Fast power system analysis via implicit linearization of the power flow manifold

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Power flow equations

Basic **ingredients** in an n-bus power system

- complex bus voltage $u_h = v_h e^{j\theta_h} \in \mathbb{C}$
- injected complex power $s_h = p_h + jq_h$
- admittance matrix $Y \in \mathbb{C}^{n\times n}$

Underlying nonlinear **power flow equations**

$$\left.\begin{array}{l}\text{Kirchhoff's \& Ohm's laws}\\ \text{nodal power balances}\end{array}\right\} \implies \operatorname{diag}(u)\overline{Yu} = s$$

Bus model specifies variables & fixed parameters

- PV bus: p_h and v_h fixed
- PQ bus: p_h and q_h fixed
- slack bus: v_h and θ_h fixed
- ⇒ remaining ingredients are variables

A brief history of power flow approximations

for computational tractability, analytic studies, & control/optimization design

- **DC power flow**: $\Re(Y) = \mathbb{0}$, $v = \mathbb{1}$, & linearization

  B. Stott, J. Jardim, & O. Alsac, "DC Power Flow Revisited" *IEEE TPS*, 2009.

- **LinDistFlow**: parameterization in flow & v_h^2 coordinates & linearization

  M.E. Baran & F.F. Wu, Optimal sizing of capacitors placed on a radial distribution system. *IEEE PES*, 1988.

- **rectangular DC power flow**: fixed-point ball for small S^2/V_{slack}^2

  S. Bolognani & S. Zampieri, On the existence and linear approximation of the power flow solution in power distribution networks. *IEEE TPS*, 2015.

- many variations & extensions, sensitivity & Jacobian methods, etc.

main twist today: separate power flow & bus model

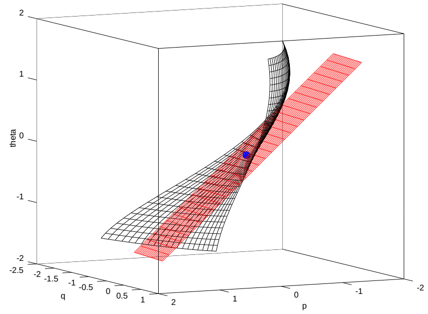
1) derivation of linear implicit model
2) relation to other approximations
3) accuracy in the three-phase case
4) some direct applications

Linear implicit model & its advantages

- **today** consider all of $x = (v, \theta, p, q)$ as variables
- **implicit model** for power flow manifold: $F(x) = \mathbb{0}$
- **linear approximant** at x^* is tangent plane: $A(x - x^*) = \mathbb{0}$

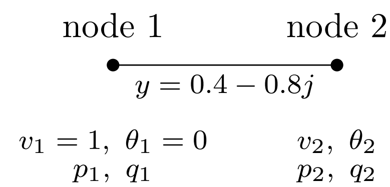
Advantages of linear implicit model:

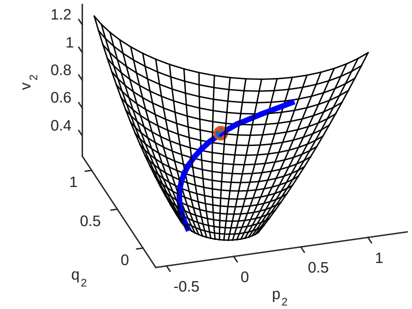
- sparsity
- ⇒ tractable for applications with high computational burden
- structure-preserving
- ⇒ prior for distributed control, optimization, estimation, etc.
- geometric methods
- ⇒ explicitly require tangent planes



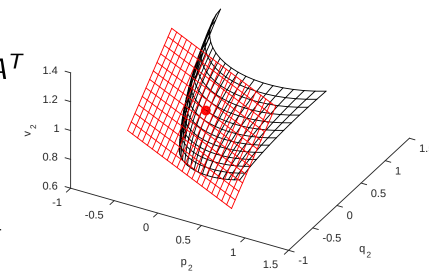
two-bus example

The power flow manifold & linear tangent approximation





1. **power flow manifold:** $F(x) = \mathbb{0}$
2. **normal space** spanned by $\left.\frac{\partial F(x)}{\partial x}\right|_{x^*} = A^T$
3. **tangent space** at x^*: $A(x - x^*) = \mathbb{0}$
4. **accuracy** depends on curvature $\frac{\partial^2 F(x)}{\partial x^2}$



Closer look at implicit formulae $A(x - x^*) = \mathbb{0}$

$$\left[\underbrace{\left(\langle \mathrm{diag}\,\overline{Yu^*}\rangle}_{\text{shunt loads}} + \underbrace{\langle \mathrm{diag}\,u^*\rangle N \langle Y\rangle\right)}_{\text{lossy DC flow}} \cdot \underbrace{\begin{bmatrix} \mathrm{diag}(\cos\theta^*) & -\mathrm{diag}(v^*)\,\mathrm{diag}(\sin\theta^*) \\ \mathrm{diag}(\sin\theta^*) & \mathrm{diag}(v^*)\,\mathrm{diag}(\cos\theta^*) \end{bmatrix}}_{\text{rotation} \times \text{scaling at operating point}}\right]$$

$$\times \underbrace{\begin{bmatrix} v - v^* \\ \theta - \theta^* \end{bmatrix} = \begin{bmatrix} p - p^* \\ q - q^* \end{bmatrix}}_{\text{deviation variables}}$$

where $N = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}$ is complex conjugate in real coordinates

and $\langle A\rangle = \begin{bmatrix} \Re(A) & -\Im(A) \\ \Im(A) & \Re(A) \end{bmatrix}$ is complex rotation in real coordinates.

. . . appears cumbersome at first glance

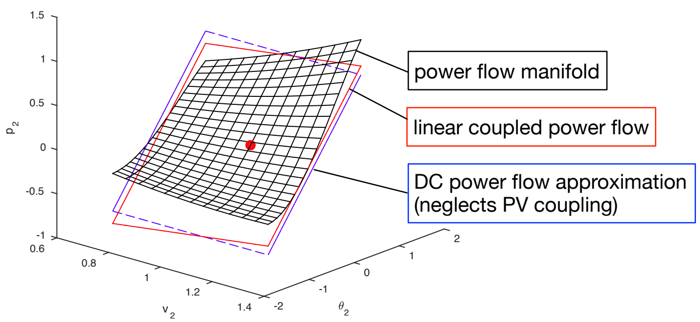
Special cases reveal some old friends I

- **flat-voltage/0-injection point:** $x^* = (v^*, \theta^*, p^*, q^*) = (\mathbb{1}, \mathbb{0}, \mathbb{0}, \mathbb{0})$

$\Rightarrow$ implicit linearization: $\begin{bmatrix} \Re(Y) & -\Im(Y) \\ -\Im(Y) & \Re(Y) \end{bmatrix} \begin{bmatrix} v \\ \theta \end{bmatrix} = \begin{bmatrix} p \\ q \end{bmatrix}$

is **linear coupled power flow** [D. Deka, S. Backhaus, & M. Chertkov, 2015]

$\Rightarrow$ $\Re(Y) = \mathbb{0}$ gives **DC power flow:** $-\Im(Y)\theta = p$ and $-\Im(Y)v = q$



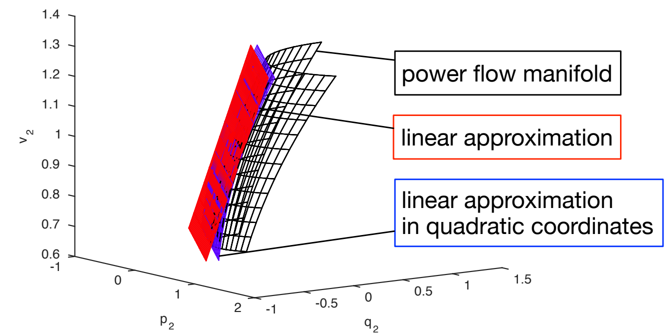
Special cases reveal some old friends II

- **flat-voltage/0-injection point:** $x^* = (v^*, \theta^*, p^*, q^*) = (\mathbb{1}, \mathbb{0}, \mathbb{0}, \mathbb{0})$

$\Rightarrow$ rectangular coord. $\Rightarrow$ **rectangular DC flow** [S. Bolognani & S. Zampieri, 2015]

- nonlinear change to **quadratic coordinates** from v_h to v_h^2

$\Rightarrow$ linearization gives (non-radial) **LinDistFlow** [M.E. Baran & F.F. Wu, 1988]



so all standard approximations are included as special cases

Extensions to more general models

Bus device models, e.g., PQ bus $s_h = p_h + jq_h = const.$

$\Rightarrow$ implicit constraint $g(x) = \mathbb{0}$ & can be absorbed in $F(x) = \mathbb{0}$

Exponential load models $s_h = const. \cdot v_h^{const.}$

$\Rightarrow$ can be handled analogously

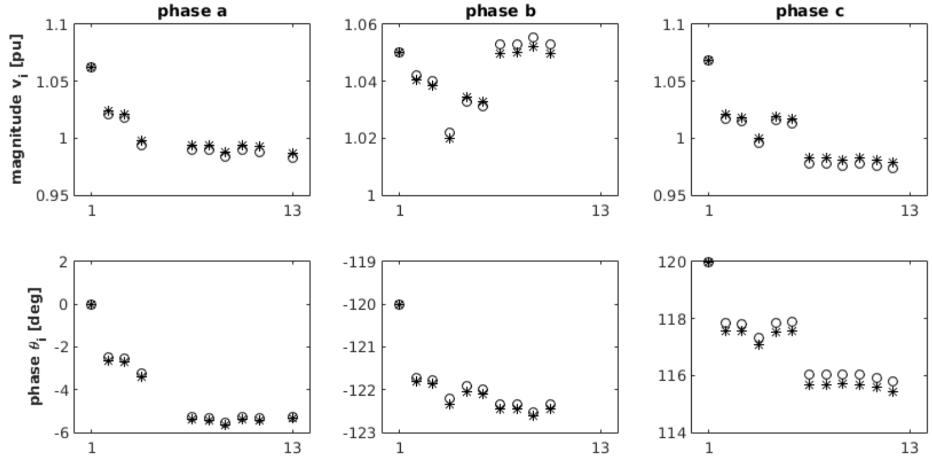
Unbalanced three-phase grids with basic ingredients

- complex voltage $u_h = [u_h^a \; u_h^b \; u_h^c]^T \in \mathbb{C}^3$
- similar definitions for other quantities

$\Rightarrow$ all previous results can be analogously re-derived

Matlab/Octave **code** available: `https://github.com/saveriob/1ACPF`

Accuracy illustrated with unbalanced three-phase IEEE13



○ exact solution ★ linear implicit model

a glorified & highly accurate linearization . . . so what?

some direct applications

Fast scenario-based decision making under uncertainties

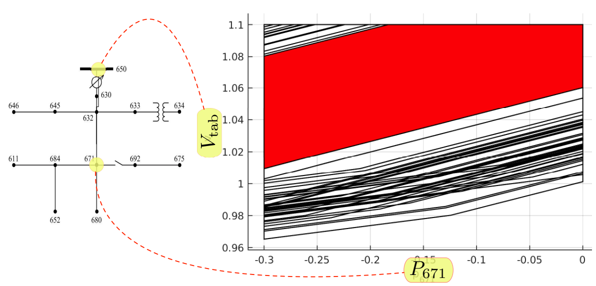
Example: feasible region for distribution network operation

$$\Big\{ \underbrace{x_{\text{dec}}}_{\text{actuation}} \;\Big|\; \text{Prob}\{ \underbrace{x_{\text{exo}} \in \mathcal{X}_{\text{exo}}}_{\text{random loads}} \;:\; \underbrace{F(x) = 0}_{\text{power flow}} \;\&\; \underbrace{Vx \leq w}_{\text{constraints}} \} \geq \underbrace{1 - \varepsilon}_{\text{chance}} \Big\}$$

Scenario-based approach:

sample x_{exo} variables & build deterministic constraints

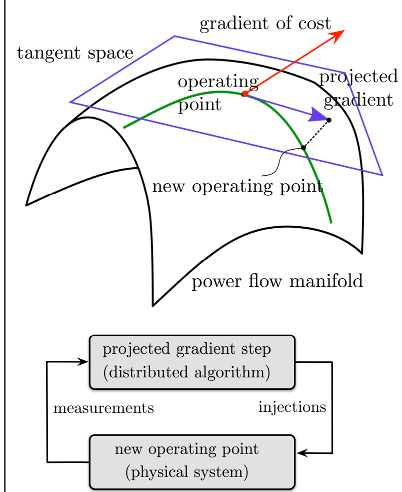
$\Rightarrow$ decision x_{dec} is feasible with high probability for *sufficiently many* samples

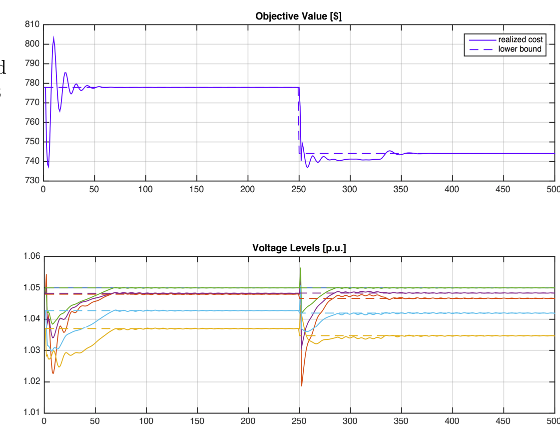


S. Bolognani & F. Dörfler. Fast scenario-based decision making in unbalanced distribution networks. *Power Systems Computation Conference (PSCC)*, June 2016.

Distributed online optimization on power flow manifold

with Adrian Hauswirth & Gabriela Hug (ETH Zürich)

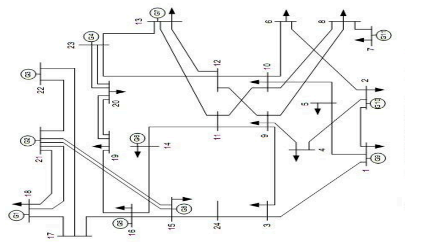




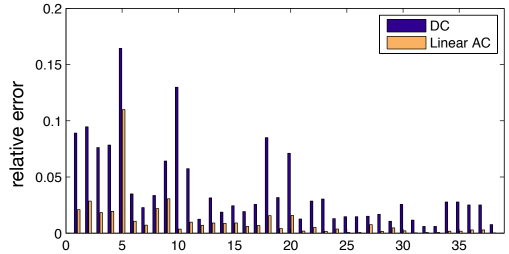
applied to optimal voltage control in IEEE 30 bus grid

Cascading failures – more accuracy for similar comp. effort

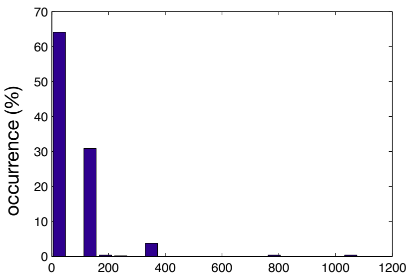
with Giovanni Sansavini & Bing Li (ETH Zürich)



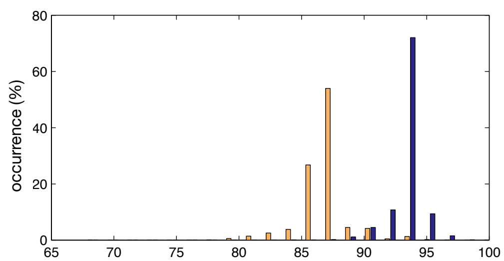
RTS24 with random loads & N-2 contingencies



line number



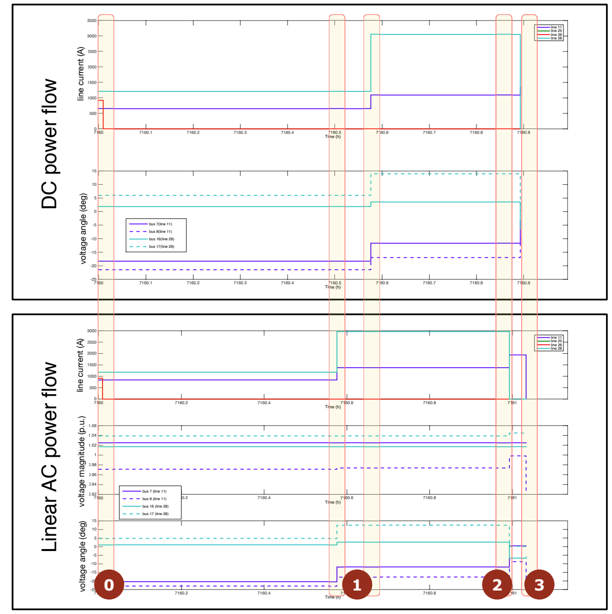
load shedding: linear AC - DC (MW)



reduced computation time over AC (%)

Cascading failures — distinct cascades under naive DC flow

with Giovanni Sansavini & Bing Li (ETH Zürich)



0 N-2 contingency

1 DC flow is optimistic compared to linear AC

2 single line outage since voltage magnitude can (partially) compensate for other remaing line

3 blackout

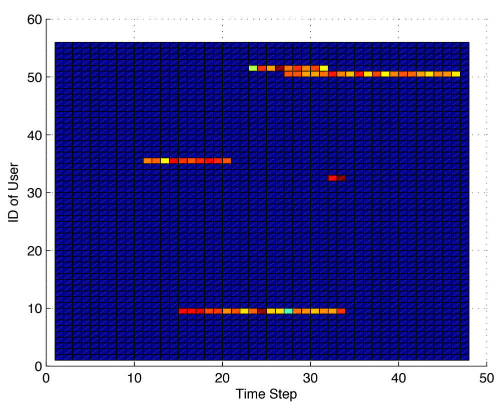
Monitoring, state estimation, learning, & detection

Consistency equation:

$$\underbrace{y_k}_{\text{measurement}} = \underbrace{H \cdot x_k}_{\text{system model}} + \underbrace{\varepsilon_k}_{\text{noise}} + \underbrace{d_k}_{\text{attack}}$$

Attack detection: collect measurements y_k over time & look for a consistent low-rank (ID $\times$ time space) input d_k

Results for IEEE 123 model: harder to trick an operator relying on a linearized AC model

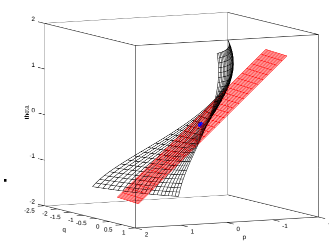


D. Drzajic, S. Bolognani, & F. Dörfler. Energy theft detection using compressive sensing methods. *ETH Zürich Semester project*, August 2015.

Conclusions

Summary

- linear, sparse, & structure-preserving model
- includes all DC & LinDistFlow approximations
- applicable to unbalanced three-phase systems
- apps: monitoring, control, decision-making, . . .



Ongoing & future work

- theory: error bounds & coord trafos
- applications: further develop apps

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