Control of Low-Inertia Power Systems: Naive & Foundational Approaches

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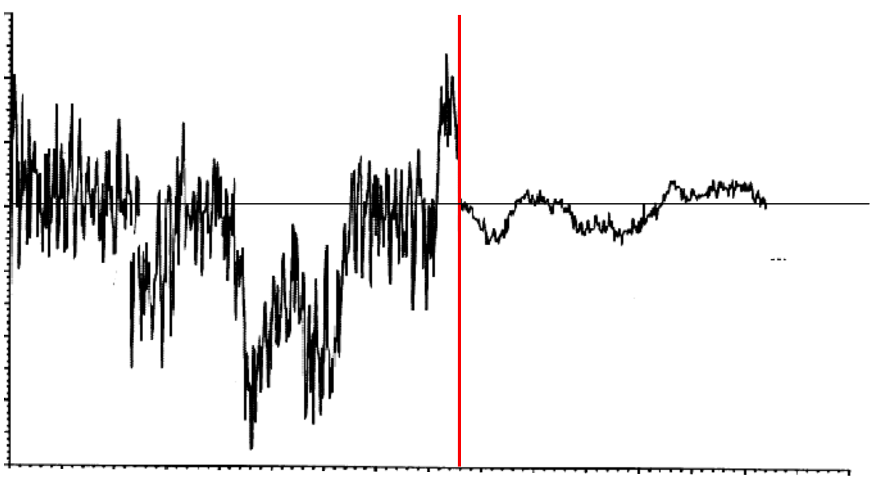
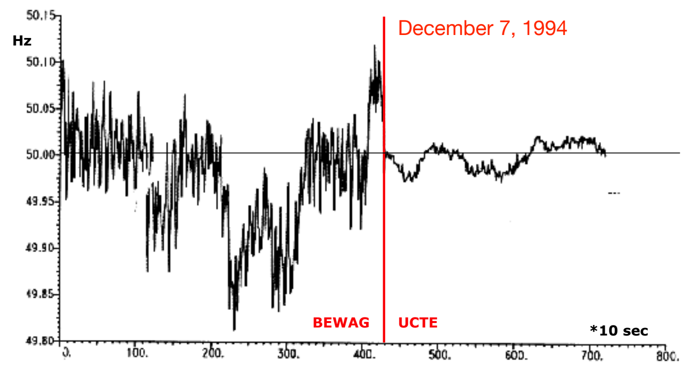
What do we see here?



Frequency of West Berlin when re-connecting to Europe

Source: *Energie-Museum Berlin*



before re-connection: islanded operation based on batteries & single boiler

afterwards connected to European grid based on synchronous generation

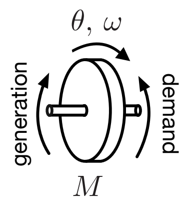
Essentially, the pre/post West Berlin curves date back to...

Fact: all of AC power systems built around **synchronous machines**!

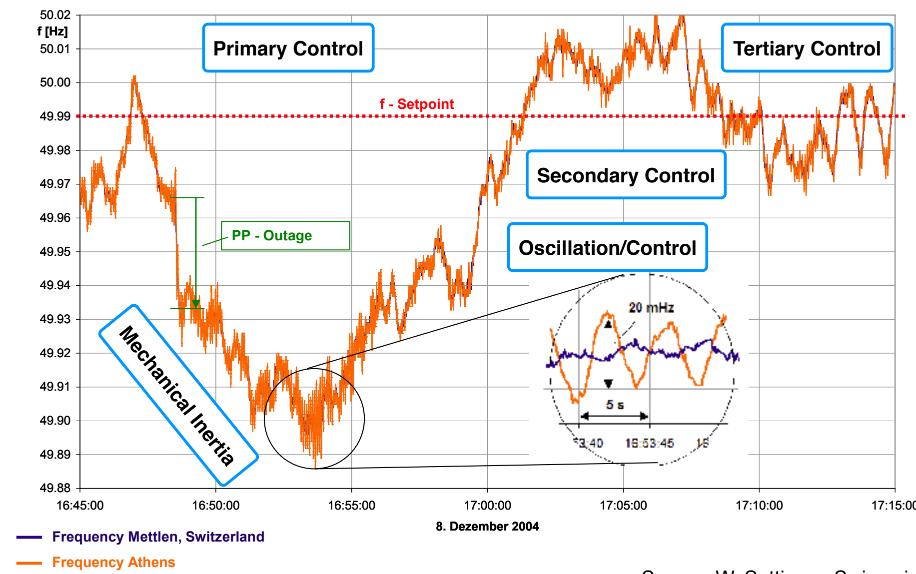
At the heart of it is the generator **swing equation**:

$$M \frac{d}{dt}\omega(t) = P_{\text{generation}}(t) - P_{\text{demand}}(t)$$

change of kinetic energy = instantaneous power balance



Operation centered around bulk synchronous generation

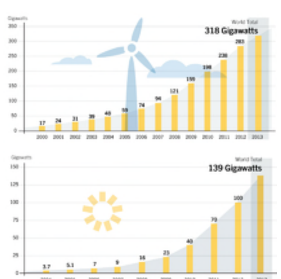


Source: W. Sattinger, Swissgrid

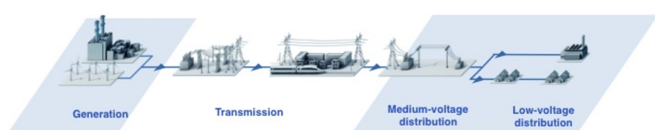
Renewable/distributed/non-rotational generation on the rise

synchronous generator | new workhorse | scaling

new primary sources



location & distributed implementation



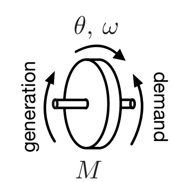
Almost all operational problems can principally be resolved ... **but one (?)**

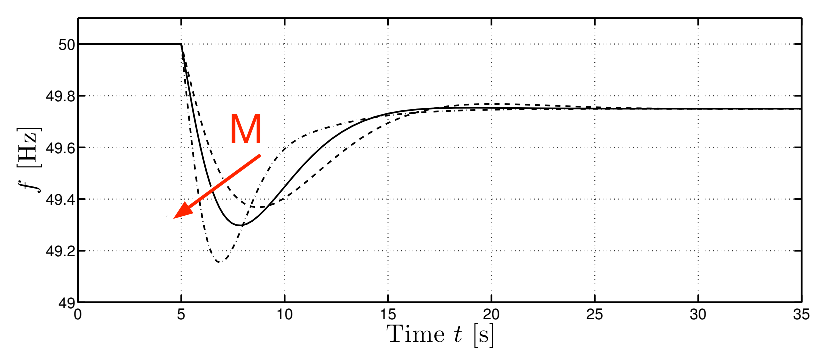
Fundamental challenge: operation of low-inertia systems

We slowly loose our giant electromechanical low-pass filter:

$$\mathbf{M} \frac{d}{dt}\omega(t) = P_{\text{generation}}(t) - P_{\text{demand}}(t)$$

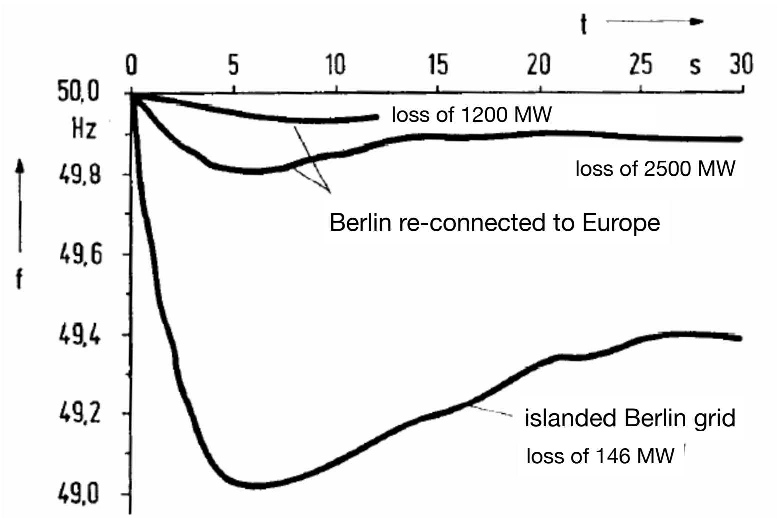
change of kinetic energy = instantaneous power balance



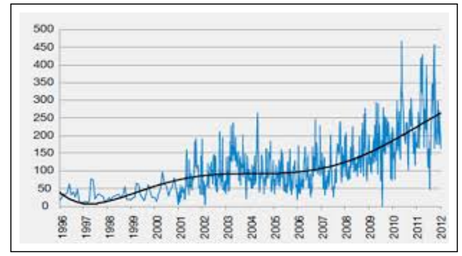


Berlin curves before and after re-connecting to Europe

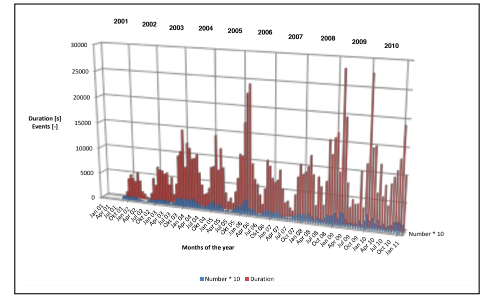
Source: *Energie-Museum Berlin*



Low-inertia stability: # 1 problem of distributed generation



frequency violations in Nordic grid (source: ENTSO-E)



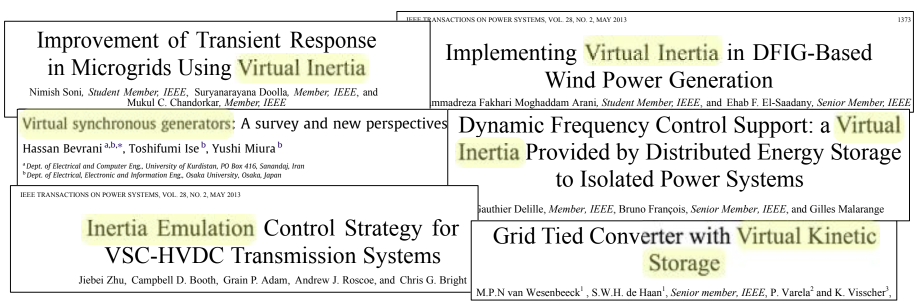
same in Switzerland (source: Swissgrid)

inertia is shrinking, time-varying, localized, ... & increasing disturbances

Solutions in sight: none really ... other than **emulating virtual inertia** through fly-wheels, batteries, super caps, HVDC, demand-response, ...

Virtual inertia emulation

devices commercially available, required by grid-codes, or incentivized through markets



Improvement of Transient Response in Microgrids Using Virtual Inertia
Nimish Soni, *Student Member, IEEE*, Suryanarayana Doolla, *Member, IEEE*, and Mukul C. Chandorkar, *Member, IEEE*

IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 28, NO. 2, MAY 2013
Implementing Virtual Inertia in DFIG-Based Wind Power Generation
...mmadreza Fakhari Moghaddam Arani, *Student Member, IEEE*, and Ehab F. El-Saadany, *Senior Member, IEEE*

Virtual synchronous generators: A survey and new perspectives
Hassan Bevrani, Toshifumi Ise, Yushi Miura

Dynamic Frequency Control Support: a Virtual Inertia Provided by Distributed Energy Storage to Isolated Power Systems
...authier Delille, *Member, IEEE*, Bruno François, *Senior Member, IEEE*, and Gilles Malarange

IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 28, NO. 2, MAY 2013
Inertia Emulation Control Strategy for VSC-HVDC Transmission Systems
Jiebei Zhu, Campbell D. Booth, Grain P. Adam, Andrew J. Roscoe, and Chris G. Bright

Grid Tied Converter with Virtual Kinetic Storage
M.P.N van Wesenbeeck, S.W.H. de Haan, *Senior member, IEEE*, P. Varela and K. Visscher

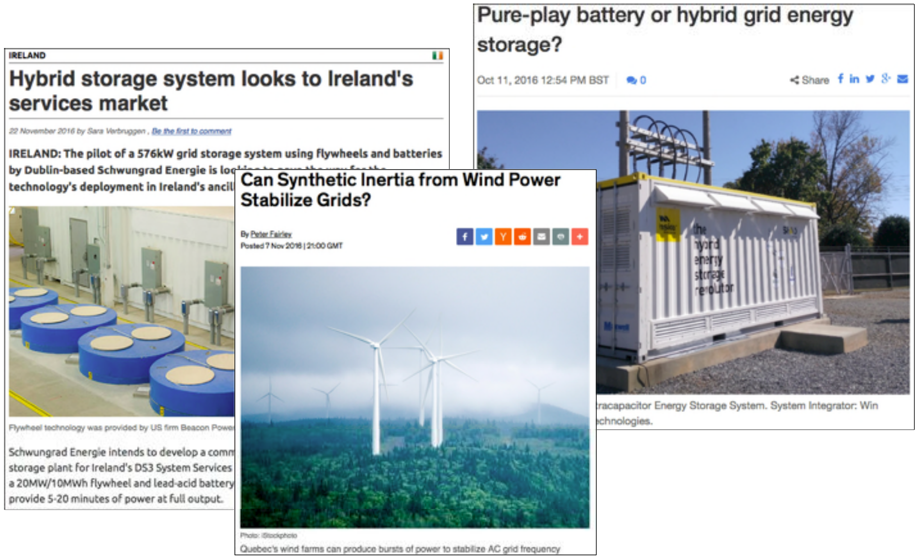
$$M \frac{d}{dt}\omega(t) = P_{\text{generation}}(t) - P_{\text{demand}}(t) \approx \textbf{derivative control on } \omega(t)$$

⇒ **focus today: where** to do it? **how** to do it properly? **what else**?

Outline

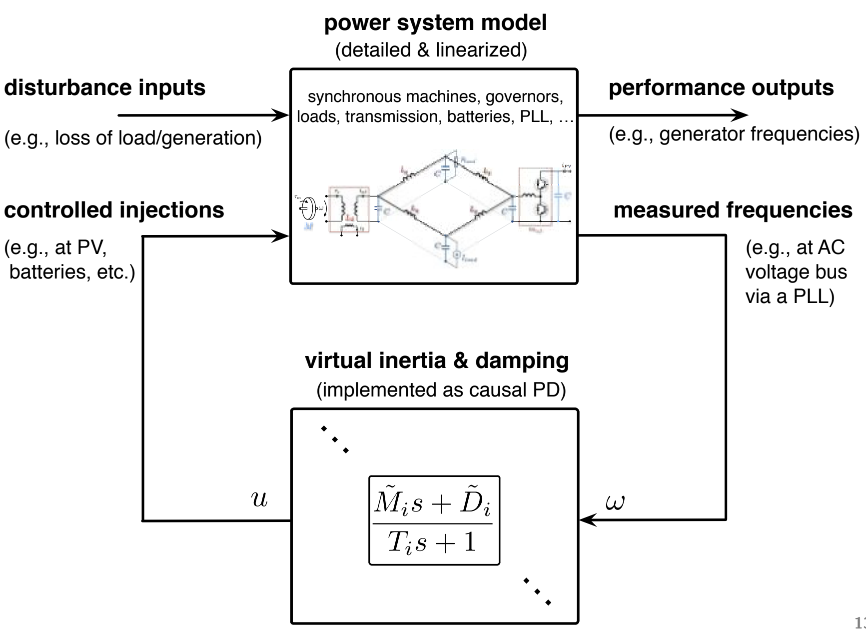
Virtual inertia is becoming a technology and a product

so let's see how we can make use of it

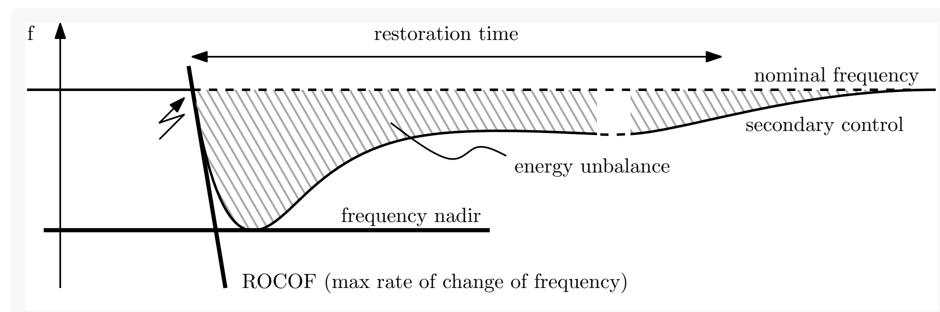


optimal placement of virtual inertia

General power system & inertia emulation model



Performance metrics for low-inertia systems

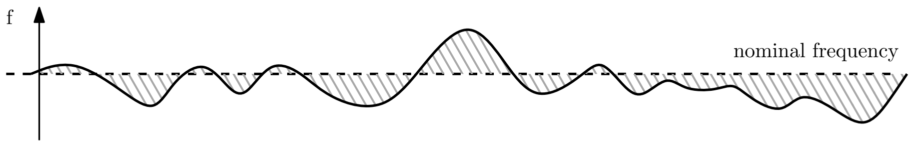


System norm quantifying signal amplifications

disturbances: impulse (fault), step (loss of unit), white noise (renewables) → system → **performance outputs:** integral, peak, ROCOF, restoration time, . . .

Integral-quadratic coherency performance metric

$$\int_0^\infty x(t)^T Q\, x(t)\, dt$$

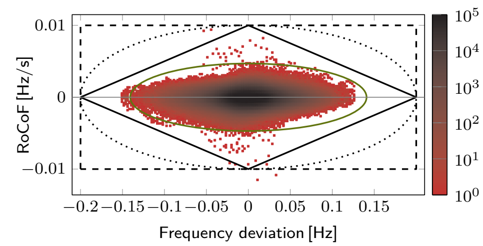


$\mathcal{H}_2$ **system norm** interpretation: $\eta \longrightarrow$ system $\longrightarrow y$

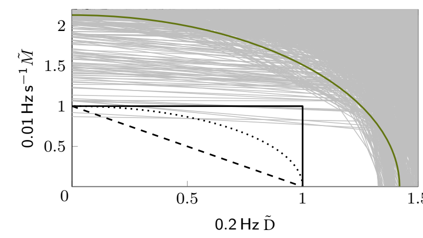
1. **performance output**: $\mathbf{y} = Q^{1/2} x$
2. **impulsive** η (faults) $\longrightarrow$ output energy $\int_0^\infty \mathbf{y}(t)^\mathsf{T}\, \mathbf{y}(t)\, dt$
3. **white noise** η (renewables) $\longrightarrow$ output variance $\lim_{t\to\infty} \mathbb{E}\left(\mathbf{y}(t)^\mathsf{T}\, \mathbf{y}(t)\right)$

Constraints on control inputs

1. **energy constraint**: $\int_0^\infty u^T R\, u\, dt$ directly captured in $\mathcal{H}_2$ framework
2. **power constraint**: $u_i = \tilde{M}_i\, \dot{\omega}_i + \tilde{D}_i\, \omega_i$ bounded: $\|u_i(t)\|_{\ell_\infty} \le \overline{u_i}$



European frequency data (source: RTE)



corresponding bounds on gains

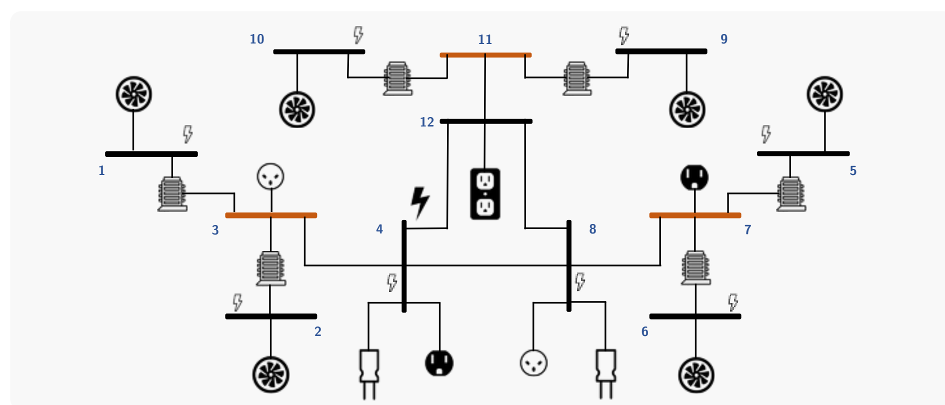
$\Rightarrow$ $\|(\omega_i(t), \dot{\omega}_i(t))\|_p$, $\|(\tilde{D}_i, \tilde{M}_i)\|_q$ bounded ($\frac{1}{p}+\frac{1}{q}=1$) $\Rightarrow$ $\|u_i(t)\|_{\ell_\infty}$ bounded

3. **budget constraint** for finitely many devices: $\sum_i \overline{u_i} = const.$

(sub)optimize performance and see what we learn

Modified Kundur case study: 3 areas & 12 buses

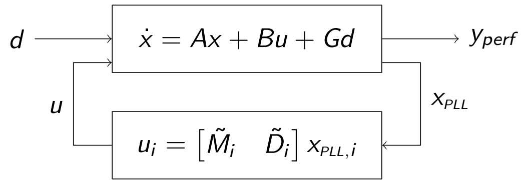
added governors (droop) at generators & PLLs to obtain frequency for inertia emulation



Test case

- **inertia emulation** control via PLL & batteries:

$$u_i = \begin{bmatrix}\tilde{M}_i & \tilde{D}_i\end{bmatrix} x_{PLL,i}$$



- **dynamics:** swing equation, droop via governor & turbine, and PLL

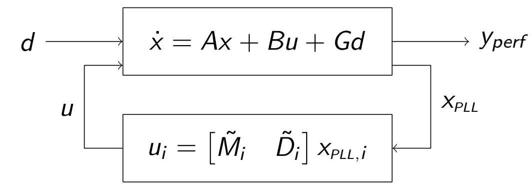
$$\begin{bmatrix}\dot{\delta}\\ \dot{\omega}\\ \dot{x}_{gov}\\ \dot{x}_{PLL}\end{bmatrix} = \underbrace{\begin{bmatrix}A_{sw} & B_{sw}K_{gov} & \mathbb{0}\\ B_{gov} & A_{gov} & \mathbb{0}\\ B_{PLL} & \mathbb{0} & A_{PLL}\end{bmatrix}}_{=A} x + \underbrace{\begin{bmatrix}B_{sw}\\ \mathbb{0}\\ \mathbb{0}\end{bmatrix}}_{=B} u + \underbrace{\begin{bmatrix}B_{sw}\\ \mathbb{0}\\ \mathbb{0}\end{bmatrix}}_{=G} d$$

- **cost** penalizes frequencies, droop control, & inertia emulation effort:

$$\underbrace{\begin{bmatrix}\omega\\ u_{gov}\\ u\end{bmatrix}}_{y_{perf}} = \underbrace{\begin{bmatrix}\mathbb{0} & I & \mathbb{0} & \mathbb{0}\\ \mathbb{0} & \mathbb{0} & K_{gov} & \mathbb{0}\\ \mathbb{0} & \mathbb{0} & \mathbb{0} & \mathbb{0}\end{bmatrix}}_{=Q^{1/2}} x + \underbrace{\begin{bmatrix}\mathbb{0}\\ \mathbb{0}\\ I\end{bmatrix}}_{=R^{1/2}} u$$

Algorithmic approach to desperate & non-convex problem

- **structured** state-feedback with **constraints** on gains
- **computation** $\mathcal{H}_2$ norm, gradient, & projections:



1. observability and controllability Gramians via **Lyapunov equations**

$$(A - BK)^\top P + P(A - BK) + Q + K^\top RK = 0$$
$$(A - BK)L + L(A - BK)^\top + GG^\top = 0$$

2. $\mathcal{H}_2$ **norm** $J = \mathrm{Trace}(G^\top PG)$ and **gradient** $\nabla_K J = 2(RK - B^\top P)L$
3. **projection** on structural constraint: $\nabla_{\tilde{M},\tilde{D}} J = \Pi_{\tilde{M},\tilde{D}}[\nabla_K J]$

$\Rightarrow$ $\tilde{M}$ and $\tilde{D}$ can be optimized by **first-order methods**, IPM, SQP, etc.

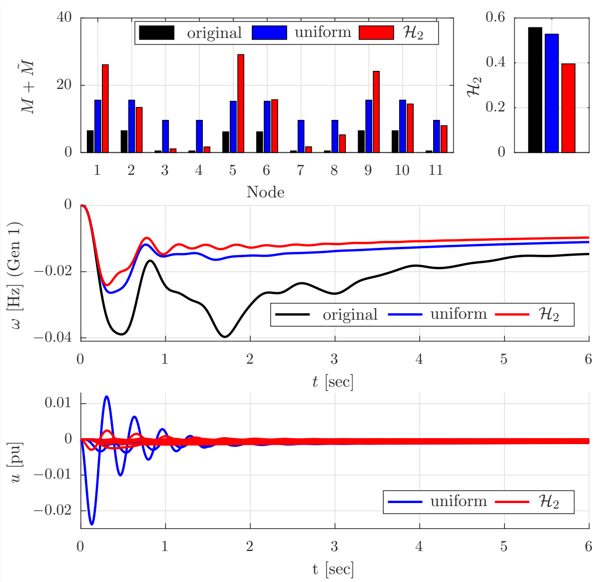
Results & insights

Optimal allocation:

- location of inertia & damping matters
- outperforms heuristic uniform allocation
- need penalty on droop control effort
- power constraint results in $\tilde{D} \approx 2\tilde{M}$

Fault at bus #4:

- strong reduction of frequency deviation
- much less control effort than heuristic



placement & metrics matter!

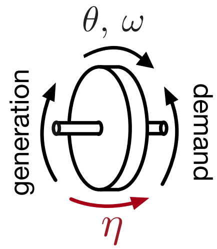
can we get analytic insights ?

Inertia placement in swing equations

- simplified network swing equation **model**:

$$m_i \ddot{\theta}_i + d_i \dot{\theta}_i = p_{gen,i} - p_{dem,i}$$
generator swing equations

$$p_{dem,i} \approx \sum_j b_{ij} (\theta_i - \theta_j)$$
linearized DC power flow



- likelihood of **disturbance** at $\#i$: $t_i \geq 0$ (available from TSO data)
- $\mathcal{H}_2$ performance **metric**: $\int_0^\infty \sum_{i,j} a_{ij}(\theta_i - \theta_j)^2 + \sum_i s_i \dot{\theta}_i^2 \, dt$
- **decision variable** is inertia: $m_i \in [\underline{m_i}, \overline{m_i}]$
  (additional nonlinearity: enters as m_i^{-1} in constraints & objective)

Closed-form results for cost of primary control

recall: primary control $d_i \dot{\theta}_i$ **effort** was crucial

$$\int_0^\infty \dot{\theta}(t)^\top D \, \dot{\theta}(t) \, dt$$

(computations show that insights *roughly* generalize to other costs)

allocation: the primary control effort $\mathcal{H}_2$ optimization reads equivalently as

$$\text{minimize}_{m_i} \quad \sum_i \frac{t_i}{m_i}$$
$$\text{subject to} \quad \sum_i m_i \leq m_{\text{bdg}}$$
$$\underline{m_i} \leq m_i \leq \overline{m_i}$$

key take-away is **disturbance matching**:

- optimal allocation $m_i^\star \propto \sqrt{t_i}$ or $m_i^\star = \min\{m_{\text{bdg}}, \overline{m_i}\}$

⇒ disturbance profile known from historic data, but rare events are crucial

- suggests **robust** $\min_m \max_t$ **allocation** to prepare for worst case

⇒ valley-filling solution: $t_i^\star / m_i^\star = const.$ (up to constraints)

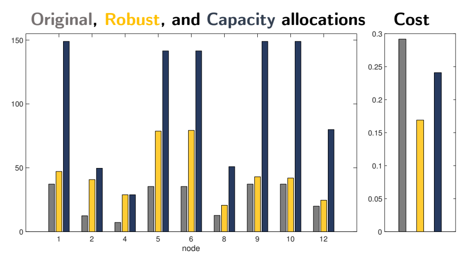
Robust min - max allocation for three-area case study

Scenario: fault (impulse) can occur at any single node

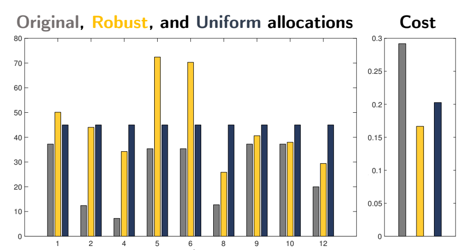
- disturbance set
  $T \in \mathbb{T} = \{\mathbb{e}_1 \cup \cdots \cup \mathbb{e}_{12}\}$

⇒ **min/max** over convex hull

- inertia **capacity constraints**
- robust inertia allocation **outperforms heuristic** max-capacity allocation
- results become **intuitive**: valley-filling property
- same for uniform allocation



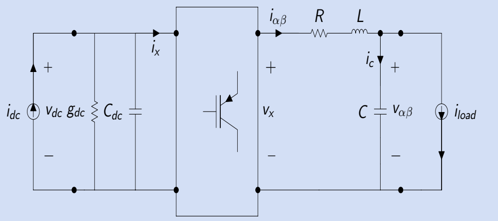
allocation subject to capacity constraints



allocation subject to the budget constraint

Outline

Averaged power converter model



DC cap & AC filter equations:

$$C_{dc}\dot{v}_{dc} = -G_{dc}v_{dc} + i_{dc} - \frac{1}{2}m^\top i_{\alpha\beta}$$
$$L\dot{i}_{\alpha\beta} = -Ri_{\alpha\beta} + \frac{1}{2}mv_{dc} - v_{\alpha\beta}$$
$$C\dot{v}_{\alpha\beta} = -i_{load} + i_{\alpha\beta}$$

modulation: $v_x = \frac{1}{2}mv_{dc}$, $i_x = \frac{1}{2}m^\top i_{\alpha\beta}$

passive: $(i_{dc}, i_{load}) \rightarrow (v_{dc}, v_{\alpha\beta})$

synchronous generator: mechanical + stator flux + AC cap

$$\dot{\theta} = \omega$$
$$M\dot{\omega} = -D\omega + \tau_m + i_{\alpha\beta}^\top L_m i_f \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \end{bmatrix}$$
$$L_s\dot{i}_{\alpha\beta} = -Ri_{\alpha\beta} - v_{\alpha\beta} - \omega L_m i_f \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \end{bmatrix}$$
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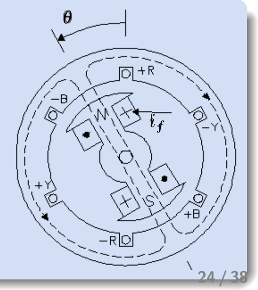
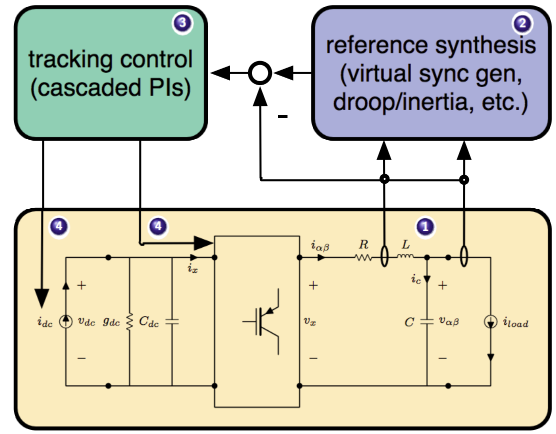


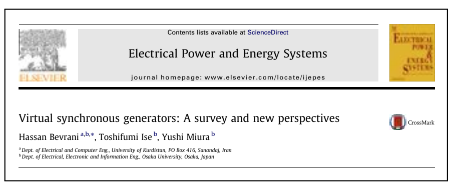
Standard power electronics control would continue by



1. acquiring & processing of **AC measurements**
2. synthesis of **references** (voltage/current/power)
3. **track** error signals at converter terminals
4. **actuation** via modulation (inner loop) and/or via DC source (outer loop)

I guess you can see the **problems building up** . . .

Challenges in power converter implementations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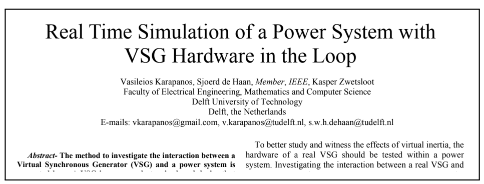
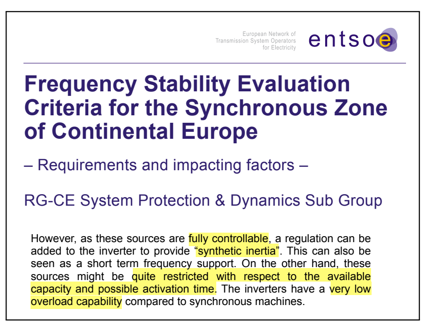
Virtual synchronous generators: A survey and new perspectives
Hassan Bevrani, Toshifumi Ise, Yushi Miura

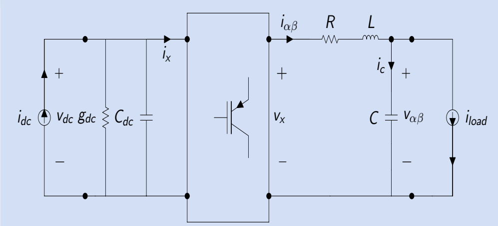

Real Time Simulation of a Power System with VSG Hardware in the Loop
Vasileios Karapanos, Sjoerd de Haan, Member, IEEE, Kasper Zwetsloot
Faculty of Electrical Engineering, Mathematics and Computer Science
Delft University of Technology
Delft, the Netherlands

1. **delays** in measurement acquisition, signal processing, & actuation
2. **accuracy** in AC measurements (averaging over multiple cycles)
3. **constraints** on currents, voltages, power, etc.
4. **certificates** on stability, robustness, & performance


entsoe
Frequency Stability Evaluation Criteria for the Synchronous Zone of Continental Europe
– Requirements and impacting factors –
RG-CE System Protection & Dynamics Sub Group
However, as these sources are fully controllable, a regulation can be added to the inverter to provide "synthetic inertia". This can also be seen as a short term frequency support. On the other hand, these sources might be quite restricted with respect to the available capacity and possible activation time. The inverters have a very low overload capability compared to synchronous machines.

let's do **something smarter** . . .

See the similarities & the differences ?



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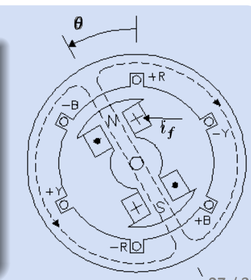
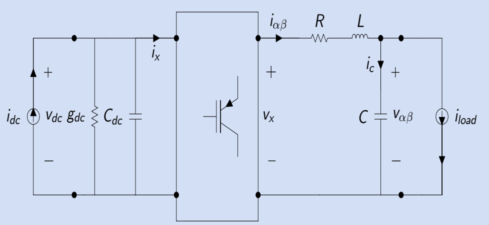


Model matching ($\neq$ emulation) as inner control loop



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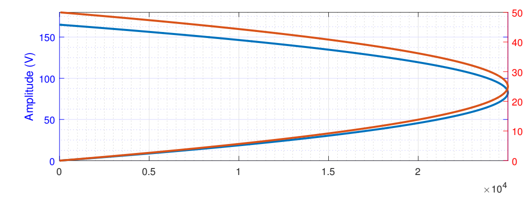
matching control: $\dot{\theta} = \eta \cdot v_{dc}$, $m = \hat{m} \cdot \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \end{bmatrix}$ with $\eta, \hat{m} > 0$

$\Rightarrow$ **matched machine** with inertia $M = \frac{C_{dc}}{\eta^2}$, droop/dissipation $D = \frac{G_{dc}}{\eta^2}$, torque $\tau_m = \frac{i_{dc}}{\eta}$, field current $i_f = \frac{\hat{m}}{\eta L_m}$, & imbalance signal $\omega = \eta \cdot v_{dc}$

$\Rightarrow$ **pros:** uses physical storage, uses DC measurements, & remains passive

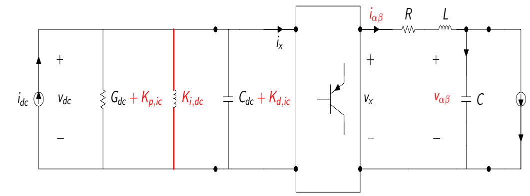
Further properties of machine matching control

1. stationary P vs. $(|V|, \omega)$ **nose curves** reveal $(P, \omega, |V|)$ **droop** slopes



2. base for **outer loops**

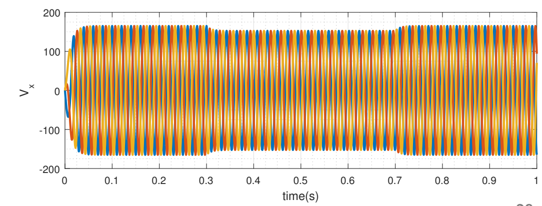
$\Rightarrow$ i_{dc} = **PID**(v_{dc}) gives virtual inertia & damping



3. reformulation as virtual & adaptive **oscillator**:

$$\dot{m} = \eta\, v_{dc} \cdot \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} m$$

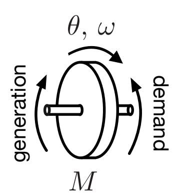
(we'll later find out that this is a profound insight)



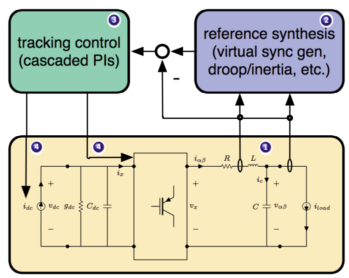
Summary: bottlenecks to inertia emulation

power system model on **grid level:**

$$M\frac{d}{dt}\omega = P_{\text{generation}} - P_{\text{demand}}$$



inertia emulation on **device level:**

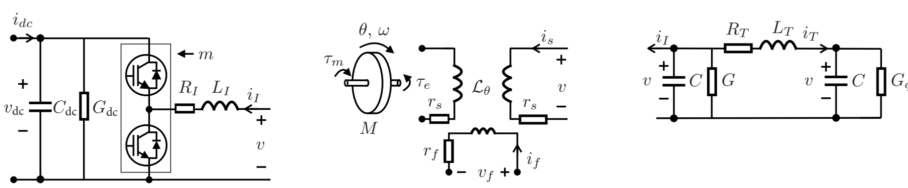


- **I/O mismatch:** none of the converter inputs or outputs are present in the swing-equation, e.g., frequency is not a state in the converter
- **inertia emulation** à la PD problematic both in theory & practice

$\Rightarrow$ maybe **matching control** $\dot{m} = \eta\, v_{dc} \cdot \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} m$ was quite clever ?

Outline

Low-inertia power system model from first principles



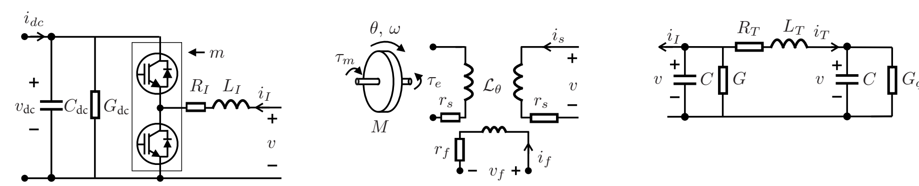
- balanced three-phase system
  - (α, β) coordinates
- synchronous machines
  - first principle
- DC/AC inverters
  - averaged-switched
- nonlinear loads $G(\|v\|)$
- voltage bus charge dynamics
- dynamic transmission lines: Π-model

Port-Hamiltonian model

$$\dot{x} = \Big(J(x,u) - R(x)\Big)\nabla H(x) + g(x)u$$

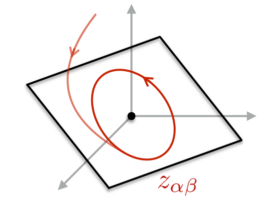
nonlinear & large, but insightful

Desired steady-state locus & control specifications



steady-state specifications for nonlinear system:

- synchronous frequency
- constant amplitude
- three-phase balanced



AC quantities v, i_s, i_I, i_T:

$$\dot{z}_{\alpha\beta} = \omega_0 \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} z_{\alpha\beta}$$

rotor angles: $\dot{\theta} = \omega_0$

DC quantities v_{dc}, v_f, ω: $\dot{z} = 0$

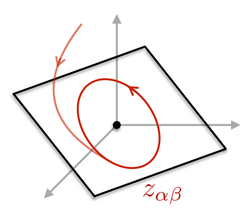
desired dynamics: $\dot{x} = f_{\text{des}}(x, \omega_0)$

controls i_{dc}, m, τ_m, i_f to be found

Proving the obvious (?)

- **steady-state locus**: physics & desired closed-loop vector field coincide (point-wise in time) on set

$$\mathcal{S} := \left\{(x, u, \omega_0) : \ f_{\text{phys}}(x, u) = f_{\text{des}}(x, \omega_0)\right\}$$



- **control-invariance**: steady-state operation $(x, u, \omega_0) \in \mathcal{S}$ for all time **if and only if**

  1. **synchronous frequency** ω_0 is constant
  2. at each **generator**: constant torque τ_m & excitation i_f
  3. at each **inverter**: constant DC current i_{dc} & inverter duty cycle with constant amplitude & synchronous frequency: $\dot{m} = \omega_0 \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} m$
  4. **network** satisfies power flow equations with impedances $R + \omega_0 JL$

  ⇒ explicit feedforward **input-to-steady-state map**

Reduction to a tractable model for synthesis

- **internal oscillator model** for inverter duty cycle with inputs ω_m, $\hat{m}$

$$\dot{\theta}_I = \omega_m, \quad m = \hat{m}\begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \end{bmatrix}$$

- **model reduction** steps

  1. **rotating coordinate frame** with synchronous frequency ω_0

     ⇒ time scales of AC quantities scaled by $1/\omega_0$

  2. **DC/AC time-scale separation** via singular perturbation arguments

     slow DC variables: $x_r = (\theta, \omega, i_f, \theta_I, v_{dc})$, $\dot{x}_r = f_z(x_r, z_{\alpha,\beta}, u)$

     fast AC variables: $z_{\alpha,\beta} = (i_s, i_I, v, i_T)$, $\epsilon\,\dot{z}_{\alpha,\beta} = f_{\alpha,\beta}(x_r, z_{\alpha,\beta}, u)$

  3. reformulation via **relative angles** δ with respect to synchronous motion
  4. **linearization** around unique steady-state

Insights from reduced model: $v_{dc} \propto$ power imbalance

- **nonlinear reduced order model** in rotating frame:

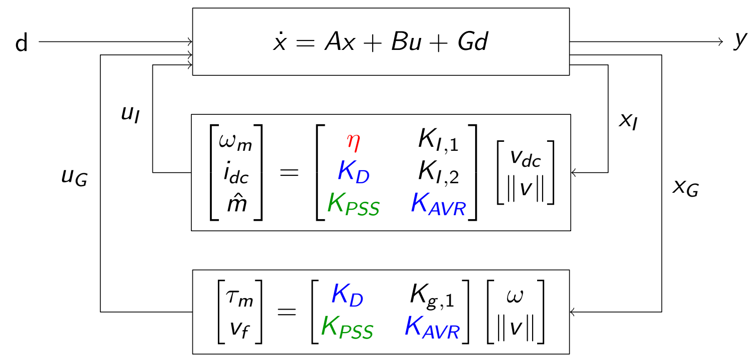
$$\begin{aligned} \dot{\theta} &= \omega \\ M\dot{\omega} &= -D\omega + \tau_m - \tau_e(x_r, u) \\ L_f \dot{i}_f &= -R_f i_f + v_f - v_{EMF}(x_r, u) \\ \dot{\theta}_I &= \omega_m \\ C_{dc}\dot{v}_{dc} &= -G_{dc}v_{dc} + i_{dc} - i_{sw}(x_r, u) \end{aligned}$$

- **interconnection** via τ_e, i_{sw}, v_{EMF}
- **analogies** & interpretation:

generator	inverter	interpretation
$\frac{1}{2}M\omega^2$	$\frac{1}{2}C_{dc}v_{dc}^2$	energy stored in device
τ_m	i_{dc}	energy supply
τ_e	i_{sw}	energy flow to grid
ω	v_{dc}	power imbalance

MIMO converter/generator control architecture

- **decentralized** converter/generator controls

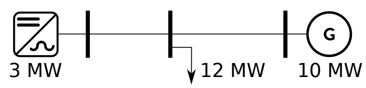


- **states:** $x = (\delta, \omega, i_f, v_{dc}, \|v\|)$
- included measurement devices for **AC voltage magnitude** $\|v\|$

Some first results ... to be continued

test case:

- generator & inverter
- impedance load
- 10% load increase at $t = 0$

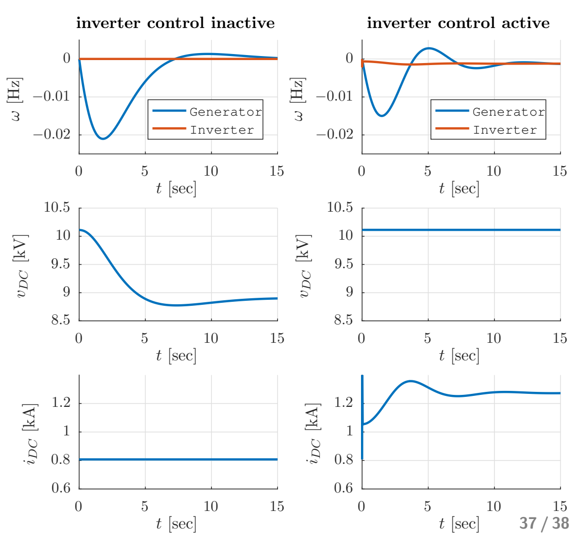


no inverter control:

- ω_m and i_{dc} constant
- power imbalance: ω_G, v_{dc}
- governor stabilizes ω_G

controlled inverter:

- reduced peak in ω_G
- v_{dc} stabilized via i_{dc}
- ω_m and ω_G synchronize



conclusions

Conclusions on virtual inertia emulation

Where to do it?

1. $\mathcal{H}_2$-optimal (non-convex) allocation
2. numerical approach via gradient computation
3. closed-form results for cost of primary control

How to do it?

1. down-sides of naive inertia emulation
2. novel machine matching control
3. reveals power imbalance visible in DC voltage

What else to do?

1. first-principle low-inertia system model
2. nonlinear steady-state control specifications
3. reduction to tractable model for synthesis
4. first promising controllers . . . to be continued

appendix

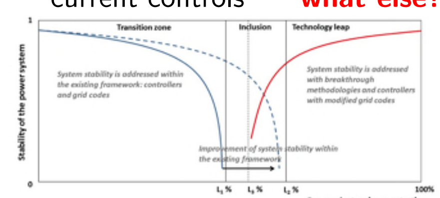
Low inertia issues have been broadly recognized

by TSOs, device manufacturers, academia, funding agencies, etc.

MIGRATE project: **M**assive **I**nte**GRAT**ion of power **E**lectronic devices

"The question that has to be examined is: how much power electronics can the grid cope with?" (European Commission)

current controls **what else?**



optimization of practically relevant power system engineering metrics

The practical engineering metrics for low-inertia systems

disturbance inputs:

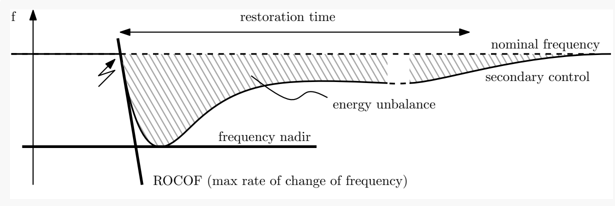
- step (loss of load/generation)
- impulse (line open-/closing)
- noise (renewables & loads)

performance outputs:

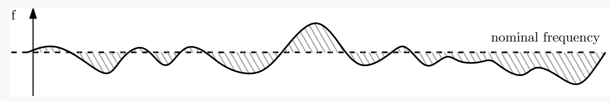
- overshoot (peak signals after fault)
- ROCOF (rate of change of frequency)
- spectrum (damping ratio cones)

re-evaluate scenario?

- not practical for optimization & control design
- metrics & faults justified only in a system dominated by machines

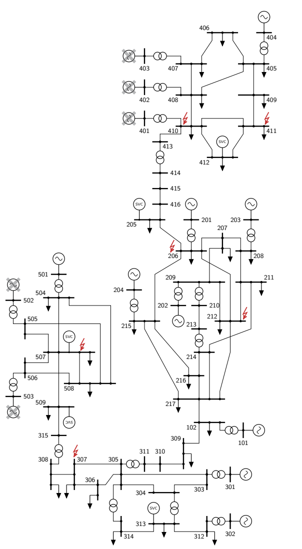


post-fault response in a low-inertia system?

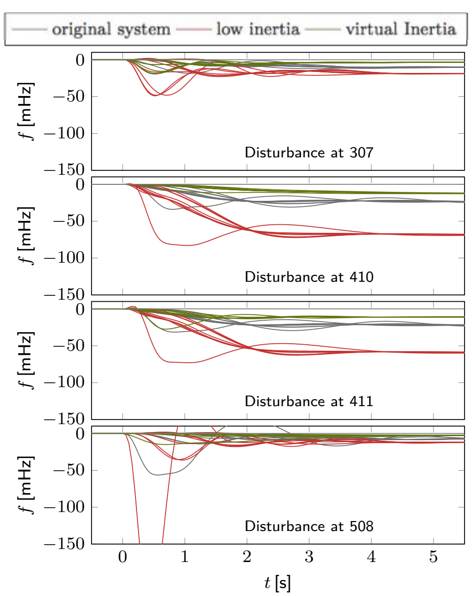


Gradient algorithms also scale up to large systems

low-inertia version of Eastern-Australian grid



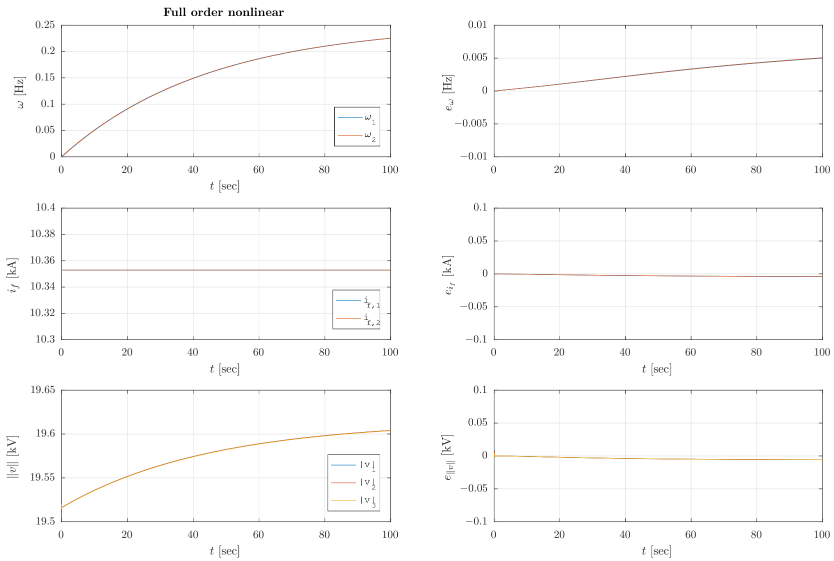
post-fault response with/without virtual inertia



accuracy of tractable low-inertia system model

Full order nonlinear vs. reduced order linearized

- two generators connected to an impedance load
- 10% increase in $\tau_{m,1}$ at $t = 0$



inertia placement in swing equations

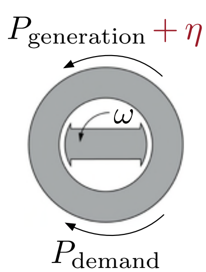
Network swing equation model

$$m_i \ddot{\theta}_i + d_i \dot{\theta}_i = p_{in,i} - p_{e,i}$$

generator swing equations

$$p_{e,i} \approx \sum_{j \in \mathcal{N}} b_{ij}(\theta_i - \theta_j)$$

linearized power flows



likelihood of **disturbance** at $\#i$: $t_i \geq 0$

state space representation:

$$\begin{bmatrix} \dot{\theta} \\ \dot{\omega} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & I \\ -M^{-1}L & -M^{-1}D \end{bmatrix}}_{A} \begin{bmatrix} \theta \\ \omega \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} T^{1/2}}_{B} \eta$$

where M, D, & T are diagonal & $L = L^{\mathsf{T}}$ (Laplacian)

Algebraic characterization of the $\mathcal{H}_2$ norm

Lemma: $\mathcal{H}_2$ norm via observability Gramian

$$\|G\|_2^2 = \text{Trace}(B^{\mathsf{T}} P B)$$

where P is the observability Gramian $P = \int_0^\infty e^{A^{\mathsf{T}} t} Q e^{At}\, dt$

- P solves a Lyapunov equation: $PA + A^{\mathsf{T}} P + Q = 0$
- A has a zero eigenvalue $\rightarrow$ restricts choice of Q

$$y = \begin{bmatrix} Q_1^{1/2} & 0 \\ 0 & Q_2^{1/2} \end{bmatrix} \begin{bmatrix} \theta \\ \omega \end{bmatrix} \qquad Q_1^{1/2} \mathbb{1} = \mathbb{0}$$

- P is unique for $P\,[\mathbb{1}\ \mathbb{0}] = [\mathbb{0}\ \mathbb{0}]$

Problem formulation

$$\begin{aligned} &\underset{P,\, m_i}{\text{minimize}} && \text{Trace}(B^{\mathsf T} P B) && \to \text{performance metric} \\ &\text{subject to} && P A + A^{\mathsf T} P + Q = 0 && \to \text{Lyapunov equation} \\ &&& P\,[\mathbb{1}\ \mathbb{0}] = [\mathbb{0}\ \mathbb{0}] && \to \text{uniqueness} \\ &&& \textstyle\sum_i m_i \le m_{\text{bdg}} && \to \text{budget constraint} \\ &&& \underline{m_i} \le m_i \le \overline{m_i}\,, && \to \text{capacity constraint} \end{aligned}$$

1. m appears as m^{-1} in system matrices A, B
2. product of $B(m)$ & P in the objective
3. product of $A(m)$ & P in the constraint

$\Rightarrow$ **large-scale & non-convex**

---

where would you place the inertia?

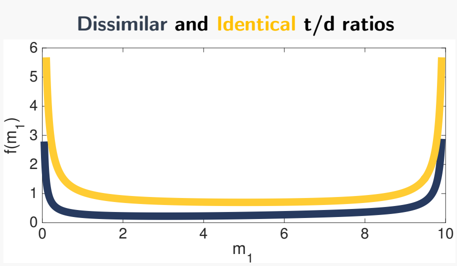
uniform, max capacity, near disturbance?

the more inertia the better?

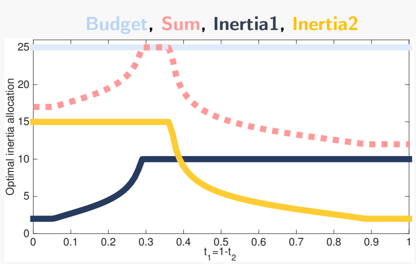
Building the intuition: results for two-area networks

Fundamental learnings

1. explicit closed-form solution is rational function
2. sufficiently uniform t_i/d_i $\to$ strongly **convex** & fairly **flat** cost
3. non trivial in the presence of capacity constraints



performance metric



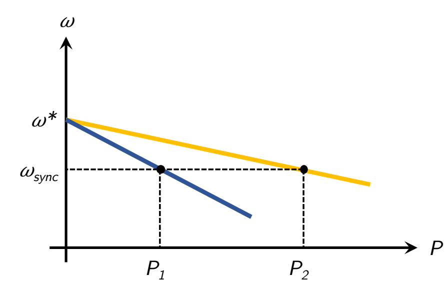
optimal inertia allocation

Closed-form results for cost of primary control

$P/\dot\theta$ primary droop control

$$(\omega_m - \omega^*) \propto (P_i^* - P_i(\theta))$$
$$\Updownarrow$$
$$D_i \dot\theta_i = P_i^* - P_i(\theta)$$

(can also model effect of PSS control)



Primary control effort $\to$ accounted for by integral quadratic cost

$$\int_0^\infty \dot\theta(t)^{\mathsf T} D\, \dot\theta(t)\, dt$$

which is the $\mathcal{H}_2$ performance for the penalties $Q_1^{1/2} = 0$ and $Q_2^{1/2} = D$

Primary control ... cont'd

Theorem: the primary control effort optimization reads equivalently as

$$\begin{aligned} \underset{m_i}{\text{minimize}} \quad & \sum\nolimits_{i=1}^{n} \frac{t_i}{m_i} \\ \text{subject to} \quad & \sum\nolimits_{i=1}^{n} m_i \leq m_{\text{bdg}} \\ & \underline{m_i} \leq m_i \leq \overline{m_i}, \quad i \in \{1, \dots, n\} \end{aligned}$$

Key take-away is **disturbance matching**:

- optimal allocation $\propto \sqrt{t_i}$ or $m_i = \min\{m_{\text{bdg}}, \overline{m_i}\}$
- optimal allocation independent of network topology

Location & **strength** of disturbance are **crucial** solution ingredients

Robust inertia allocation

empirical disturbance distributions available but we want to prepare for "rare events"

$$\begin{aligned} \underset{P,\, m_i}{\text{minimize}} \ \underset{\mathbf{t_i}}{\textbf{maximize}} \quad & \text{Trace}(B(\mathbf{t_i}^{1/2})^{\mathsf{T}} P B(\mathbf{t_i}^{1/2})) & \rightarrow \text{robust performance} \\ \text{subject to} \quad & T \in \mathbb{T} & \rightarrow \text{disturbance family} \\ & t_i \geq 0 \, \forall i \ \ \& \ \sum\nolimits_{i=1}^{n} t_i = 1 & \rightarrow \text{normalization} \\ & \text{inertia budget, capacities, \& Lyapunov equation} & \end{aligned}$$

Key insights:

- inner maximization problem is **linear** in T

$\Rightarrow$ min - max can be converted to minimization by duality

- **valley filling** solution for primary control metric:

  $t_i^\star / m_i^\star = const.$ (up to constraints)

numerical method for the general case

Taylor & power series expansions

Key idea: scalar series expansion at m_i in direction μ_i:

$$\frac{1}{m_i + \delta\mu_i} = \frac{1}{m_i} - \frac{\delta\mu_i}{m_i^2} + \mathcal{O}(\delta^2)$$

$\Rightarrow$ expand system matrices via **Taylor series** in direction μ:

$$\mathbf{A}(m + \delta\mu) = A^{(0)}_{(m,\mu)} + A^{(1)}_{(m,\mu)}\delta + \mathcal{O}(\delta^2) \quad , \quad \mathbf{B}(m + \delta\mu) = \dots$$

$\Rightarrow$ expand observability Gramian via **power series** in direction μ:

$$\mathbf{P}(m + \delta\mu) = P^{(0)}_{(m,\mu)} + P^{(1)}_{(m,\mu)}\delta + \mathcal{O}(\delta^2)$$

Magic happens: the Lyapunov equation decouples

$$\begin{aligned} \mathbb{0} = \delta^0 & \left(P^{(0)}A^{(0)} + A^{(0)\top}P^{(0)} + Q\right) + \\ \delta^1 & \left(P^{(1)}A^{(0)} + A^{(0)\top}P^{(1)} + \left(P^{(0)}A^{(1)} + A^{(1)\top}P^{(0)}\right)\right) + \mathcal{O}(\delta^2) \end{aligned}$$

Explicit gradient computation

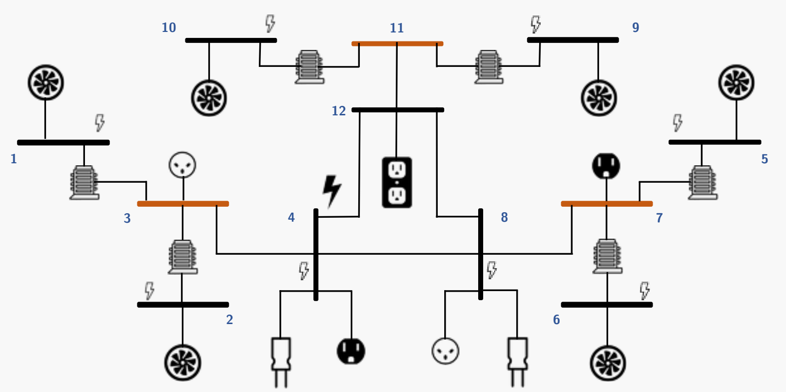
1. nominal Lyapunov equation for $\mathcal{O}(\delta^0)$:
$$\mathbf{P}^{(0)} = \mathbf{Lyap}(\mathbf{A}^{(0)}, \mathbf{Q})$$
2. perturbed Lyapunov equation for $\mathcal{O}(\delta^1)$ terms:
$$\mathbf{P}^{(1)} = \mathbf{Lyap}(\mathbf{A}^{(0)}, \mathbf{P}^{(0)}\mathbf{A}^{(1)} + \mathbf{A}^{(1)^{\mathsf{T}}}\mathbf{P}^{(0)})$$
3. expand objective at m in direction μ:
$$\mathrm{Trace}(B(m)^{\mathsf{T}} P(m) B(m)) = \mathrm{Trace}((\ldots) + \delta(\ldots)) + \mathcal{O}(\delta^2)$$
4. gradient: $\mathrm{Trace}(2 * B^{(1)^{\mathsf{T}}} P^{(0)} B^{(0)} + B^{(0)^{\mathsf{T}}} P^{(1)} B^{(0)})$

$\Rightarrow$ use favorite method for reduced optimization problem
with explicit gradient & without Lyapunov constraint

results for a three-area case study

Modified Kundur case study: 3 areas & 12 buses

transformer reactance 0.15 p.u., line impedance (0.0001+0.001i) p.u./km



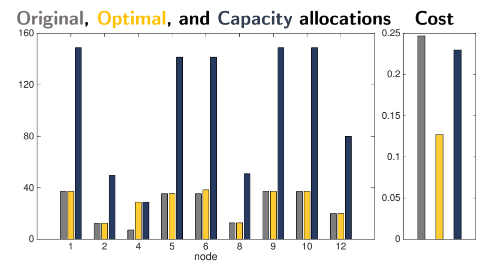
uniform deviation from sync as **performance metric**: $Q = \begin{bmatrix} I_n - \frac{1}{n}\mathbb{1}\mathbb{1}^T & \\ & I_n \end{bmatrix}$

Heuristics outperformed by $\mathcal{H}_2$ - optimal allocation

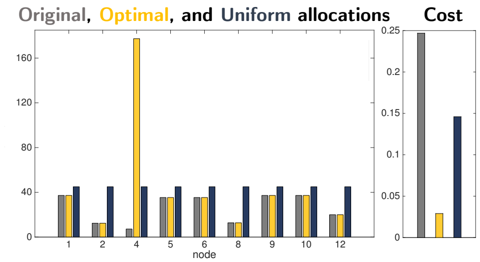
Scenario: disturbance at #4

- locally optimal solution **outperforms heuristic** max/uniform allocation
- optimal allocation ≈ **matches disturbance**
- inertia emulation at all undisturbed nodes is actually **detrimental**

$\Rightarrow$ **location** of disturbance & inertia emulation matters

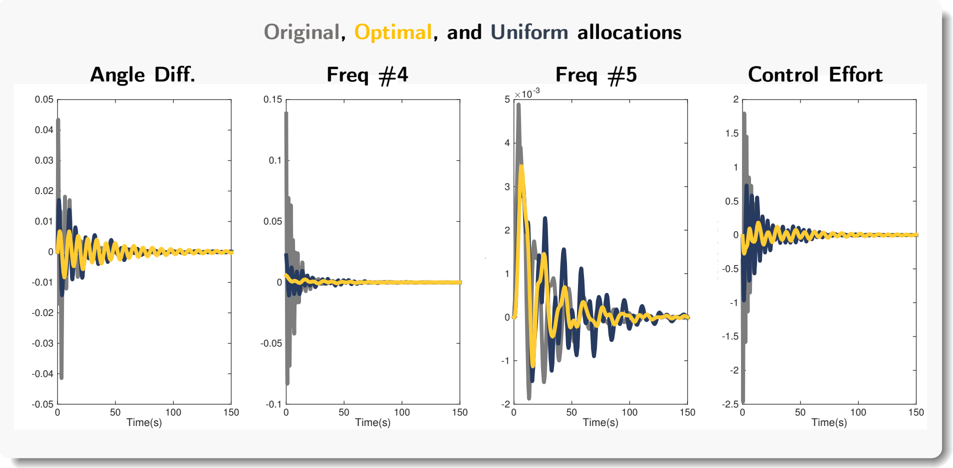


allocation subject to capacity constraints



allocation subject to the budget constraint

Eye candy: time-domain plots of post fault behavior



Take-home messages:

best oscillation performance	smallest peak frequency at #4	undisturbed sites are irrelevant	minimal control effort $m_i \cdot \ddot{\theta}_i$

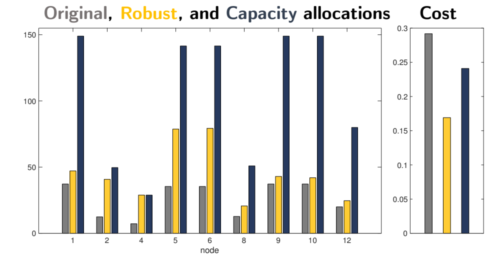
Robust min - max allocation

Scenario: fault (impulse) can occur at any single node

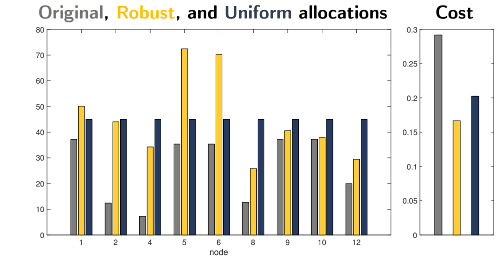
- disturbance set
  $T \in \mathbb{T} = \{\mathbb{e}_1 \cup \cdots \cup \mathbb{e}_{12}\}$

$\Rightarrow$ min / max over convex hull

- robust inertia allocation **outperforms** heuristics
- results become **more intuitive**: the more inertia (capacity & budget) the better & valley-filling property



allocation subject to capacity constraints



allocation subject to the budget constraint