_j B_{ij}\sin(\theta_i - \theta_j)$

passive loads/inverters: $0 = P_i^* - \sum_j B_{ij}\sin(\theta_i - \theta_j)$

Closed-loop stability under droop control

Theorem: stability of droop control [J. Simpson-Porco, FD, & F. Bullo, '12]

$\exists$ unique & exp. stable frequency sync $\iff$ active power flow is feasible

Main **proof ideas** and some **further results**:

- synchronization frequency: ($\propto$ power balance)

$$\omega_{\text{sync}} = \omega^* + \frac{\sum_{\text{sources}} P_i^* + \sum_{\text{loads}} P_i^*}{\sum_{\text{sources}} D_i}$$

- steady-state power injections: (depend on D_i & P_i^*)

$$\mathcal{P}_i = \begin{cases} P_i^* & (\#i \text{ passive}) \\ P_i^* - D_i(\omega_{\text{sync}} - \omega^*) & (\#i \text{ active}) \end{cases}$$

- stability via incremental Lyapunov [Zhao, Mallada, & FD '14, J. Schiffer & FD '15]

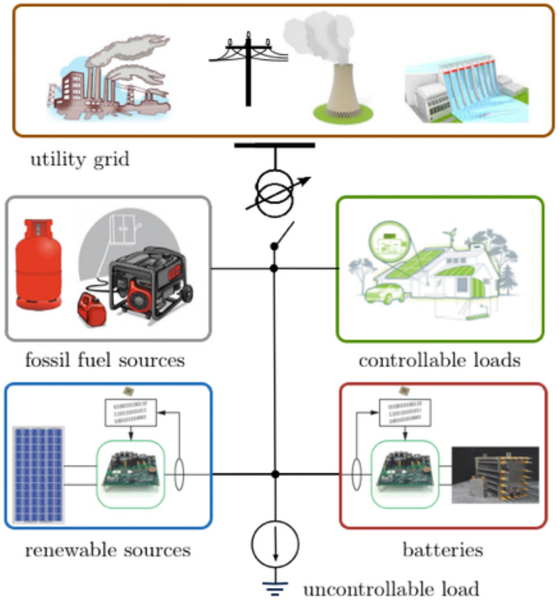
$$\mathcal{V}(x) = \text{kinetic energy} + \text{DAE potential energy} + \varepsilon \cdot \text{Chetaev cross term}$$

tertiary control

(energy management)

Tertiary control & energy management

an offline resource allocation & scheduling problem



Tertiary control & energy management

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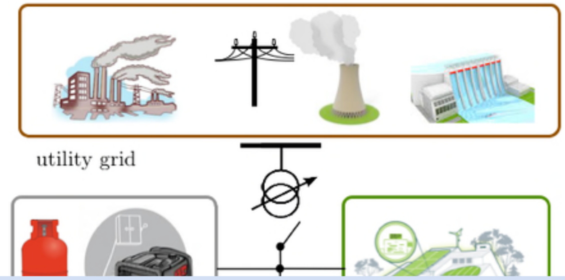


minimize	{cost of generation, losses, ...}
subject to	
equality constraints:	power balance equations
inequality constraints:	flow/injection/voltage constraints
logic constraints:	commit generators yes/no
	⋮

Objective: economic generation dispatch

minimize the total accumulated generation (many variations possible)

$$\text{minimize}_{\theta \in \mathbb{T}^n,\, u \in \mathbb{R}^{n_I}} \quad J(u) = \sum\nolimits_{\text{sources}} \alpha_i u_i^2$$

subject to

- source power balance: $P_i^* + u_i = P_i(\theta)$
- load power balance: $P_i^* = P_i(\theta)$
- branch flow constraints: $|\theta_i - \theta_j| \leq \gamma_{ij} < \pi/2$

Unconstrained case: identical marginal costs $\alpha_i u_i^\star = \alpha_j u_j^\star$ at optimality

In conventional power system operation, the economic dispatch is

- solved **offline**, in a **centralized** way, & with a **model & load forecast**

In a grid with distributed energy resources, the economic dispatch should be

- solved **online**, in a **decentralized** way, & **without knowing a model**

Objective: decentralized dispatch optimization

Insight: droop-controlled system = decentralized primal/dual algorithm

Theorem: optimal droop [FD, Simpson-Porco, & Bullo '13, Zhao, Mallada, & FD '14]

The following statements are equivalent:

(i) the economic dispatch with cost coefficients α_i is strictly feasible with global minimizer $(\theta^\star, u^\star)$.

(ii) $\exists$ droop coefficients D_i such that the power system possesses a unique & locally exp. stable sync'd solution θ.

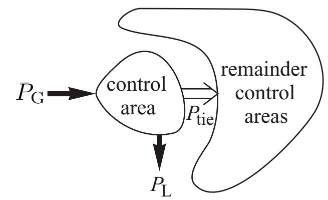
If (i) & (ii) are true, then $\theta_i \sim \theta_i^\star$, $u_i^\star = -D_i(\omega_{\text{sync}} - \omega^*)$, & $\boxed{D_i \alpha_i = D_j \alpha_j}$.

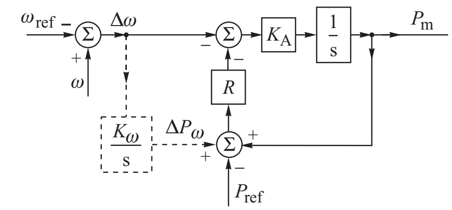
- similar results for **non-quadratic** (strictly convex) cost & **constraints**
- similar results in transmission ntwks with DC flow [E. Mallada & S. Low, '13] & [N. Li, L. Chen, C. Zhao, & S. Low '13] & [X. Zhang & A. Papachristodoulou, '13] & [M. Andreasson, D. V. Dimarogonas, K. H. Johansson, & H. Sandberg, '13] & ...

secondary control (frequency regulation)

Conventional secondary frequency control in power systems

iInterconnected systems	isolated systems
• **centralized** automatic generation control (AGC)	• **decentralized** PI control





compatible with econ. dispatch [N. Li, L. Chen, C. Zhao, & S. Low '13]

is *globally* stabilizing [C. Zhao, E. Mallada, & FD, '14]

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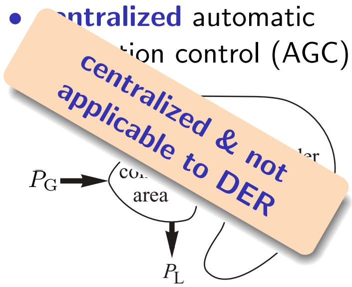
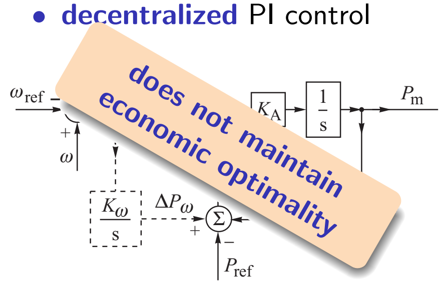




compatible with econ. dispatch [N. Li, L. Chen, C. Zhao, & S. Low '13]

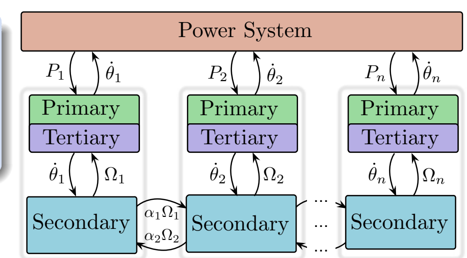
is *globally* stabilizing [C. Zhao, E. Mallada, & FD, '14]

Distributed energy resources require **distributed** (**!**) secondary control.

Distributed Averaging PI (DAPI) control

$$D_i\dot{\theta}_i = P_i^* - P_i(\theta) - \Omega_i$$

$$k_i\dot{\Omega}_i = D_i\dot{\theta}_i - \sum_{j \subseteq \text{sources}} a_{ij} \cdot (\alpha_i\Omega_i - \alpha_j\Omega_j)$$



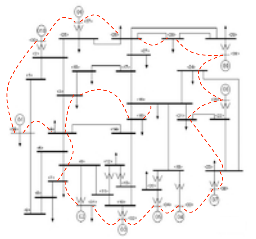
- no tuning & no time-scale separation: $k_i, D_i > 0$
- recovers optimal dispatch
- distributed & modular: connected comm. network
- has seen many extensions [C. de Persis et al., H. Sandberg et al., J. Schiffer et al., M. Zhu et al., ...]

Theorem: stability of DAPI

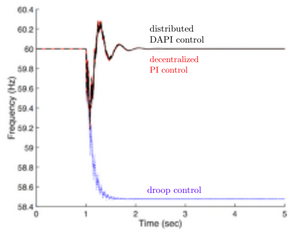
[J. Simpson-Porco, FD, & F. Bullo '12]
[C. Zhao, E. Mallada, & FD '14]

primary droop controller works
$\Longleftrightarrow$
secondary DAPI controller works

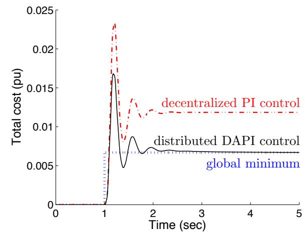
Some quick simulations & extensions



IEEE 39 New England with distributed DAPI control

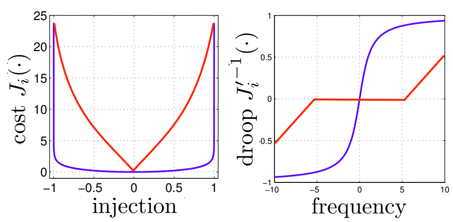


decentralized PI & DAPI control regulate frequency



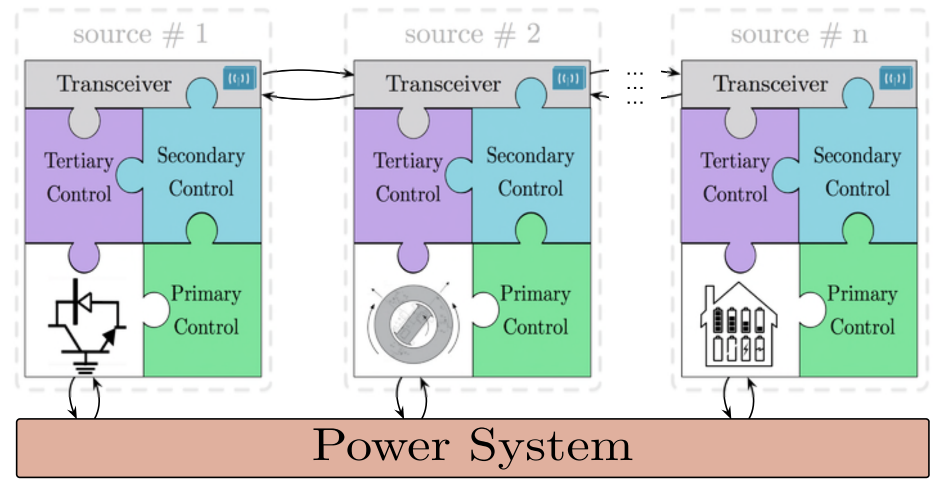
DAPI control minimizes cost with little effort

- ⇒ strictly convex & differentiable cost $J(u) = \sum_{\text{sources}} J_i(u_i)$
- ⇒ non-linear frequency droop curve $J_i'^{-1}(\dot\theta_i) = P_i^* - P_i(\theta)$
- ⇒ include dead-bands, saturation, etc.



Plug'n'play architecture

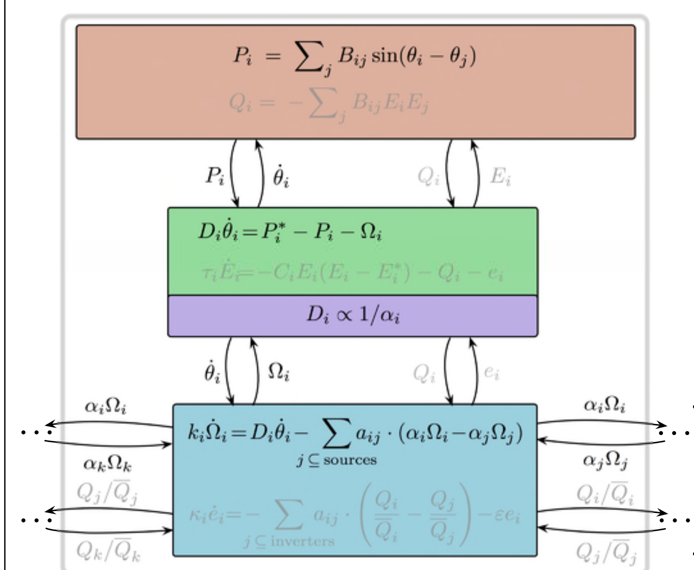
flat hierarchy, distributed, no time-scale separations, & model-free



plug-and-play experiments

Plug'n'play architecture

recap of detailed signal flow (active power only)



Power system: physics & loadflow

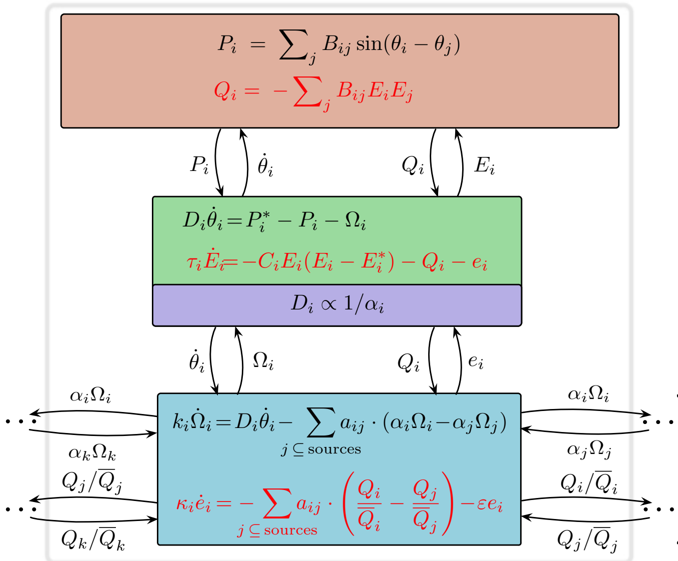
Primary control: mimic oscillators & polyn. symmetry

Tertiary control: marginal costs $\propto$ 1 /control gains

Secondary control: diffusive averaging of optimal injections

Plug'n'play architecture

similar results for decoupled reactive power flow [J. Simpson-Porco, FD, & F. Bullo '13 - '15]



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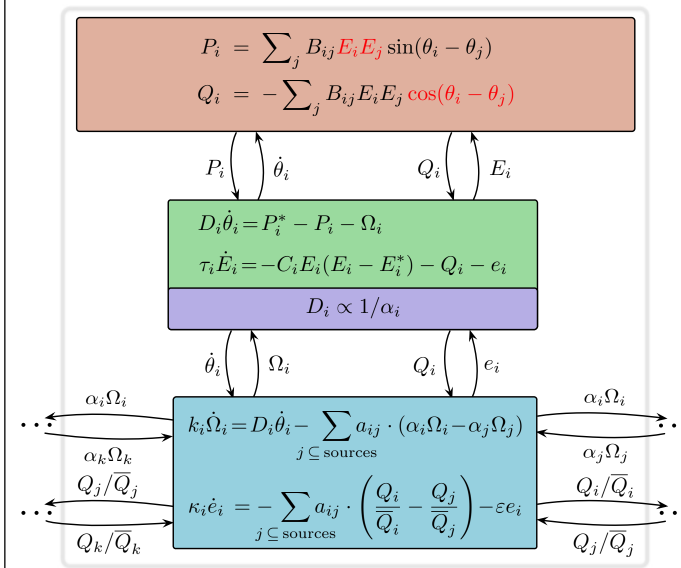
Plug'n'play architecture

can all be proved also in the coupled case [J. Schiffer, FD, N. Monshizadeh C. de Persis, '15]



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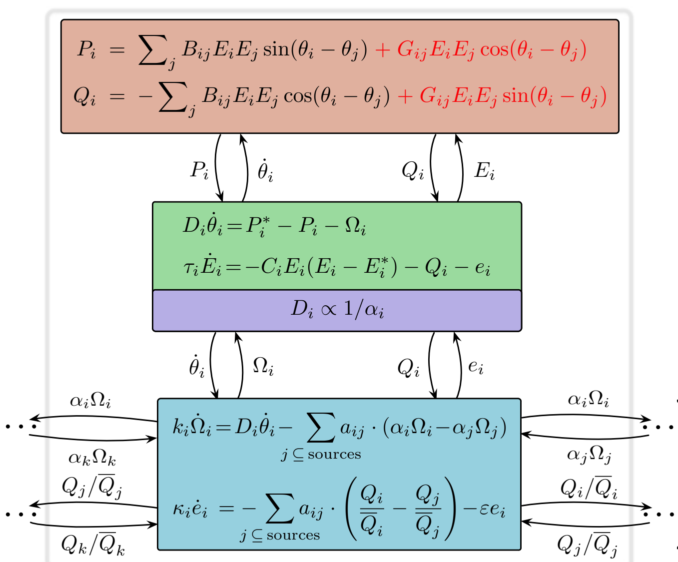
Plug'n'play architecture

experiments also work well in the lossy case



Power system: physics & loadflow

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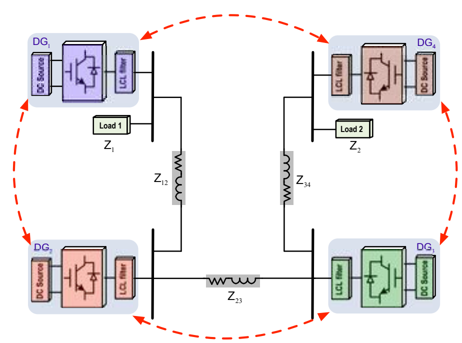
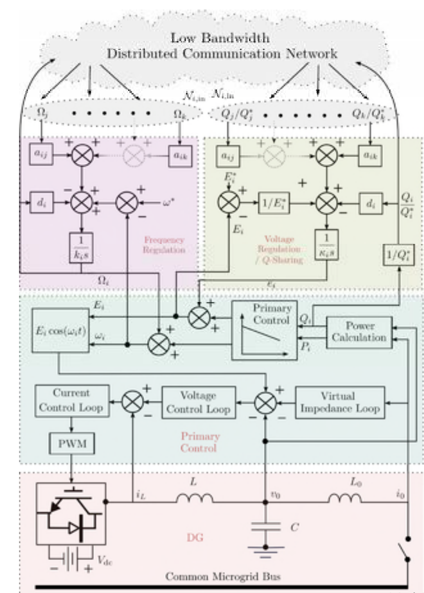
Secondary control: diffusive averaging of optimal injections

Experimental validation

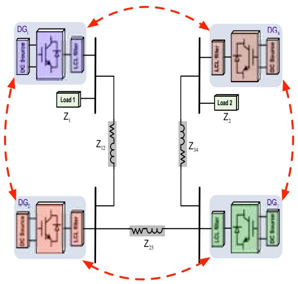
in collaboration with Q. Shafiee & J.M. Guerrero @ Aalborg University

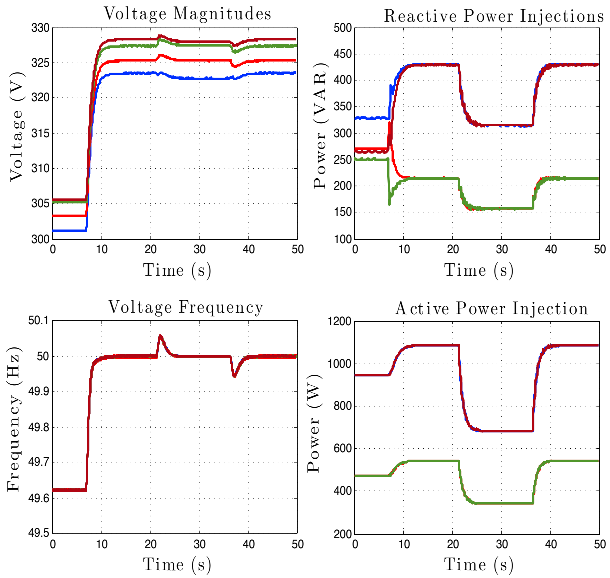




Experimental validation

frequency/voltage regulation & active/reactive load sharing





$t \in [0s, 7s]$: primary & tertiary control

$t = 7s$: secondary control activated

$t = 22s$: load # 2 unplugged

$t = 36s$: load # 2 plugged back

what can we do better?

algorithms, detailed models, cyber-physical aspects, ...

many groups out there push all these directions heavily

fact: most controllers are essentially nonlinear/distributed/optimal PID emulating synchronous machines

$$\underbrace{M\ddot{\theta}(t)}_{\text{virtual inertia}} = \underbrace{P^*}_{\text{set-point}} - \underbrace{D\dot{\theta}(t)}_{\text{droop control}} - \underbrace{\int_0^t \dot{\theta}(\tau)\, d\tau}_{\text{secondary control}}$$

now: **do things differently**

Variation I:

VOC: virtual oscillator control

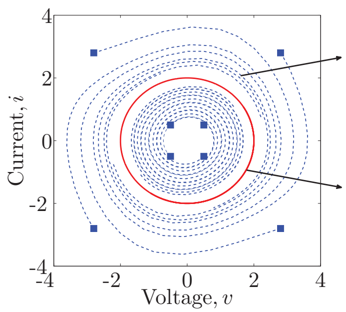
instead of primary droop control

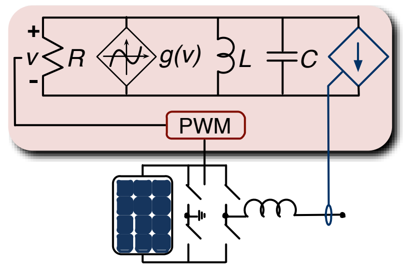
Removing the assumptions of droop control

- **idealistic assumptions:** quasi-stationary operation & phasor coordinates

⇒ **future grids:** more power electronics, more renewables, & less inertia

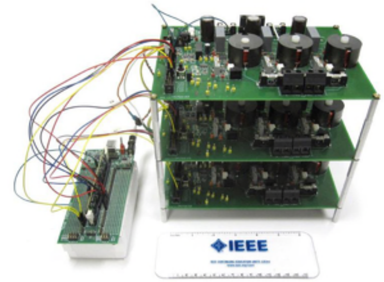
⇒ **Virtual Oscillator Control:** control inverters as limit cycle oscillators
[Torres, Moehlis, & Hespanha '12, Johnson, Dhople, Hamadeh, & Krein '13]

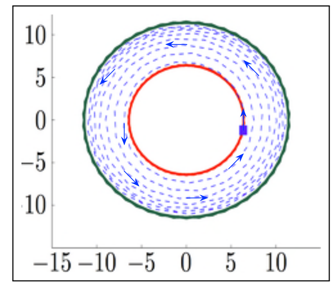




digitally implemented VOC

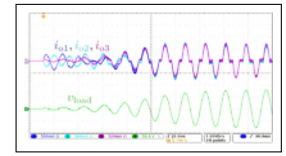
Plug'n'play **V**irtual **O**scillator **C**ontrol (VOC)



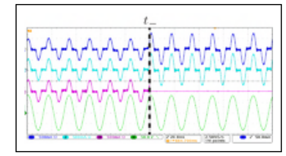


change of setpoint

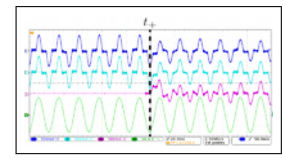
Oscilloscope plots:



emergence of synchrony



removal of inverter



addition of inverter

Crash course on planar limit cycle oscillators

$$L\frac{d}{dt}i = v$$
$$C\frac{d}{dt}v = -Rv - g(v) - i - i_{\text{grid}}$$

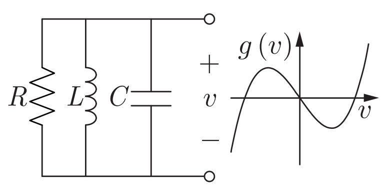
⇒ normalized coordinates

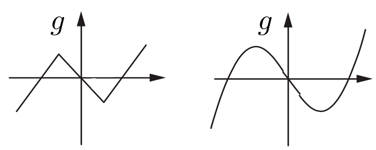
$$\ddot{v} + v + \varepsilon k_1 g'(v)\cdot\dot{v} = \varepsilon k_2 u$$

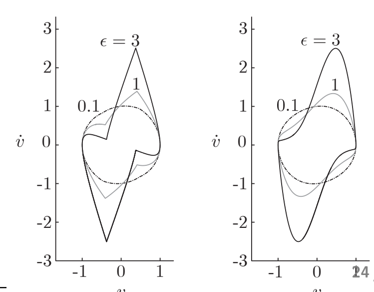
Liénard's limit cycle condition
for virtual oscillator with $u = 0$:

if $\varepsilon = \sqrt{L/C} \to 0$
⇒ $\mathcal{O}(\varepsilon)$ close to harmonic oscillator

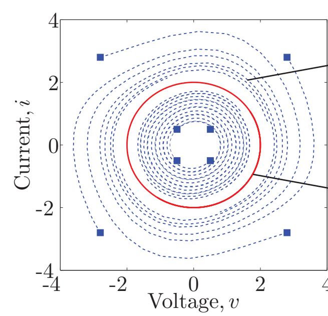
if damping $g'(v)$ is negative near origin & positive elsewhere
⇒ unique & stable limit cycle

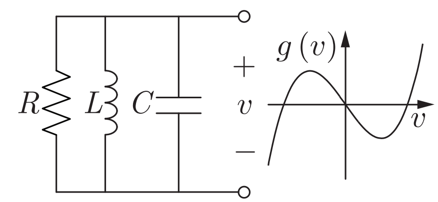






Backward compatibility to droop [M. Sinha, FD, B. Johnson, & S. Dhople, '14]





⇒ transf. to polar coordinates, averaging, & generalized power definitions

Thm: in vicinity of the limit cycle:
VOC ⊃ droop:

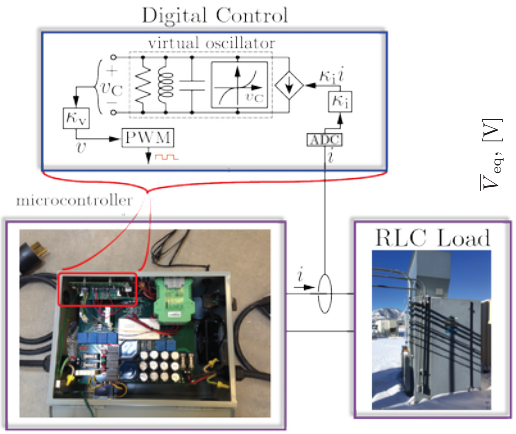
$$\dot{\theta} = constant \cdot (\text{reactive power})$$
$$r - r^* = constant \cdot (P^* - \text{active power})$$

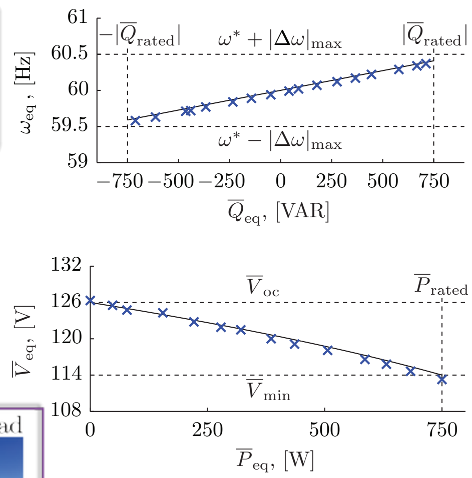
Experimental validation [B. Johnson, M. Sinha, N. Ainsworth, FD, & S. Dhople, '15]

❶ **VOC ⊃ droop:**

$$\dot{\theta} = \mathit{constant} \cdot (\text{reactive power})$$

$$r - r^{*} = \mathit{constant} \cdot (P^{*} - \text{active power})$$





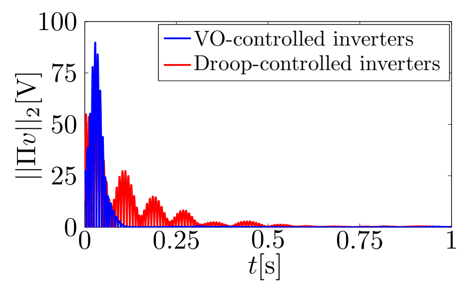
analytic vs. measured droop curves of VOC

Experimental validation [B. Johnson, M. Sinha, N. Ainsworth, FD, & S. Dhople, '15]

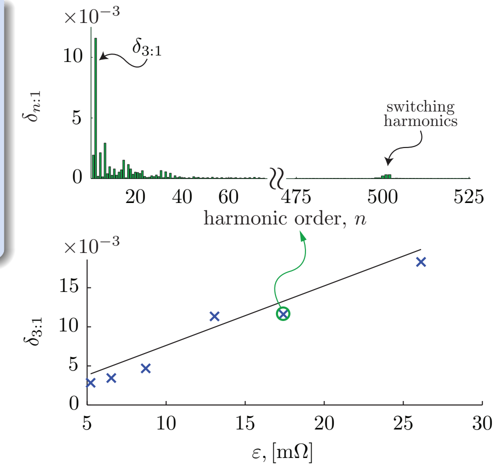
❶ **VOC ⊃ droop**

❷ **VOC $\xrightarrow{\varepsilon \to 0}$ harmonic oscillator**
with $\varepsilon/8$ harmonic ratio 3:1

❸ **VOC: faster & better transients**
than droop-controlled inverters



synchronization error: VOC vs. droop

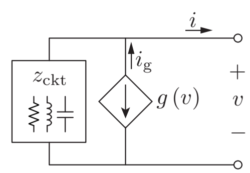


$\varepsilon/8$ harmonic ratio 3:1

Analysis of VOC system [S. Dhople, B. Johnson, FD, & A. Hamadeh '13]

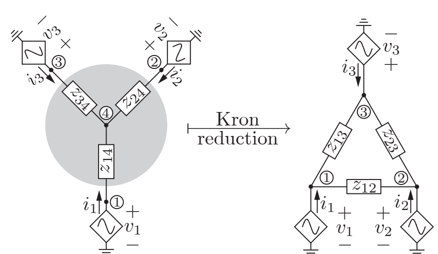
Nonlinear oscillators:

- passive circuit impedance $z_{\text{ckt}}(s)$
- active current source $g(v)$



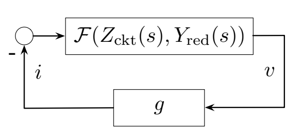
Co-evolving network:

- RLC network & loads are LTI
- Kron reduction: eliminate loads



Stability analysis:

- homogeneity assumption:
  identical reduced oscillators
- Lure system formulation
- incremental IQC analysis



⇝ sync for strong coupling

Variation II:

CH: no centralized dispatch but power trade in **energy markets**

⇓

game-theoretic formulation
of optimal secondary control

Market formulation of secondary control [FD & S. Grammatico '15]

Competitive spot market:

1. given a prize λ, player i bids
   $u_i^\star = \underset{u_i}{\text{argmin}} \{J_i(u_i) - \lambda u_i\} = J_i'^{-1}(\lambda)$
2. market clearing prize $\lambda^\star$ from
   $0 = \sum_i P_i^* + u_i^\star = \sum_i P_i^* + J_i'^{-1}(\lambda^\star)$

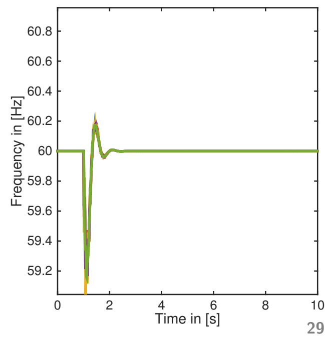
Broadcast controller:

1. convex measurement:
   $k \cdot \dot{\lambda}(t) = \sum_i C_i \dot{\theta}_i(t)$
2. local allocation:
   $u_i(t) = J_i'^{-1}(\lambda(t))$

Auction (dual decomposition):

1. $u_i^+ = \underset{u_i}{\text{argmin}} \{J_i(u_i) - \lambda u_i\} = J_i'^{-1}(\lambda)$
2. $\lambda^+ = \lambda - \epsilon \left(\sum_i P_i^* + u_i^+\right) = \lambda - \epsilon \cdot \omega_{\text{sync}}$

$\Rightarrow$ converges to optimal economic dispatch



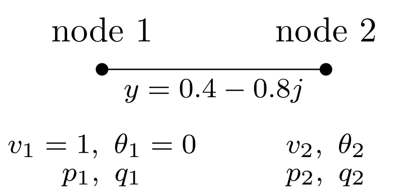
Variation III:

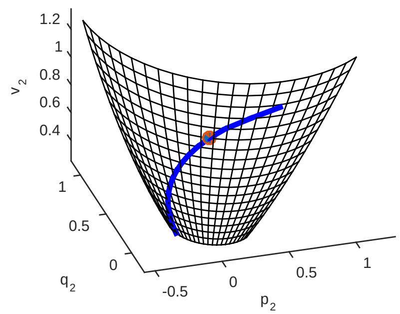
can we turn tertiary optimization directly into continuous control?

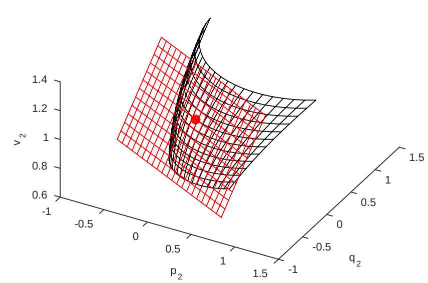
$\Downarrow$

preview on **online optimization**

The power flow manifold & linear tangent approximation







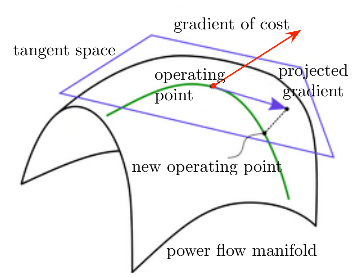
1. **power flow manifold:** $F(x) = \mathbb{0}$
2. **normal space** spanned by $\left.\frac{\partial F(x)}{\partial x}\right|_{x^*}$
3. **tangent space**: $\left.\frac{\partial F(x)}{\partial x}\right|_{x^*}^T (x - x^*) = \mathbb{0}$

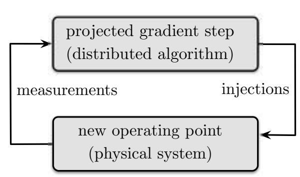
$\Rightarrow$ sparse & implicit model is **structure-preserving** $\rightarrow$ distributed control

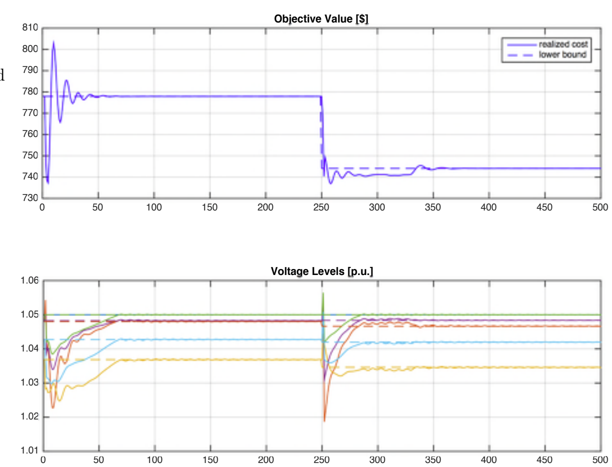
Online optimization on power flow manifold

with Adrian Hauswirth, Saverio Bolognani, & Gabriela Hug

- **manifold optimization** $\rightarrow$ gradient flow on power flow manifold
- **online optimization** $\rightarrow$ controller realizes gradient flow in closed loop







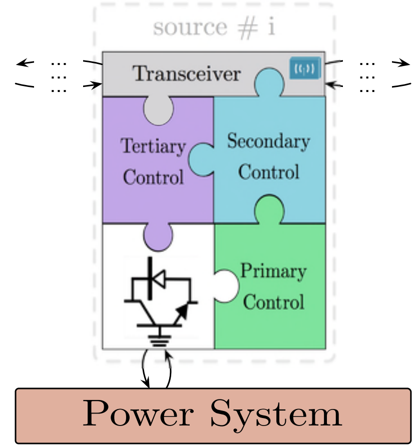
applied to optimal voltage control in IEEE 30 grid

conclusions

Conclusions

Summary

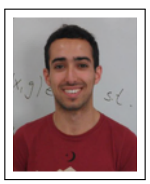
- primary decentralized droop
- distributed secondary control
- economic dispatch optimization
- experimental validation
- beyond emulation & PID strategies
  - primary virtual oscillator control
  - markets turned into controllers
  - control via online optimization



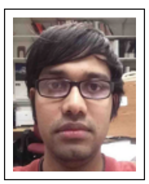
Ongoing work & next steps

- better models & sharper analysis
- optimize transient control behavior
- alternatives not based on emulation of synchronous machines & PID

Acknowledgements


J. Simpson-Porco


Q. Shafiee


M. Sinha


B. Gentile


A. Hamadeh


S. Dhople


B. Johnson


S. Zampieri

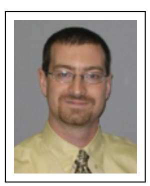

J. Guerrero


F. Bullo


J. Zhao


J. Schiffer


S. Grammatico


N. Ainsworth


S. Bolognani