Decentralized and optimal control of inter-area oscillations in power networks

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Electro-mechanical oscillations in power networks

Dramatic consequences: blackout of August 10, 1996, resulted from instability of the 0.25 Hz mode in the Western interconnected system















Outline

Introduction

Slow Coherency Modeling

Conventional Wide-Area Analysis & Control

Variance Amplification as Performance Metric

Sparsity-Promoting Wide-Area Control

Fully Decentralized & Optimal Control

Large-Scale Case Study: NE-NY grid

Conclusions

inter-area oscillations

Dominant electro-mechanical swing dynamics coarse-grained power network = coupled, forced, & heterogeneous pendula

generator swing equations:

$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_i - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

linearized at equilibrium $(\theta^*, \dot{\theta}^*, P^*)$: $M\ddot{\theta} + D\dot{\theta} + L\theta = P$



where M, D are diagonal inertia and damping matrices & L is a Laplacian:

$$L = \begin{bmatrix} \vdots & \ddots & \vdots & \ddots & \vdots \\ -B_{i1}\cos(\theta_i^* - \theta_1^*) & \cdots & \sum_{j=1}^n B_{ij}\cos(\theta_i^* - \theta_j^*) & \cdots & -B_{in}\cos(\theta_i^* - \theta_n^*) \\ \vdots & \ddots & \vdots & \ddots & \vdots \end{bmatrix}$$
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The main controllers to dampen oscillations

Automatic Voltage Regulator (AVR) Power System Stabilizer (PSS)

- objective: voltage = *const*.
- objective: net damping > 0

 \Rightarrow diminishing damping

 \Rightarrow damping of oscillations



HVDC (high voltage DC) & **FACTS** (flexible AC transmission systems): control by modulating lines

Control-induced oscillations

- fact: multi-machine power systems have unstable zeros
 - $\Rightarrow\,$ multiple local controllers interact in an adverse way
 - $\Rightarrow\,$ numerous tuning rules & heuristics for PSS design



slow coherency modeling with D. Romeres & F. Bullo



3 design of remedial actions [Xu et. al. '11] & wide-area control

(later)



Linear transformation & time-scale separation

swing equation $\stackrel{I}{\longleftrightarrow}$ singular perturbation standard form $M\ddot{\theta} + D\dot{\theta} + L\theta = 0 \qquad \overleftarrow{T}_{T^{-1}} \qquad \left\{ \begin{array}{cc} \frac{d}{dt_s} \\ \frac{y}{\sqrt{\delta}} z \\ \sqrt{\delta} z \\ \sqrt{\delta} z \\ \sqrt{\delta} z \\ \frac{z}{\sqrt{\delta}} z \end{array} \right\} = \left[\begin{array}{ccc} \ddots & \vdots & \ddots \\ \cdots & A & \cdots \\ \vdots & \vdots & \ddots \\ \frac{y}{\sqrt{\delta}} z \\ \frac{z}{\sqrt{\delta}} z \end{array} \right]$

slow motion given by center of inertia:

$$y_{\alpha} = \frac{\sum_{i \in \alpha} M_i \theta_i}{\sum_{i \in \alpha} M_i}$$

fast motion given by intra-area differences:

 $z_{i=1}^{\alpha} = \theta_i - \theta_1$

slow time scale: $t_s = \delta \cdot t \cdot$ "max internal area degree"

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Area 2

-Reduced Mode

Area * Area 2

Area 3



Area aggregation & approximation

- singular perturbation standard form:
- aggregated swing equations obtained by $\delta \downarrow 0$:

Properties of aggregated model







Spectral analysis reveals the critical modes & areas
recall solution of x = Ax: x(t) = ∑i vie^{λit} · wi^Tx0 mode #i contribution from x0
analyze eigenvectors & participation factors of weakly damped modes
spectral partitioning reveals coherent groups in eigenvectors polarities







Challenges in wide-area control

- **1** signal selection is combinatorial
- 2 control design is suboptimal
- **identification** of critical modes is somewhat *ad hoc*

what information do you want to extract from the spectrum of a non-normal matrix?

Example:
$$\dot{x} = \begin{bmatrix} -1 & 10^2 \\ 0 & -1 \end{bmatrix} x$$



Today:

- \Rightarrow performance metric: variance amplification of stochastic system
- \Rightarrow simultaneously optimize performance & control architecture
- \Rightarrow fully decentralized & nearly optimal controller

Input-output analysis in \mathcal{H}_2 - metric

- ... complementing/improving modal analysis
- ▶same metric used later for control synthesis
- linear system with white noise input: $\dot{x} = Ax + B_1 \eta$
- energy of homogeneous network as **performance output**: $z = Q^{1/2}x$
- power spectral density quantified by Hilbert-Schmidt norm

$$\|G(\mathbf{j}\,\omega)\|_{\mathrm{HS}}^2 \,=\, \mathrm{trace}\,(G(\mathbf{j}\,\omega)\cdot G^*(\mathbf{j}\,\omega)) \,=\, \sum\nolimits_i \sigma_i^2(G(\mathbf{j}\,\omega))$$

• steady-state variance of the output quantified by \mathcal{H}_2 -norm

$$\|G\|_{\mathcal{H}_2}^2 := \lim_{t \to \infty} \mathbb{E}\left(x(t)^T Q x(t)\right) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \|G(\mathbf{j}\omega)\|_{\mathrm{HS}}^2 d\omega$$

variance amplification as performance metric

with X. Wu & M. Jovanović

Slow coherency performance objectives

• recall sources for inter-area oscillations:



• linearized swing equation: $M\ddot{\theta} + D\dot{\theta} + L\theta = P$

• mechanical energy: $\frac{1}{2}\dot{\theta}M\dot{\theta} + \frac{1}{2}\theta^{T}L\theta$

heterogeneities in topology, power transfers,
 & machine responses (inertia & damp)

 \Rightarrow performance **objectives** = energy of homogeneous network:

 $x^{T}Qx = \dot{\theta}^{T}M\dot{\theta} + \theta^{T}(I_{n} - (1/n) \cdot \mathbb{1}_{n \times n})\theta$

• other choices possible: center of inertia, inter-area differences, etc.

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Power spectral density ... reveals inter-area modes & local mode # 4





Variance amplification

via diagonal elements of output covariance matrix ... reveal #1 & #9 as crucial



sparsity-promoting optimal control

by F. Lin, M. Fardad, & M. Jovanović

Optimal linear quadratic regulator (LQR)

• model: linearized ODE dynamics $\dot{x}(t) = Ax(t) + B_1\eta(t) + B_2u(t)$

- control: memoryless linear state feedback u = -Kx(t)
- optimal centralized control with quadratic \mathcal{H}_2 performance index:

minimize $J(K) \triangleq \lim_{t \to \infty} \mathbb{E}\left\{x(t)^T Q x(t) + u(t)^T R u(t)\right\}$ subject to linear dynamics: $\dot{x}(t) = Ax(t) + B_1\eta(t) + B_2u(t)$, linear control: u(t) = -Kx(t), stability: $(A - B_2 K)$ Hurwitz.

(no structural constraints on K)



Sparsity-promoting optimal LQR simultaneously optimize performance & architecture

[Lin, Fardad, & Jovanović '13]

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minimize $\lim_{t \to \infty} \mathbb{E} \left\{ x(t)^T Q x(t) + u(t)^T R u(t) \right\} + \gamma \operatorname{card}(K)$ subject to linear dynamics: $\dot{x}(t) = Ax(t) + B_1\eta(t) + B_2u(t)$, linear control: u(t) = -Kx(t), stability: $(A - B_2 K)$ Hurwitz.

 \Rightarrow for $\gamma = 0$: standard optimal control (typically not sparse) \Rightarrow for $\gamma > 0$: sparsity is promoted (problem is combinatorial) \Rightarrow card(K) convexified by weighted ℓ_1 -norm $\sum_{i,j} w_{ij} |K_{ij}|$

Parameterized family of feedback gains $K(\gamma) = \arg\min_{\nu} \left(J(K) + \gamma \cdot \sum_{i,i} w_{ij} |K_{ij}| \right)$ $I\left(K(\gamma)\right)$ CENTRALIZED LOCALTZED FULLY DECENTRALIZED

Algorithmic approach to sparsity-promoting control

• equivalent formulation via **observability Gramian** *P*:

minimize $J_{\gamma}(K) \triangleq \operatorname{trace} (B_1^T P B_1) + \gamma \cdot \sum_{i,j} w_{ij} |K_{ij}|$ subject to $(A - B_2 K)^T P + P(A - B_2 K)$ $= -(Q + K^T R K)$

- **2** warm-start at optimal centralized \mathcal{H}_2 controller with $\gamma = 0$
- **③ homotopy path:** continuously increase γ until the desired value $\gamma_{\rm des}$
- **3 ADMM:** iterative solution for each value of $\gamma \in [0, \gamma_{des}]$
- **6** update weights: update w_{ij} in each ADMM step: $w_{ij} \mapsto \frac{1}{|K_{ii}| + \varepsilon}$
- **o polishing:** structured optimization with desired sparsity pattern

Some ADMM details

- minimize $f(K) + \gamma \cdot g(K) = \mathcal{H}_2$ performance + $\gamma \cdot$ sparsity
- additional variable/constraint decoupling smooth & separable objectives:

minimize
$$f(K) + \gamma \cdot g(L)$$

subject to $K - L = 0$

Introduce augmented Lagrangian

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$$\mathcal{L}_{\rho}(K,L,\Lambda) = f(K) + \gamma \cdot g(L) + \operatorname{trace}(\Lambda(K-L)) + \frac{\rho}{2} \|K - L\|_{F}^{2}$$

- alternating direction method of multipliers (ADMM):

 K⁺ ≜ argmin_K L_ρ(*K*, L, Λ) (iteratively via smooth method)
 L⁺ ≜ argmin_L L_ρ(*K*⁺, L, Λ) (analytically via soft-thresholding)
 Λ⁺ ≜ Λ + ρ · (*K*⁺ − L⁺)

 guarantees: stabilizing gains (always) & convergent (if locally convex)
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- Regularization of rotational symmetry
 - rotational symmetry of power flow (absence of reference angle)

$$M_i\ddot{ heta}_i + D_i\dot{ heta}_i = P_i - \sum_j B_{ij}\sin(heta_i - heta_j)$$

- \Rightarrow [θ r] = [$\mathbb{1}_n$ O] is eigenvector of linearized power system models
- \Rightarrow eigenvector is not detectable: $[\mathbb{1}_n \mathbb{O}]^T Q [\mathbb{1}_n \mathbb{O}] = 0$
- \Rightarrow no numeric LQR solution with standard Ricatti solvers
- regularization: $x^T Q x = \dot{\theta}^T M \dot{\theta} + \theta^T ((1+\varepsilon)I_n (1/n) \cdot \mathbb{1}_{n \times n}) \theta$
- \Rightarrow resulting feedback requires absolute angle: $\mathcal{K}_{\varepsilon}[\mathbb{1}_n \mathbb{O}] = \varepsilon \cdot [\star \mathbb{O}]$

sparsity-promoting control of inter-area oscillations

with M. Jovanović, M. Chertkov, & F. Bullo







too much plain vanilla need a closer look at **rotational symmetry**

fully decentralized & optimal control

with X. Wu & M. Jovanović

Taking the rotational symmetry into account

• structural constraint: there is no absolute angle

open-loop:
$$A\begin{bmatrix} 1\\ 0\end{bmatrix} = \begin{bmatrix} 0\\ 0\end{bmatrix} \implies$$
 closed-loop: $(A - B_2 K)\begin{bmatrix} 1\\ 0\end{bmatrix} = \begin{bmatrix} 0\\ 0\end{bmatrix}$

 \Rightarrow elimination of the average mode 1

$$x = \begin{bmatrix} \theta \\ r \end{bmatrix} = \underbrace{\begin{bmatrix} U & 0 \\ 0 & I \end{bmatrix}}_{T} \xi + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \overline{\theta}$$

where U is orthonormal with columns \perp span (1)

• embedding in ADMM to promote sparsity in original coordinates

minimize
$$f(K) + \gamma \cdot g(L)$$
 subject to $KT^{T} - L = 0$

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Summary & conclusions

- ${\small \bigcirc}$ analysis of inter-area dynamics via slow coherency theory
- ② sparsity-promoting distributed optimal wide-area control
- $\Rightarrow\,$ trade-off: sparse control architecture vs. performance
- 0 extensions to rotational symmetry & block sparsity
- $\Rightarrow\,$ yields fully decentralized & nearly optimal controllers
- illustrations with New England & New York power grid models

Code available online

- sparsity-promoting wide-area control: http://www.ece.umn.edu/users/mihailo/software/lqrsp/wac.html
- extensions to rotational symmetry & block sparsity:
 www.umn.edu/~mihailo/software/lqrsp/matlab-files/lqrsp_wac.zip
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