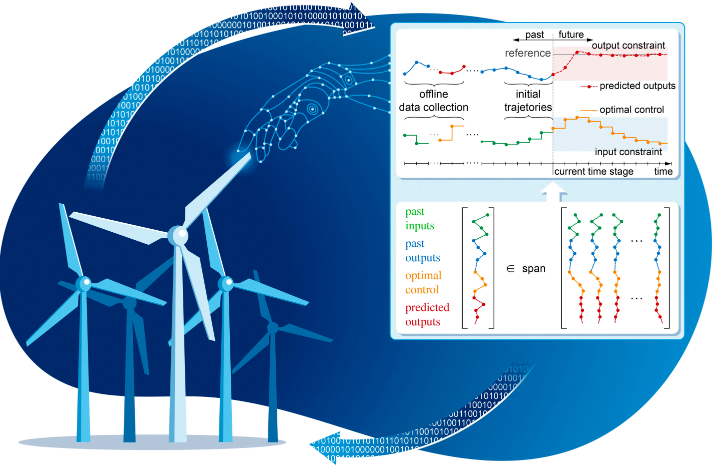
Data-Enabled Predictive Control of Autonomous Energy Systems



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...

& many master students

Big, deep, intelligent & so on

- ***unprecedented availability*** of computation, storage, & data
- ***theoretical advances*** in statistics, optimization, & machine learning
- …and ***big-data*** frenzy

→ increasing importance of ***data-centric methods*** in all of science / engineering

> Make up your own opinion, but machine learning works too well to be ignored.

From Pixels to Torques: Policy Learning with Deep Dynamical Models

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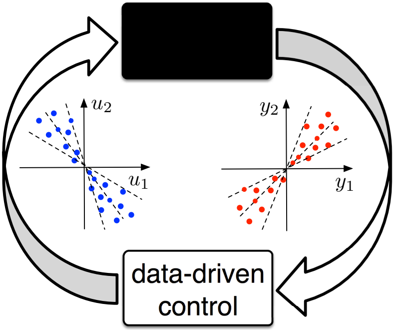
Thoughts on data-driven control

- ***indirect data-driven control*** via models:
  data $\xrightarrow{\text{SysID}}$ model + uncertainty $\rightarrow$ control
- growing trend: ***direct data-driven control***
  by-passing models . . . (again) hyped, why ?



The direct approach is ***viable alternative***

- for some ***applications*** : model-based approach is too complex to be useful

  $\rightarrow$ too complex models, environments, sensing modalities, specifications (e.g., wind farm)

- due to (well-known) ***shortcomings of ID***

  $\rightarrow$ too cumbersome, models not identified for control, incompatible uncertainty estimates, ...

- when ***brute force*** data/compute available

Central promise: *It is often easier to learn a control policy from data rather than a model.*

Example 1973: autotuned PID

Abstraction reveals pros & cons

indirect (model-based) ***data-driven control***

minimize	control cost (u, x)	} outer optimization
subject to	(u, x) satisfy state-space model	
where	x estimated from (u, y) & model	} middle opt.
where	model identified from (u^d, y^d) data	} inner opt.

outer optimization + middle opt. } separation & certainty equivalence (→ LQG case)

inner opt. } **no** separation (→ ID-4-control)

→ nested multi-level optimization problem

direct (black-box) ***data-driven control***

minimize	control cost (u, y)
subject to	(u, y) consistent with (u^d, y^d) data

→ ***trade-offs***

modular vs. end-2-end

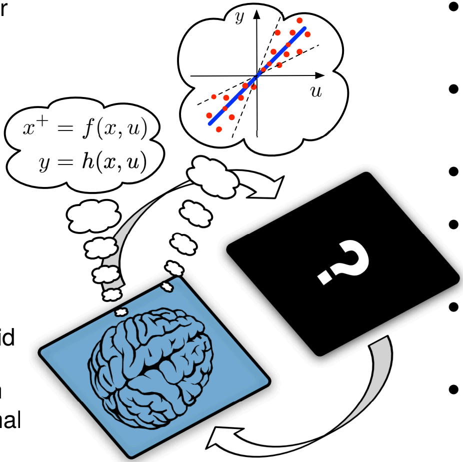
suboptimal (?) vs. optimal

convex vs. non-convex (?)

Additionally: account for ***uncertainty*** (hard to propagate in indirect approach)

Indirect (models) vs. direct (data)

- models are useful for design & beyond
- modular → easy to debug & interpret
- id = noise filtering
- id = projection on model class
- harder to propagate uncertainty through id
- no robust separation principle → suboptimal
- …



- some models too complex to be useful
- end-to-end → suitable for non-experts
- design handles noise
- harder to inject side info but no bias error
- transparent: no unmodeled dynamics
- possibly optimal but often less tractable
- …

there are ***no universal conclusions*** & plenty of counterexamples

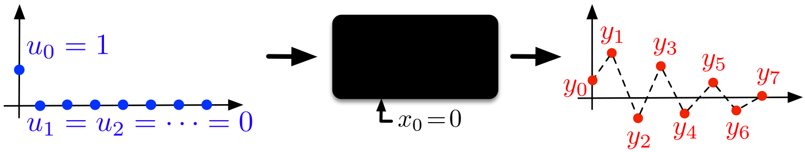
A direct approach: dictionary + MPC

① trajectory ***dictionary learning***

- motion primitives / basis functions
- theory: Koopman & Liouville
  practice: (E)DMD & particles

② ***MPC*** optimizing over dictionary span

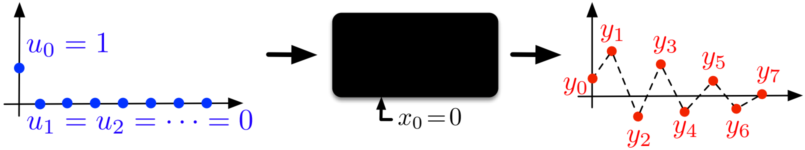
→ huge ***theory vs. practice*** gap
→ back to basics: ***impulse response***



- $\left.\begin{array}{l} u_0 = \delta_0 \\ x_0 = 0 \end{array}\right\} \rightsquigarrow y(t) = \text{impulse response} =: g(t) = \{g_0, g_1, g_2 \ldots\}$
- response to any other input $u(t)$ given by convolution with $g(t)$:

$$y(t) = g(t) * u(t) = \sum_{\tau=0}^{t-1} g(t-\tau)\, u(\tau) = g(0)\,u(t) + g(1)\,u(t-1) + \ldots$$

↑ convolution



Now what if we had the impulse response recorded in our data-library?

$$\begin{bmatrix} g_0 & g_1 & g_2 & \dots \end{bmatrix} = \begin{bmatrix} y_0^d & y_1^d & y_2^d & \dots \end{bmatrix}$$

⇝ can predict any future input $y_{\text{future}}(t)$ to any future input $u_{\text{future}}(t)$ by convolution with $g(t) = y^d(t)$: $y_{\text{future}}(t) = y_0^d u_{\text{future}}(t) + y_1^d u_{\text{future}}(t-1) + \dots$

in vector form

$\longrightarrow$ dynamic matrix control (Shell, 1970s): ***predictive control from raw data***

$$y_{\text{future}}(t) = \begin{bmatrix} y_0^d & y_1^d & y_2^d & \dots \end{bmatrix} \cdot \begin{bmatrix} u_{\text{future}}(t) \\ u_{\text{future}}(t-1) \\ u_{\text{future}}(t-2) \\ \vdots \end{bmatrix}$$

today: arbitrary, finite, & corrupted data, …stochastic & nonlinear ?

Today's menu

1. behavioral system theory: ***fundamental lemma***
2. ***DeePC***: data-enabled predictive control
3. robustification via salient ***regularizations***
4. cases studies from ***wind*** & ***power systems***

blooming literature (2-3 ArXiv / week)

$\rightarrow$ survey & tutorial to get started:

DATA-DRIVEN CONTROL BASED ON BEHAVIORAL APPROACH: FROM THEORY TO APPLICATIONS IN POWER SYSTEMS

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Review article

Behavioral systems theory in data-driven analysis, signal processing, and control

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[link] to related publications

Organization of the “school”

- I will ***teach the basics*** & provide pointers to more sophisticated research material ⟶ study cutting-edge papers yourself
- it’s a school: so we will spend time on the ***board*** ⟶ take notes

take notes in these boxes: hello world ☺

- We teach this material also in the ETH Zürich bachelor & have plenty of ***background material*** + implementation experience ⟶ please reach out if you need anything
- we will take a ***break*** after 90 minutes ⟶ coffee ☺

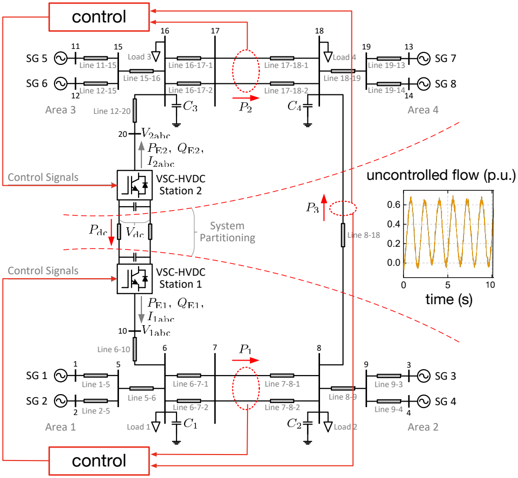
Preview

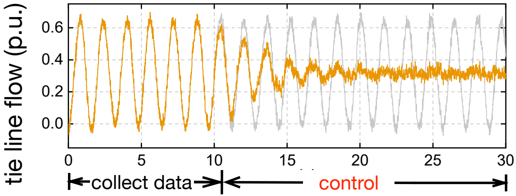
complex 4-area power ***system***:

large ($n = 208$), few sensors (8), nonlinear, noisy, stiff, input constraints, & decentralized control specifications

control objective: oscillation damping without a model

(grid has many owners, models are proprietary, operation in flux, . . .)





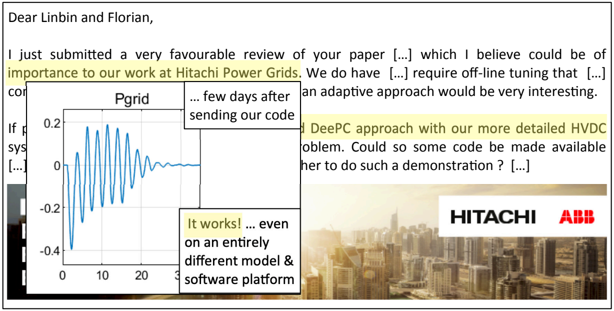
seek a method that ***works reliably***, can be ***efficiently*** implemented, & ***certifiable***

→ automating ourselves

Reality check: black magic or hoax ?

surely, nobody would put apply such a **shaky data-driven method**

- on the **world's most complex engineered system** (the electric grid),
- using the **world's biggest actuators** (Gigawatt-sized HVDC links),
- and subject to **real-time, safety, stability, constraints** . . . right?



at least someone believes that our method is practically useful . . .

LTI system representations

- ARX model (auto regressive, exogenous): $y(t+2) + 2y(t+1) + 3y(t) = 4u(t)$

- ARX to state space: $x(t) = \begin{bmatrix} y(t) \\ y(t+1) \end{bmatrix} \rightsquigarrow \begin{cases} \underbrace{x^+}_{x(t+1)} = \underbrace{\begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix}}_{=A} x + \underbrace{\begin{bmatrix} 0 \\ 4 \end{bmatrix}}_{=B} u \\ y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{=C} x \end{cases}$

- ARX to transfer function via z-transform

$y(t) \longrightarrow Y(z)$

$y(t+1) \longrightarrow z \cdot Y(z)$

$\rightsquigarrow Y(z) = \frac{4}{z^2 + 2z + 3} U(z)$

These are all parametric representations of an LTI system.
They are all so-called kernel representations since they nullify signals. E.g., for the ARX model, let z denote the forward time shift, e.g., $z \cdot y(t) = y(t+1)$, then we can write the model as

$$\underbrace{\begin{bmatrix} z^2 + 2z + 3, & 4 \end{bmatrix}}_{\text{"kernel"}} \cdot \underbrace{\begin{bmatrix} y(t) \\ u(t) \end{bmatrix}}_{\text{signal}} = 0$$

Behavioral view on dynamical systems

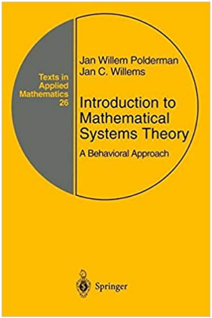
Definition: A discrete-time ***dynamical system*** is a 3-tuple $(\mathbb{Z}_{\geq 0}, \mathbb{W}, \mathscr{B})$ where

(i) $\mathbb{Z}_{\geq 0}$ is the *discrete-time* axis,

(ii) $\mathbb{W}$ is the *signal space*, &

(iii) $\mathscr{B} \subseteq \mathbb{W}^{\mathbb{Z}_{\geq 0}}$ is the *behavior*.

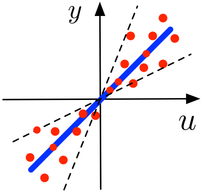
} $\mathscr{B}$ is the set of all trajectories



Definition: The dynamical system $(\mathbb{Z}_{\geq 0}, \mathbb{W}, \mathscr{B})$ is

(i) ***linear*** if $\mathbb{W}$ is a vector space & $\mathscr{B}$ is a subspace of $\mathbb{W}^{\mathbb{Z}_{\geq 0}}$

(ii) & ***time-invariant*** if $\mathscr{B} \subseteq \sigma\mathscr{B}$, where $\sigma w_t = w_{t+1}$.



LTI system $=$ shift-invariant subspace of trajectory space

$\longrightarrow$ abstract perspective suited for ***data-driven control***

Properties of the LTI trajectory space

- solution to $x^+ = A\underbrace{x}_{\in \mathbb{R}^n} + B\underbrace{u}_{\in \mathbb{R}^m}$ with $x(0) = x_{ini}$ is

  $x(0) = x_{ini}$, $x(1) = A x_{ini} + B u(0)$, $x(2) = A^2 x_{ini} + A B u(0) + B u(1)$, ...

  $\rightsquigarrow x(t) = A^t x_{ini} + \sum_{\tau=0}^{t-1} A^\tau B u(t-\tau-1)$

- solution to $x^+ = Ax + B$, $\underbrace{y}_{\in \mathbb{R}^p} = Cx + Du$, $x(0) = x_{ini}$ is

$$y(t) = C A^t x_{ini} + \underbrace{\sum_{\tau=0}^{t-1} C A^\tau B u(t-\tau-1) + D u(t)}_{g(t) * u(t)}$$

- in vector notation for $t \in \{0, 1, \dots, T\}$

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ \vdots \\ y(T) \end{bmatrix} = \underbrace{\begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^T \end{bmatrix}}_{\mathcal{O}_T} x_{ini} + \underbrace{\begin{bmatrix} D & & & \\ CB & D & & \\ CAB & CB & D & \\ \ddots & \ddots & \ddots & \\ CA^{T-1}B \cdots & CAB & CB & D \end{bmatrix}}_{\mathcal{C}_T} \begin{bmatrix} u(0) \\ u(1) \\ u(2) \\ \vdots \\ u(T) \end{bmatrix}$$

$\mathcal{O}_T$: extended observability matrix $\quad$ $\mathcal{C}_T$: convolution (or impulse response) matrix

- compactly: $y = \mathcal{O}_T x_{ini} + \mathcal{C}_T u$ (*)
  $\mathcal{O}_T \in \mathbb{R}^{pT \times n}$

- <u>observability:</u> x_{ini} can be uniquely reconstructed from (*)
  $\Leftrightarrow \; n = \operatorname{rank} \mathcal{O}_T = \operatorname{rank} \begin{bmatrix} C \\ CA \\ \vdots \\ CA^T \end{bmatrix}$

  ⇝ the smallest T so that $\operatorname{rank} \mathcal{O}_T = n$ is called the <u>lag</u> ℓ of the system. In the SISO case ($p=1$), we have $\ell = n$; for $p \geq 1$, we have $\ell \leq n$ (MIMO case).

  ⇝ given past data $y_{ini} \in \mathbb{R}^{pT_{ini}}$, $u_{ini} \in \mathbb{R}^{mT_{ini}}$ of length $T_{ini} \geq \ell$, we can uniquely recover x_{ini} from
  $$y_{ini} = \mathcal{O}_{T_{ini}} x_{ini} + \mathcal{C}_{T_{ini}} x_{ini}$$

- **dimension** of the LTI trajectory space: for any x_{ini} what is the dimension $y = \begin{bmatrix} y(0) \\ \vdots \\ y(T) \end{bmatrix}$ and $u = \begin{bmatrix} u(0) \\ \vdots \\ u(T) \end{bmatrix}$?

$$\underbrace{\begin{bmatrix} u \\ y \end{bmatrix}}_{u \in \mathbb{R}^{mT},\; y \in \mathbb{R}^{pT}} = \begin{bmatrix} 0 & I \\ \mathcal{O}_T & \mathcal{C}_T \end{bmatrix} \begin{bmatrix} x_{ini} \\ u \end{bmatrix} \quad x_{ini} \in \mathbb{R}^n,\; u \in \mathbb{R}^{mT}$$

↑ this column has full rank mT (second column)

↑ this colum has full rank n provided $T \geq \ell$ (first column)

⇝ **dimension $\begin{bmatrix} u \\ y \end{bmatrix} = mT + n$ for $T \geq \ell$**

LTI systems & matrix time series

foundation of state-space subspace system ID & signal recovery algorithms

$\big(u(t), y(t)\big)$ satisfy recursive ***difference equation***

$$b_0 u_t + b_1 u_{t+1} + \ldots + b_n u_{t+n} + a_0 y_t + a_1 y_{t+1} + \ldots + a_n y_{t+n} = 0$$

(ARX / kernel representation)

$$\Rightarrow$$

$\Leftarrow$

under assumptions

$\begin{bmatrix} 0 & b_0 & a_0 & b_1 & a_1 & \ldots & b_n & a_n & 0 \end{bmatrix}$ in left nullspace of ***trajectory matrix*** (collected data)

$$\mathscr{H}\begin{pmatrix} u^d \\ y^d \end{pmatrix} = \begin{bmatrix} \begin{pmatrix} u^d_{1,1} \\ y^d_{1,1} \end{pmatrix} & \begin{pmatrix} u^d_{1,2} \\ y^d_{1,2} \end{pmatrix} & \begin{pmatrix} u^d_{1,3} \\ y^d_{1,3} \end{pmatrix} & \cdots \\ \begin{pmatrix} u^d_{2,1} \\ y^d_{2,1} \end{pmatrix} & \begin{pmatrix} u^d_{2,2} \\ y^d_{2,2} \end{pmatrix} & \begin{pmatrix} u^d_{2,3} \\ y^d_{2,3} \end{pmatrix} & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ \begin{pmatrix} u^d_{T,1} \\ y^d_{T,1} \end{pmatrix} & \begin{pmatrix} u^d_{T,2} \\ y^d_{T,2} \end{pmatrix} & \begin{pmatrix} u^d_{T,3} \\ y^d_{T,3} \end{pmatrix} & \cdots \end{bmatrix}$$

where $y^d_{t,i}$ is tth sample from ith experiment

Fundamental Lemma

Given: data $\begin{pmatrix} u_i^d \\ y_i^d \end{pmatrix} \in \mathbb{R}^{m+p}$ & LTI complexity parameters $\begin{cases} \text{lag } \ell \\ \text{order } n \end{cases}$

$$\text{set of all } T\text{-length trajectories } = \left\{ (u, y) \in \mathbb{R}^{(m+p)T} : \exists x \in \mathbb{R}^{nT} \text{ s.t. } x^+ = Ax + Bu\,,\; y = Cx + Du \right\} = \text{colspan} \begin{bmatrix} \begin{pmatrix} u_{1,1}^d \\ y_{1,1}^d \end{pmatrix} & \begin{pmatrix} u_{1,2}^d \\ y_{1,2}^d \end{pmatrix} & \begin{pmatrix} u_{1,3}^d \\ y_{1,3}^d \end{pmatrix} & \cdots \\ \begin{pmatrix} u_{2,1}^d \\ y_{2,1}^d \end{pmatrix} & \begin{pmatrix} u_{2,2}^d \\ y_{2,2}^d \end{pmatrix} & \begin{pmatrix} u_{2,3}^d \\ y_{2,3}^d \end{pmatrix} & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ \begin{pmatrix} u_{T,1}^d \\ y_{T,1}^d \end{pmatrix} & \begin{pmatrix} u_{T,2}^d \\ y_{T,2}^d \end{pmatrix} & \begin{pmatrix} u_{T,3}^d \\ y_{T,3}^d \end{pmatrix} & \cdots \end{bmatrix}$$

parametric state-space model

raw data (every column is an experiment)

if and only if the trajectory matrix has rank $m \cdot T + n$ for all $T > \ell$

$$\text{set of all } T\text{-length trajectories } = \left\{ (u, y) \in \mathbb{R}^{(m+p)T} : \exists x \in \mathbb{R}^{nT} \text{ s.t. } x^+ = Ax + Bu\,,\; y = Cx + Du \right\} = \operatorname{colspan} \begin{bmatrix} \begin{pmatrix} u^d_{1,1} \\ y^d_{1,1} \end{pmatrix} & \begin{pmatrix} u^d_{1,2} \\ y^d_{1,2} \end{pmatrix} & \begin{pmatrix} u^d_{1,3} \\ y^d_{1,3} \end{pmatrix} & \cdots \\ \begin{pmatrix} u^d_{2,1} \\ y^d_{2,1} \end{pmatrix} & \begin{pmatrix} u^d_{2,2} \\ y^d_{2,2} \end{pmatrix} & \begin{pmatrix} u^d_{2,3} \\ y^d_{2,3} \end{pmatrix} & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ \begin{pmatrix} u^d_{T,1} \\ y^d_{T,1} \end{pmatrix} & \begin{pmatrix} u^d_{T,2} \\ y^d_{T,2} \end{pmatrix} & \begin{pmatrix} u^d_{T,3} \\ y^d_{T,3} \end{pmatrix} & \cdots \end{bmatrix}$$

all trajectories constructible from finitely many previous trajectories

- ***standing on the shoulders of giants***: classic Willems' result was only "if" & required further assumptions: Hankel, persistency of excitation, controllability

A note on persistency of excitation

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- terminology ***fundamental*** is justified : motion primitives, subspace SysID, dictionary learning, (E)DMD, . . . all implicitly rely on this equivalence

- many recent ***extensions*** to other ***system classes*** (bi-linear, descriptor, LPV, delay, Volterra series, Wiener-Hammerstein, . . .), other ***matrix data structures*** (mosaic Hankel, Page, . . .), & other ***proof methods***

Input design for Fundamental Lemma

Definition: The data signal $u^d \in \mathbb{R}^{mT_d}$ of length T_d is ***persistently exciting of order*** T if the Hankel matrix $\begin{bmatrix} u_1 & \cdots & u_{T_d-T+1} \\ \vdots & \ddots & \vdots \\ u_T & \cdots & u_{T_d} \end{bmatrix}$ is of full rank.

mT rows

rank $= mT$ only if $T_d - T + 1 \geq mT$ $\Rightarrow$ sufficiently rich & long data

Input design [Willems et al, '05]: Controllable LTI system & persistently exciting input u^d of order $T + n$ $\Longrightarrow$ rank $\left(\mathscr{H}\begin{pmatrix} u^d \\ y^d \end{pmatrix}\right) = mT + n$.

Data matrix structures & preprocessing

Matrix structures

$w = \begin{pmatrix} u^d \\ y^d \end{pmatrix}$

- **trajectory matrix:** $H(w) = \begin{bmatrix} \text{trajectory 1 of length } T & \vdots & \text{trajectory 2 of length } T & \vdots & \cdots \end{bmatrix}$

  requires independent experiments

- **page matrix:** $H(w) = \begin{bmatrix} w_1 & w_{T+1} & w_{2T+1} & \\ w_2 & \vdots & \vdots & \cdots \\ \vdots & & & \\ w_T & w_{2T} & w_{3T} & \end{bmatrix}$

  requires one long trajectory

- **Hankel matrix:** $H(w) = \begin{bmatrix} w_1 & w_2 & w_3 & \\ w_2 & w_3 & w_4 & \cdots \\ \vdots & \vdots & \vdots & \\ w_T & w_{T+1} & w_{T+2} & \end{bmatrix}$ uses shift invariance

  requires one short trajectory

$\rightsquigarrow$ Hankel structure (constant anti-diagonals) due to shift invariance

⇝ all these are valid matrices whose columns span the space of length-T trajectories provided that their rank is $mT+n$. One can also combine these different data structures into block matrices...

Preprocessing: if the data $w = \begin{pmatrix} u^d \\ y^d \end{pmatrix}$ is noisy, then likely all of the above matrices have full rank.

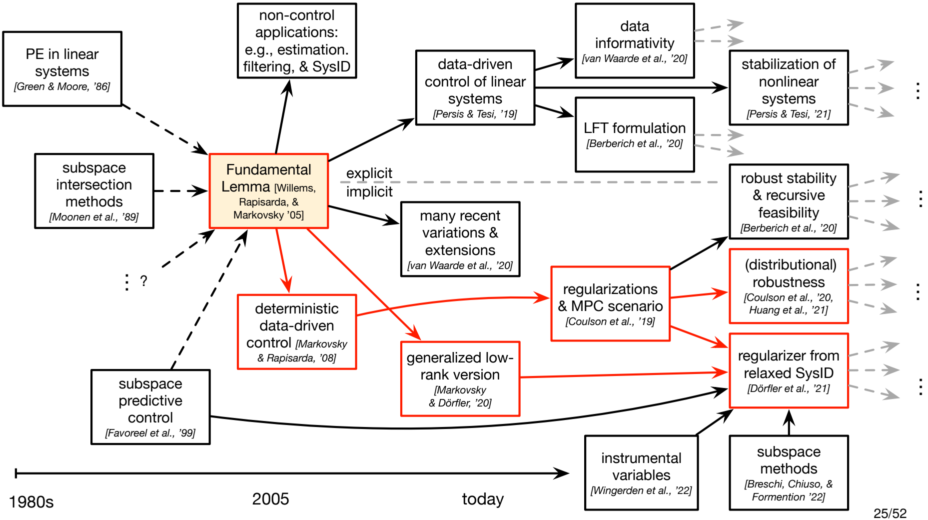
⇝ low-rank approximation: to denoise the data, find the "closest" matrix so that its rank is $mT+n$

for theory: look up the Eckhard Young theorem

⇝ practical solution is singular value thresholding: remove all but the $mT+n$ dominant singular values of the SVD

⇝ hard problem for Hankel matrices if it is desired to keep the Hankel structure (and induce shift invariance)

Bird’s view & today’s sample path through the accelerating literature



Output **M**odel **P**redictive **C**ontrol (**MPC**)

$$
\begin{aligned}
\underset{u,x,y}{\text{minimize}} \quad & \sum_{k=1}^{T_{\text{future}}} \|y_k - r_k\|_Q^2 + \|u_k\|_R^2 \\
\text{subject to} \quad & \left.\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ y_k &= Cx_k + Du_k \end{aligned}\right\} \quad \forall k \in \{1, \dots, T_{\text{future}}\} \\
& \left.\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ y_k &= Cx_k + Du_k \end{aligned}\right\} \quad \forall k \in \{-T_{\text{ini}} - 1, \dots, 0\} \\
& \left.\begin{aligned} u_k &\in \mathcal{U} \\ y_k &\in \mathcal{Y} \end{aligned}\right\} \quad \forall k \in \{1, \dots, T_{\text{future}}\}
\end{aligned}
$$

quadratic cost with $R \succ 0, Q \succeq 0$ & ref. r

model for **prediction** with $k \in [1, T_{\text{future}}]$

model for **estimation** with $k \in [-T_{\text{ini}} - 1, 0]$ & $T_{\text{ini}} \geq$ lag (many flavors)

hard operational or safety **constraints**

*"[MPC] has perhaps too little system theory and too much **brute force** [...], but MPC is an area where all aspects of the field [...] are in synergy." – Willems '07*

Elegance aside, for a LTI plant, deterministic, & with known model, MPC is the ***gold standard of control***.

Data-enabled Predictive Control (DeePC)

$$\underset{g,u,y}{\text{minimize}} \quad \sum_{k=1}^{T_{\text{future}}} \|y_k - r_k\|_Q^2 + \|u_k\|_R^2$$

$$\text{subject to} \quad \mathscr{H}\begin{pmatrix} u^{\mathsf{d}} \\ y^{\mathsf{d}} \end{pmatrix} \cdot g = \begin{bmatrix} u_{\text{ini}} \\ y_{\text{ini}} \\ \hline u \\ y \end{bmatrix}$$

$$\left.\begin{array}{l} u_k \in \mathcal{U} \\ y_k \in \mathcal{Y} \end{array}\right\} \quad \forall k \in \{1, \ldots, T_{\text{future}}\}$$

quadratic cost with $R \succ 0, Q \succeq 0$ & ref. r

non-parametric model for **prediction** and **estimation**

hard operational or safety **constraints**

- real-time measurements $(u_{\text{ini}}, y_{\text{ini}})$ for estimation — updated **online**
- trajectory matrix $\mathscr{H}\begin{pmatrix} u^{\mathsf{d}} \\ y^{\mathsf{d}} \end{pmatrix}$ from past experimental data — collected **offline** (could be adapted online)

→ ***equivalent to MPC*** in deterministic LTI case ...
but needs to be robustified in case of noise / nonlinearity !

Regularizations to make it work

$$\underset{g,u,y,\sigma}{\text{minimize}} \quad \sum_{k=1}^{T_{\text{future}}} \|y_k - r_k\|_Q^2 + \|u_k\|_R^2 + \lambda_y \|\sigma\|_p + \lambda_g h(g)$$

$$\text{subject to} \quad \mathscr{H}\begin{pmatrix} u^{\mathsf{d}} \\ y^{\mathsf{d}} \end{pmatrix} \cdot g = \begin{bmatrix} u_{\text{ini}} \\ y_{\text{ini}} \\ \hline u \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ \sigma \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{matrix} u_k \in \mathcal{U} \\ y_k \in \mathcal{Y} \end{matrix}\right\} \quad \forall k \in \{1, \ldots, T_{\text{future}}\}$$

measurement noise
→ infeasible y_{ini} estimate
→ estimation slack σ
→ moving-horizon least-square filter

noisy or nonlinear (offline) **data matrix**
→ any $\binom{u}{y}$ feasible
→ add regularizer $h(g)$

Bayesian intuition: regularization ⇔ prior, e.g., $h(g) = \|g\|_1$ sparsely selects {trajectory matrix columns} = {motion primitives} ∼ low-order basis

triangle inequality is tight since Δx can be collinear with $Ax-b$

Robustness intuition: regularization ⇔ robustifies, e.g., in a simple case

$$\min_x \max_{\Delta : \|\Delta\| \le \rho} \|(A+\Delta)x - b\| \;\underset{=}{\le}\; \min_x \max_{\|\Delta\| \le \rho} \|Ax-b\| + \|\Delta x\| = \min_x \|Ax-b\| + \rho\|x\|$$

Regularization = relaxation of bi-level ID

$$\begin{aligned}
&\text{minimize}_{u,y,g} && \text{control cost}(u,y)\\
&\text{subject to} && \begin{bmatrix} u \\ y \end{bmatrix} = \mathscr{H}\begin{pmatrix} \hat{u} \\ \hat{y} \end{pmatrix} g\\
&\text{where} && \begin{pmatrix} \hat{u} \\ \hat{y} \end{pmatrix} \in \operatorname{argmin} \left\| \begin{pmatrix} \hat{u} \\ \hat{y} \end{pmatrix} - \begin{pmatrix} u^{\mathrm{d}} \\ y^{\mathrm{d}} \end{pmatrix} \right\|\\
& && \text{subject to } \operatorname{rank}\left(\mathscr{H}\begin{pmatrix} \hat{u} \\ \hat{y} \end{pmatrix}\right) = mL + n
\end{aligned}$$

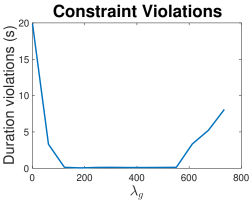
} optimal control

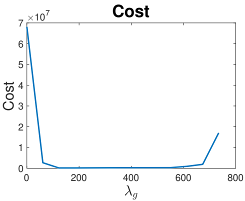
} system identification

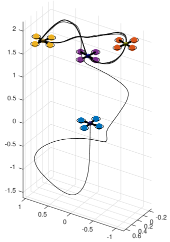
↓ sequence of convex relaxations ↓

$$\begin{aligned}
&\text{minimize}_{u,y,g} && \text{control cost}(u,y) + \lambda_g \cdot \|g\|_1\\
&\text{subject to} && \begin{bmatrix} u \\ y \end{bmatrix} = \mathscr{H}\begin{pmatrix} u^{d} \\ y^{d} \end{pmatrix} g
\end{aligned}$$

ℓ_1-regularization
$=$ relaxation of id
smooth order selection







Certainty-Equivalence Regularizer

ARX representation of predictor:

- recall $y = \mathcal{O}_T x_{ini} + \mathcal{C}_T u$

  where x_{ini} satisfies $y_{ini} = \mathcal{O}_{T_{ini}} x_{ini} + \mathcal{C}_{T_{ini}} u_{ini}$

$\rightsquigarrow y = K \cdot \begin{bmatrix} u_{ini} \\ y_{ini} \\ u \end{bmatrix}$, where K is

learned from data (certainty equiv.):

$$K = \underset{\tilde{K}}{\operatorname{argmin}} \left\| Y_f - \tilde{K} \begin{bmatrix} U_p \\ Y_p \\ U_f \end{bmatrix} \right\| = Y_f \begin{bmatrix} U_p \\ Y_p \\ U_f \end{bmatrix}^{+}$$

(note: K should also satisfy structural constraints, which we dropped)

$$\rightsquigarrow y = Y_f \cdot \begin{bmatrix} U_p \\ Y_p \\ U_f \end{bmatrix}^{+} \cdot \begin{bmatrix} u_{ini} \\ y_{ini} \\ u \end{bmatrix}$$

DeePC representation of predictor:

$$\begin{bmatrix} u_{ini} \\ y_{ini} \\ u \\ y \end{bmatrix} = \begin{bmatrix} U_p \\ Y_p \\ U_f \\ Y_f \end{bmatrix} g$$

— $H(w)$ suitably partitioned

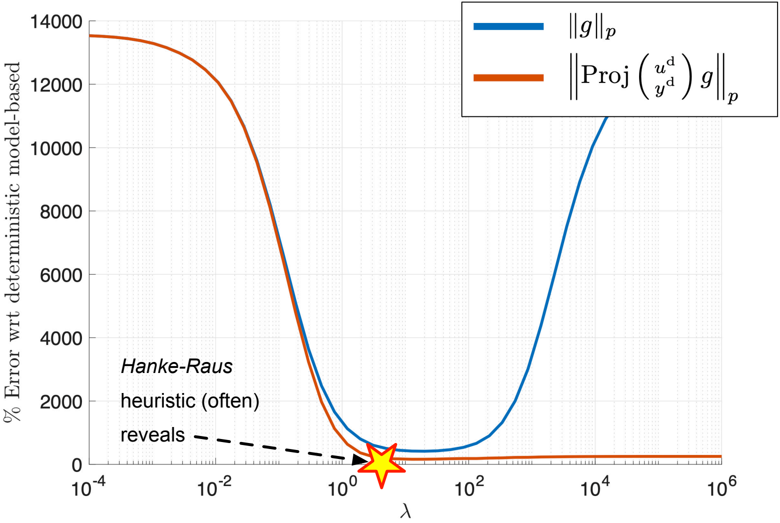
or $y = Y_f\, g$ with g from $\begin{bmatrix} U_p \\ Y_p \\ U_f \end{bmatrix} g = \begin{bmatrix} u_{ini} \\ y_{ini} \\ u \end{bmatrix}$

$$\rightsquigarrow y = \underbrace{Y_f \cdot \begin{bmatrix} U_p \\ Y_p \\ U_f \end{bmatrix}^{+} \begin{bmatrix} u_{ini} \\ y_{ini} \\ u \end{bmatrix}}_{\text{particular solution}} + \underbrace{Y_f \cdot \ker \begin{bmatrix} U_p \\ Y_p \\ U_f \end{bmatrix}}_{\text{homogeneous}}$$

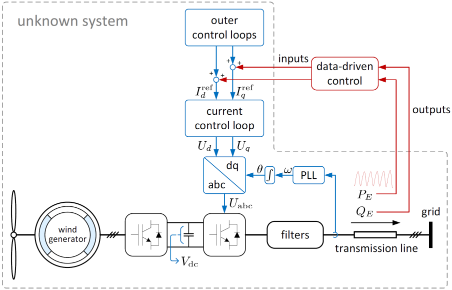
these two coincide if we force that $0 = \left\| \left(I - \begin{bmatrix} U_p \\ Y_p \\ U_f \end{bmatrix}^{+} \begin{bmatrix} U_p \\ Y_p \\ U_f \end{bmatrix} \right) g \right\|$

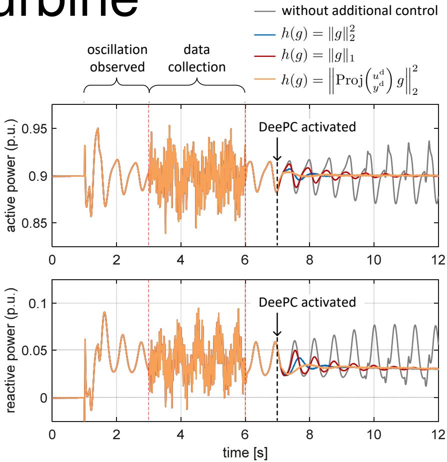
add this regularizer to DeePC → $\text{Proj}\begin{pmatrix} u^d \\ y^d \end{pmatrix}$ on nullspace

Performance of regularizers applied to a stochastic LTI system

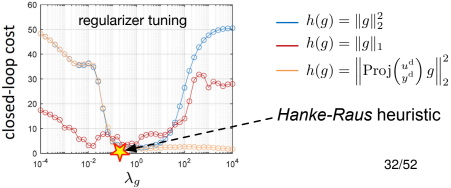


Case study: wind turbine

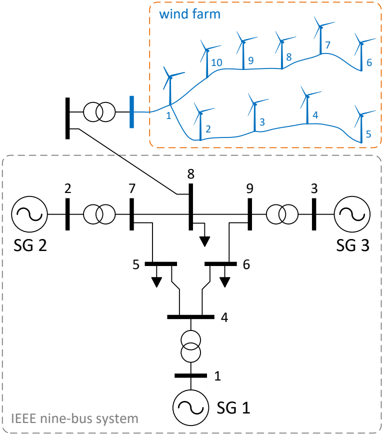


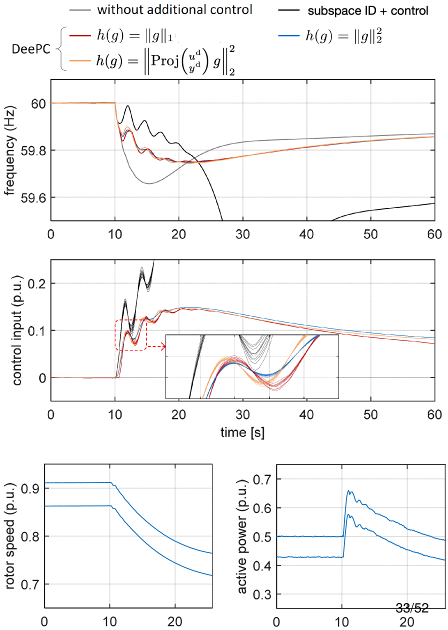


- detailed ***industrial model***: 37 states & highly nonlinear (abc $\leftrightarrow$ dq, MPTT, PLL, power specs, dynamics, etc.)
- turbine & grid model ***unknown*** to commissioning engineer & operator
- weak grid + PLL + fault $\rightarrow$ ***loss of sync***
- disturbance to be rejected by ***DeePC***



Case study ++ : wind farm

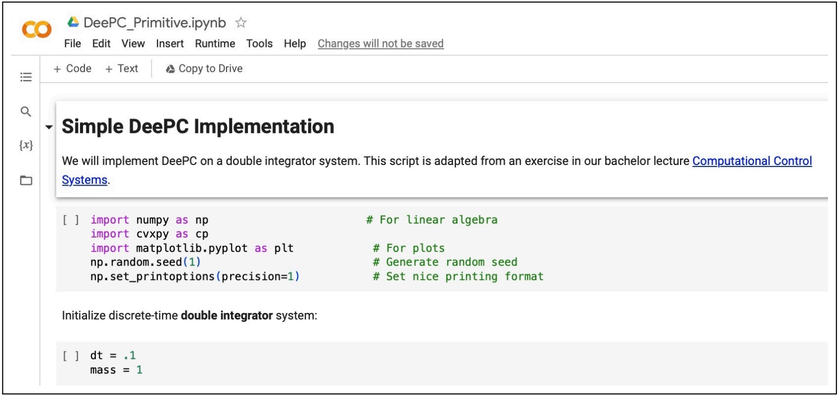




- ***high-fidelity models*** for turbines, machines, & IEEE-9-bus system
- ***fast frequency response*** via ***decentralized DeePC*** at turbines

DeePC is easy to implement → try it !

→ simple script adapted from our ETH Zürich bachelor course on *Computational control* : https://colab.research.google.com/drive/1URdRqr-Up0A6uDMjlU6gwmsoAAPl1GId?usp=sharing



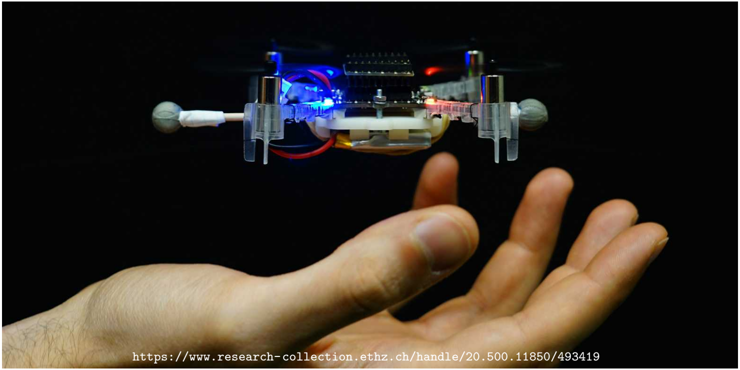
Towards a theory for nonlinear systems

idea : lift nonlinear system to large/∞-dimensional bi-/linear system

$\rightarrow$ Carleman, Volterra, Fliess, Koopman, Sturm-Liouville methods

$\rightarrow$ nonlinear dynamics can be approximated by LTI on finite horizon

regularization singles out relevant features / basis functions in data

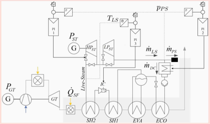


Works very well across case studies

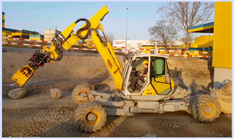
quad coptor fig-8 tracking

quadruped (by Fawcett, Afsari Amers, & Hamed)

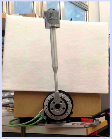
greenhouse automation (by Automatoes)



combined cycle power plant (by P Mahdavipour et. al)

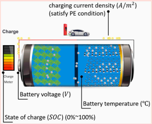


robotic excavator

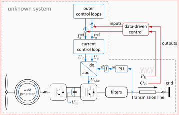


pendulum swing up

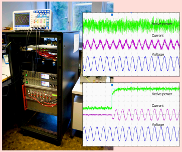
traffic coordination (by J. Wang et al.)



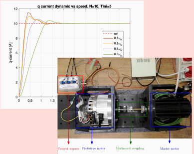
battery charging (by K. Chen et al.)



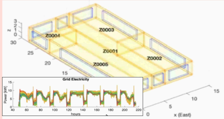
wind turbine control



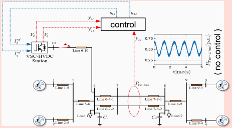
grid-connected converter



synchronous motor drive



energy hub & building automation



power system oscillation damping

Distributional robustification beyond LTI

- ***problem abstraction*** : $\min_{x \in \mathcal{X}} \; c(\widehat{\xi}, x) \;\; = \;\; \min_{x \in \mathcal{X}} \; \mathbb{E}_{\xi \sim \widehat{\mathbb{P}}}\left[c(\xi, x)\right]$

  where $\widehat{\xi}$ denotes *measured* data with *empirical distribution* $\widehat{\mathbb{P}} = \delta_{\widehat{\xi}}$

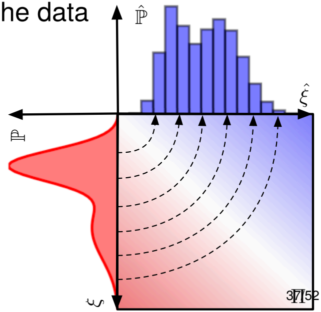
$\Rightarrow$ ***poor out-of-sample performance*** of above sample-average solution $x^\star$
for real problem: $\mathbb{E}_{\xi \sim \mathbb{P}}\left[c(\xi, x^\star)\right]$ where $\mathbb{P}$ is the *unknown distribution* of ξ

- ***distributionally robust*** formulation accounting for all (possibly nonlinear) stochastic processes that could have generated the data

$$\inf_{x \in \mathcal{X}} \; \sup_{\mathbb{Q} \in \mathbb{B}_\epsilon(\widehat{\mathbb{P}})} \; \mathbb{E}_{\xi \sim \mathbb{Q}}\left[c(\xi, x)\right]$$

where $\mathbb{B}_\epsilon(\widehat{\mathbb{P}})$ is an ***ϵ-Wasserstein ball*** centered at empirical sample distribution $\widehat{\mathbb{P}}$:

$$\mathbb{B}_\epsilon(\widehat{\mathbb{P}}) = \left\{ \mathbb{P} \,:\, \inf_{\Pi} \int \left\| \xi - \widehat{\xi} \right\|_p d\Pi \leq \epsilon \right\}$$

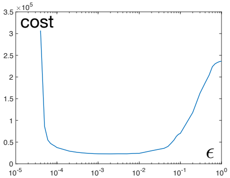


- ***distributionally robustness*** $\equiv$ ***regularization***: under minor conditions

Theorem: $\inf_{x\in\mathcal{X}} \sup_{\mathbb{Q}\in\mathbb{B}_\epsilon(\widehat{\mathbb{P}})} \mathbb{E}_{\xi\sim\mathbb{Q}}\left[c(\xi,x)\right] \equiv \min_{x\in\mathcal{X}} c\left(\widehat{\xi},x\right) + \epsilon\,\text{Lip}(c)\cdot\|x\|_p^\star$

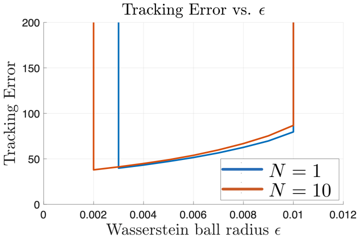
distributional robust formulation — previous regularized DeePC formulation

Cor: ℓ_∞-robustness in trajectory space $\iff$ ℓ_1-regularization of DeePC



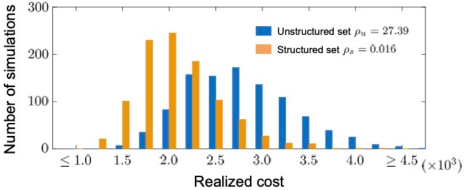
- ***measure concentration***: average matrix $\frac{1}{N}\sum_{i=1}^{N}\mathscr{H}_i(y^{\mathrm{d}})$ from i.i.d. experiments

$\Longrightarrow$ ambiguity set $\mathbb{B}_\epsilon(\widehat{\mathbb{P}})$ includes true $\mathbb{P}$ with high confidence if $\epsilon \sim 1/N^{1/\dim(\xi)}$



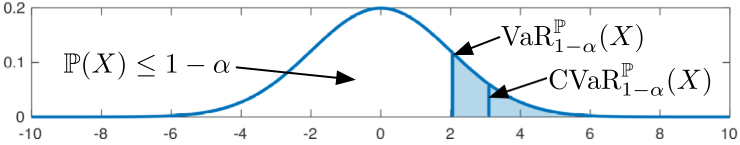
Further ingredients

- more ***structured uncertainty sets***: tractable reformulations (relaxations) & performance guarantees



- ***distributionally robust probabilistic constraints***

$\sup_{\mathbb{Q} \in \mathbb{B}_\epsilon(\widehat{\mathbb{P}})} \text{CVaR}_{1-\alpha}^{\mathbb{Q}} \iff$ averaging + regularization + tightening



- replace (finite) moving horizon estimation via $\binom{u_{\text{ini}}}{y_{\text{ini}}}$ by ***recursive Kalman filtering*** based on optimization solution $g^\star$ as hidden state . . .

how does DeePC relate to sequential SysID + control ?

surprise: **DeePC consistently beats models** across all our real-world case studies !

why ?!?

Comparison: direct vs. indirect control

indirect* ID-based *data-driven control

minimize control cost (u, y)

subject to (u, y) satisfy parametric model

where model $\in$ argmin id cost (u^d, y^d)

subject to model $\in$ LTI(n, ℓ) class } ID

ID projects data on the set of LTI models

- with parameters (n, ℓ)
- removes noise & thus lowers variance error
- suffers bias error if plant is not LTI(n, ℓ)

direct* regularized *data-driven control

minimize control cost (u, y) + $\lambda \cdot$ regularizer

subject to (u, y) consistent with (u^d, y^d) data

- ***regularization robustifies***
  $\rightarrow$ choosing λ makes it work
- ***no projection*** on LTI(n, ℓ)
  $\rightarrow$ no de-noising & no bias

hypothesis: ID wins in stochastic (variance) & DeePC in nonlinear (bias) case

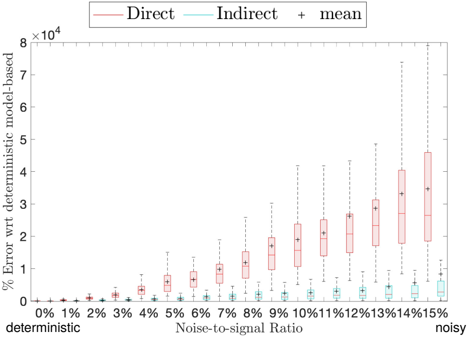
Case study: direct vs. indirect control

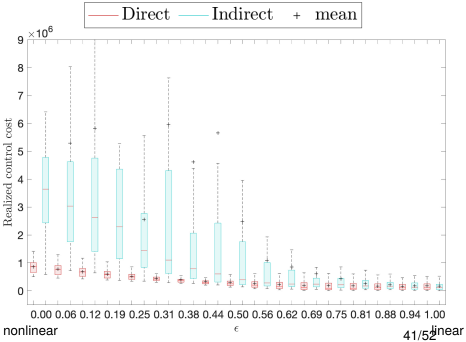
stochastic LTI case $\rightarrow$ indirect ID wins

- LQR control of 5th order LTI system
- Gaussian noise with varying noise to signal ratio (100 rollouts each case)
- ℓ_1-regularized DeePC, SysID via N4SID, & judicious hyper-parameters

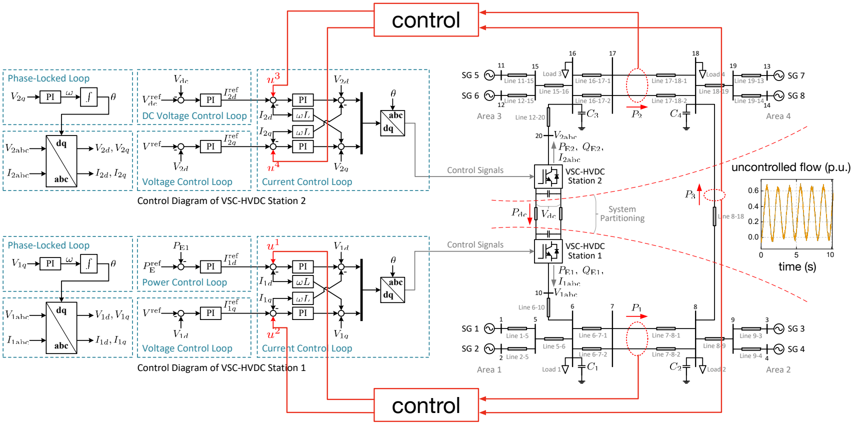
nonlinear case $\rightarrow$ direct DeePC wins

- Lotka-Volterra + control: $x^+ = f(x,u)$
- interpolated system $x^+ = \epsilon \cdot f_{\text{linearized}}(x,u) + (1-\epsilon)\cdot f(x,u)$
- same ID & DeePC as on the left & 100 initial x_0 rollouts for each ϵ



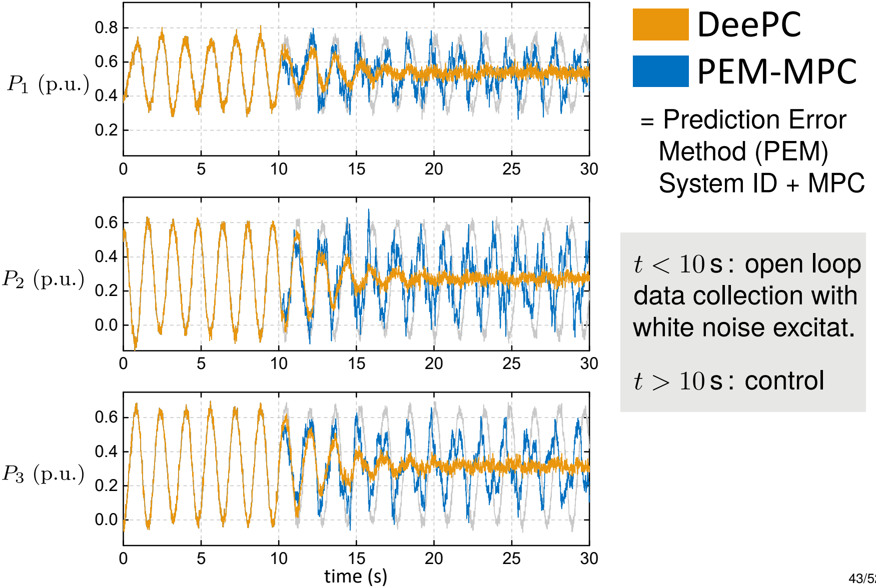


Power system case study revisited



- ***complex*** 4-area power ***system***: large ($n = 208$), few measurements (8), nonlinear, noisy, stiff, input constraints, & decentralized control
- ***control objective:*** damping of inter-area oscillations via HVDC link
- ***real-time*** MPC & DeePC prohibitive → choose T, T_{ini}, & T_{future} wisely

Centralized control

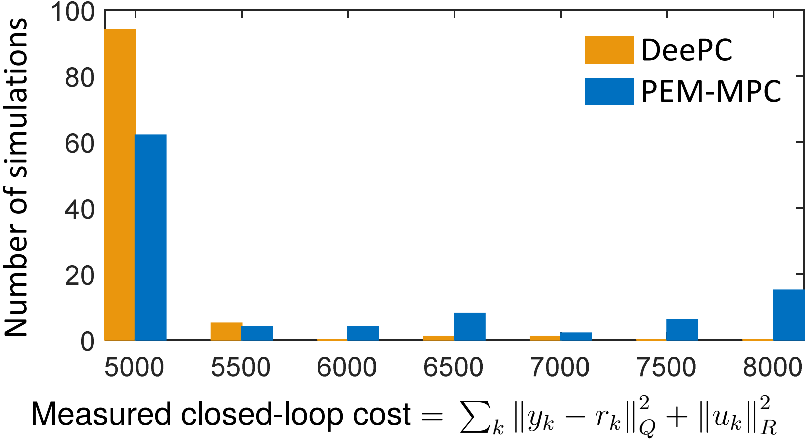


= Prediction Error Method (PEM) System ID + MPC

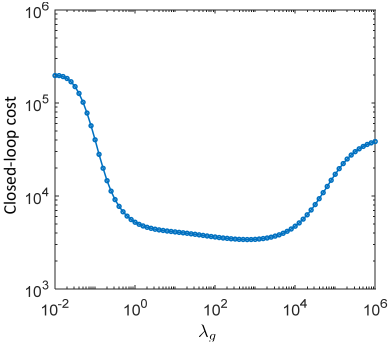
$t < 10\,$s : open loop data collection with white noise excitat.

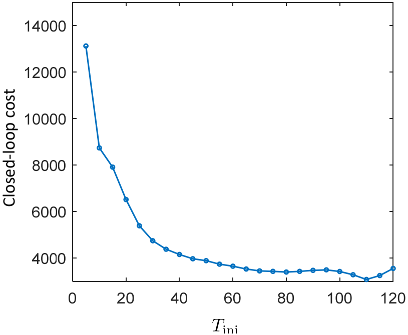
$t > 10\,$s : control

Performance: DeePC wins (clearly!)



DeePC hyper-parameter tuning





regularizer λ_g

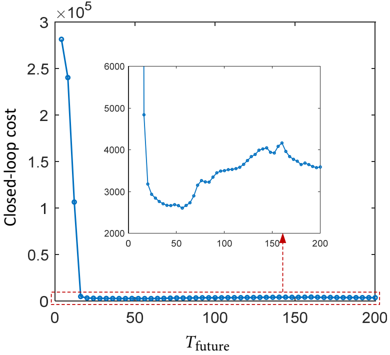
- for distributional robustness $\approx$ radius of Wasserstein ball
- wide range of sweet spots

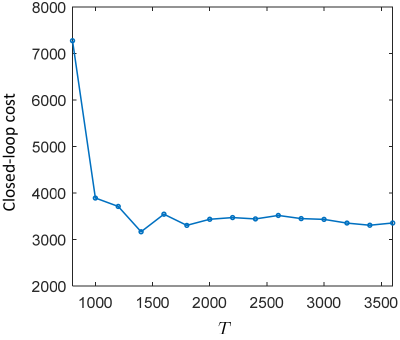
$\rightarrow$ choose $\lambda_g = 20$

estimation horizon T_{ini}

- for model complexity $\approx$ lag
- $T_{\text{ini}} \geq 50$ is sufficient & low computational complexity

$\rightarrow$ choose $T_{\text{ini}} = 60$





prediction horizon T_{future}

- nominal MPC is stable if horizon T_{future} long enough

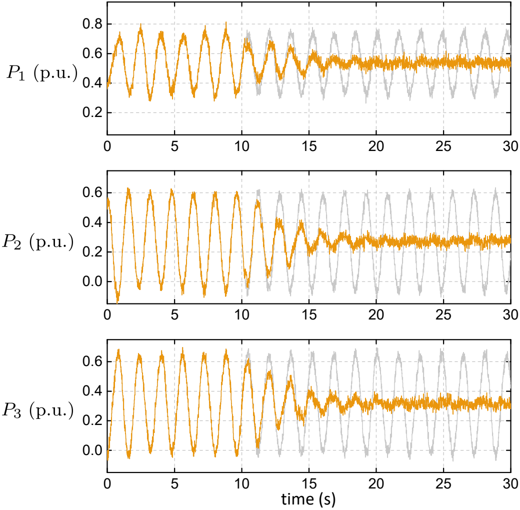
  $\rightarrow$ choose $T_{\text{future}} = 120$ and apply first 60 input steps

data length T

- long enough for low-rank condition but card(g) grows

  $\rightarrow$ choose $T = 1500$
  (data matrix $\approx$ square)

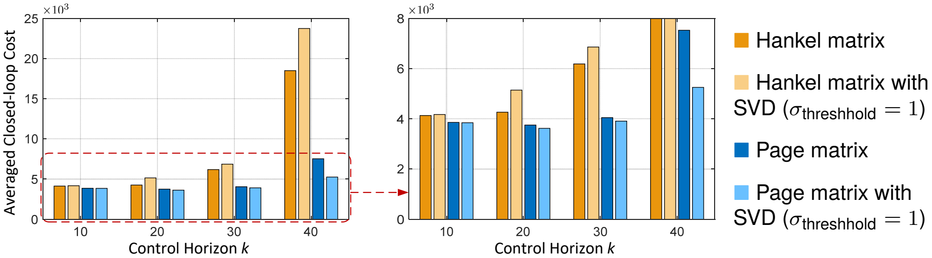
Computational cost



- $T = 1500$
- $\lambda_g = 20$
- $T_{\text{ini}} = 60$
- $T_{\text{future}} = 120$ & apply first 60 input steps
- sampling time = 0.02 s
- solver (OSQP) time = 1 s (on Intel Core i5 7200U)

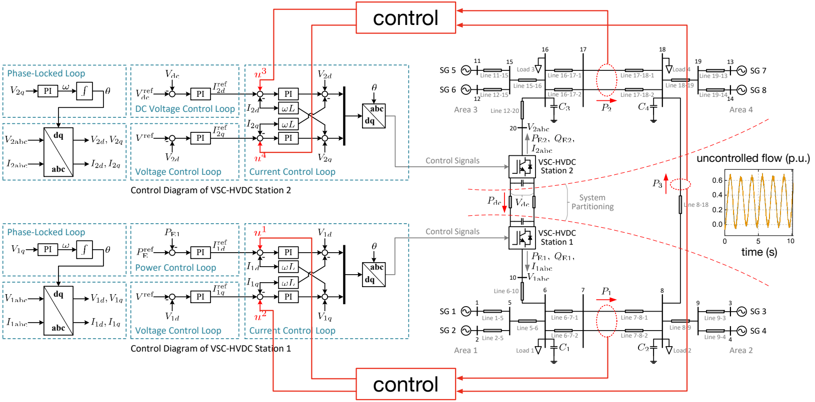
$\Rightarrow$ **implementable**

Comparison: Hankel & Page matrix



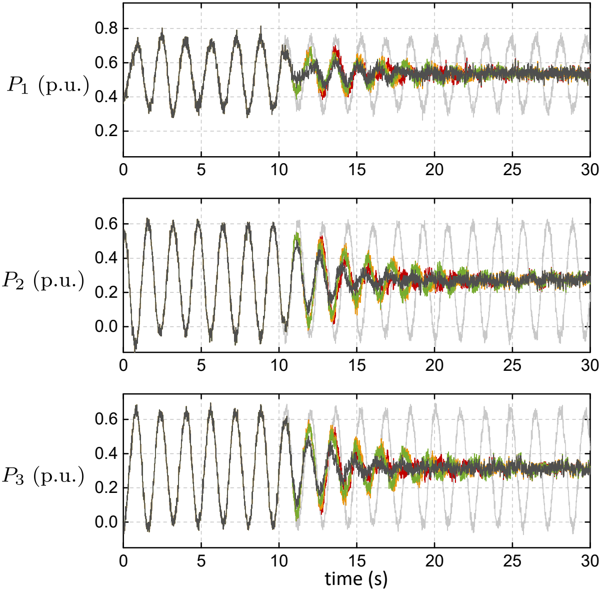
- comparison baseline: Hankel and Page matrices of ***same size***
- ***perfomance***: Page consistency beats Hankel matrix predictors
- offline ***denoising via SVD threshholding*** works wonderfully for Page though obviously not for Hankel (entries are constrained)
- effects very pronounced for ***longer horizon*** (= open-loop time)
- ***price-to-be-paid***: Page matrix predictor requires more data

Decentralized implementation



- ***plug'n'play MPC:*** treat interconnection P_3 as disturbance variable w with past disturbance w_{ini} measurable & future $w_{\text{future}} \in \mathcal{W}$ uncertain
- for each controller ***augment trajectory matrix*** with disturbance data w
- decentralized ***robust min-max DeePC:*** $\min_{g,u,y} \max_{w \in \mathcal{W}}$

Decentralized control performance



- colors correspond to different hyper-parameter settings (not discernible)
- ambiguity set $\mathcal{W}$ is ∞-ball (box)
- for computational efficiency $\mathcal{W}$ is downsampled (piece-wise linear)
- solver time ≈ 2.6 s

$\Rightarrow$ **implementable**

Conclusions

main take-aways

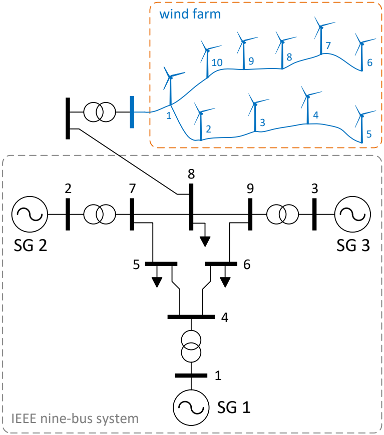
- matrix time series as predictive model
- robustness & side-info by regularization
- method that works in theory & practice

ongoing work

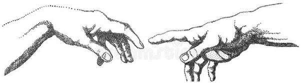
→ certificates for adaptive & nonlinear cases

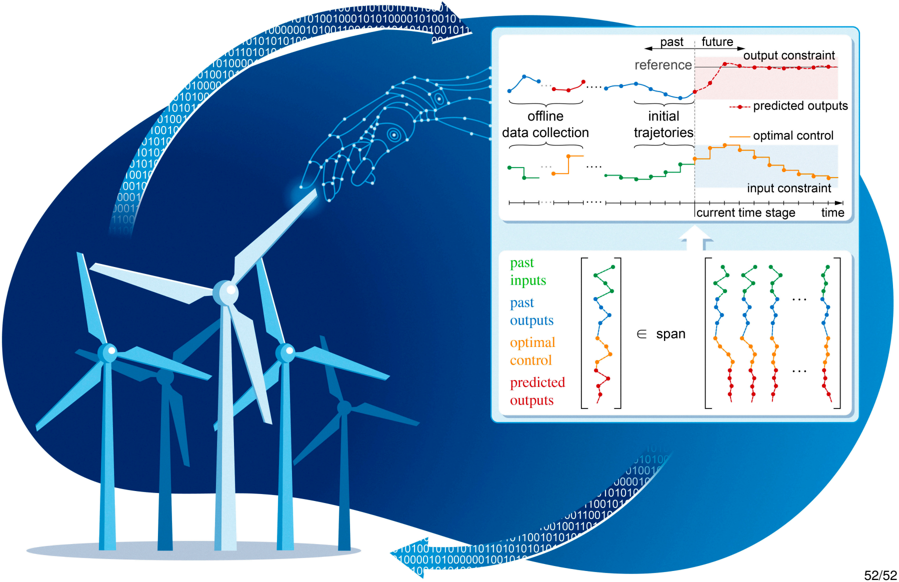
→ propagate optimal transport uncertainties

→ applications with a true “business case”



only catch (no-free-lunch) : optimization problems become large
→ models are compressed, de-noised, & tidied-up representations

Florian's version of 



Thanks !

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