# An optimization-on-manifold approach to the design of distributed feedback control in smart grids

Saverio Bolognani, Florian Dörfler

Automatic Control Laboratory
ETH Zürich

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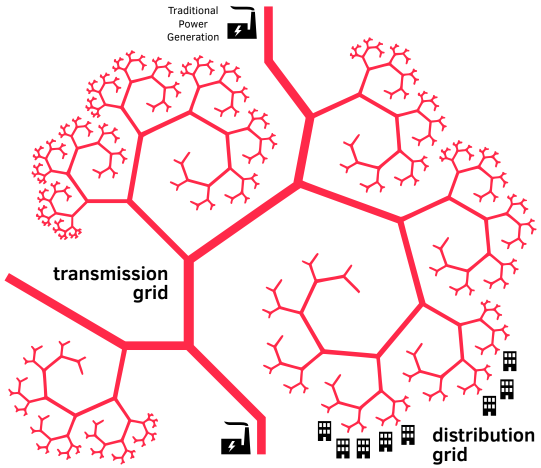
Distributed and Stochastic Optimization: Theory and Applications



## FUTURE ELECTRIC POWER DISTRIBUTION GRIDS



## Power distribution grids

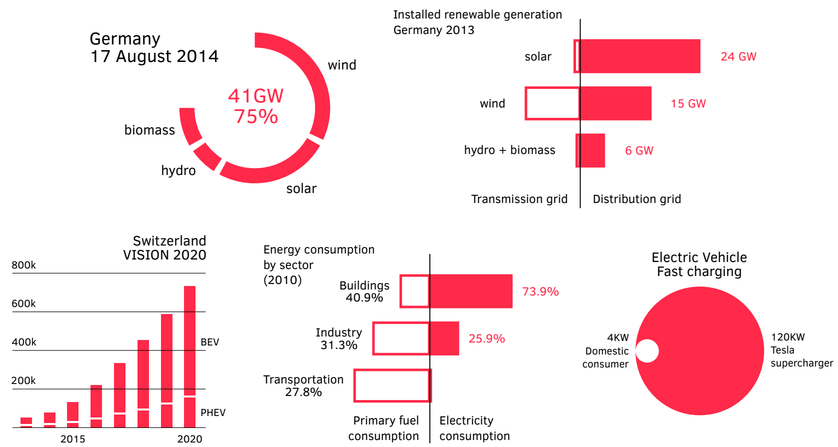



- **Distribution grid:** the “capillary system” of power networks
- It delivers power from the transmission grid to the consumers.
- Very little sensing, monitoring, actuation.
- The “easy” part of the grid: conventionally **fit-and-forget** design.

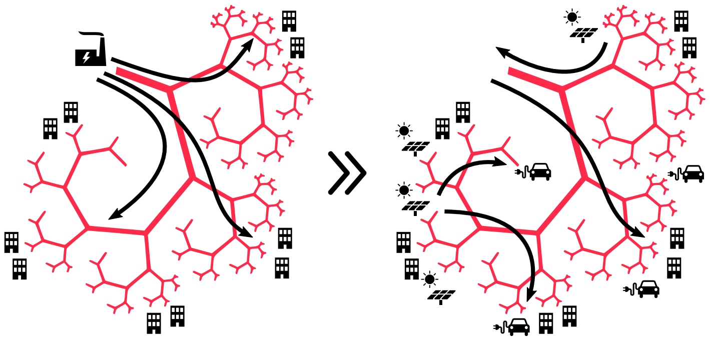


## New challenges

- **Distributed microgenerators** (conventional and renewable sources)
- **Electric mobility** (large flexible demand, spatio-temporal patterns).



## Distribution grid congestion

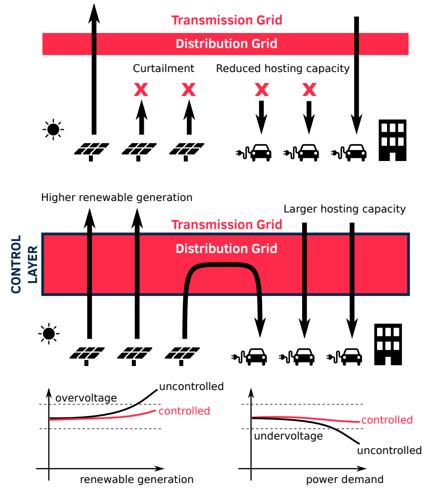


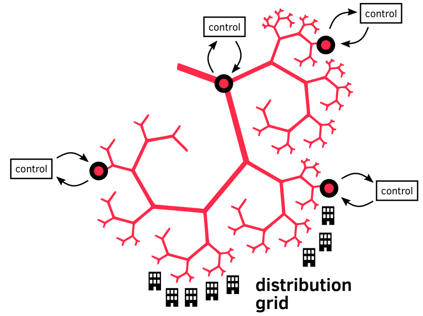
Operation of the grid close or above the **physical limits**, due to simultaneous and **uncoordinated** power demand/generation.

→ lower efficiency, blackouts
→ **curtailment of renewable generation**
→ **bottleneck to electric mobility**

*Fit-and-forget* → unsustainable **grid reinforcement**

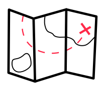





- **Virtual** grid reinforcement
  - same infrastructure
  - more sensors and intelligence
  - controlled grid = larger capacity
- **Transparent** control layer
  - invisible to the users
  - modular design





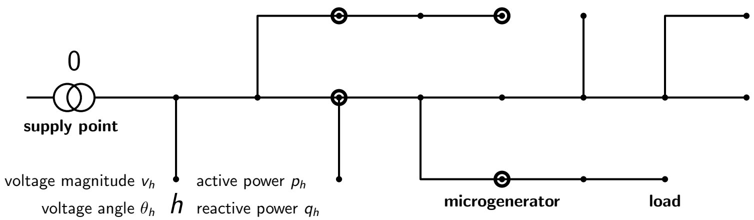
## OVERVIEW

1. A feedback control approach
2. A tractable model for control design
3. Control design example
   - Reactive power control for voltage regulation
4. Next step
   - Optimization on the power flow manifold
5. Conclusions



## A FEEDBACK CONTROL APPROACH

# Distribution grid model




**Grid equations**

$$\text{diag}(u)\overline{Yu} = s$$

where

- $u_h = v_h e^{j\theta_h}$ complex voltages
- $s_h = p_h + jq_h$ complex powers

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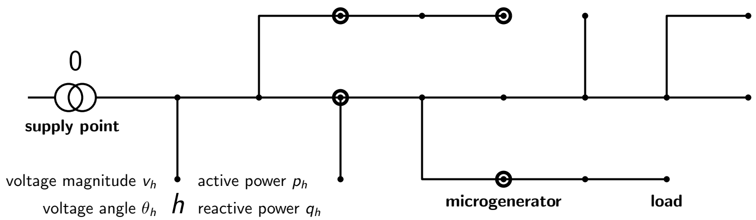
A feedback control approach

# Distribution grid model




**Actuation**

- **Tap changer / voltage regulators** – supply point voltage $v_0$
- **Reactive power compensators** – reactive power $q_h$
  - static compensators
  - power inverters of the microgenerators (when available)
- **Active power management** – active power $p_h$
  - smart building control, storage and deferrable loads
  - generator curtailment and load shedding

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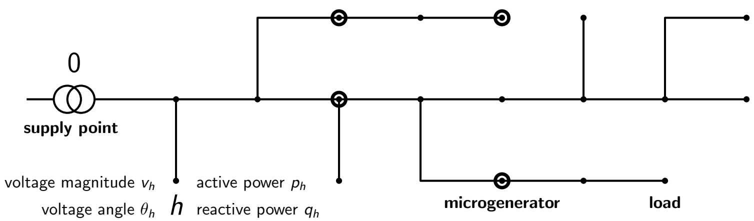
A feedback control approach

# Distribution grid model




**Sensing**

- **Power meters** – active power $p_h$ and reactive power $q_h$
- **Voltage meters** – nodal voltage $v_h$
- **Phasor measurement units (PMU)** – voltage magnitude $v_h$ and angle $\theta_h$ (PQube @ UC Berkeley, GridBox in Zürich/Bern, Smart Grid Campus @ EPFL)
- **Line currents, transformer loading, . . .**

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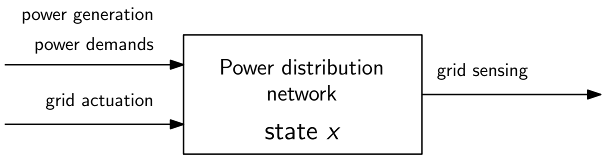
A feedback control approach

# A control framework

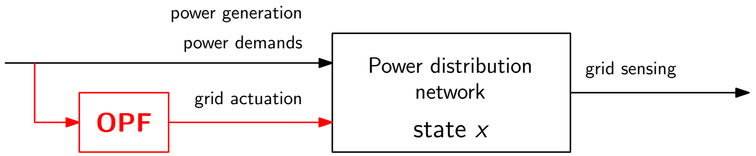



**Control objective**

Drive the system to a state $x^* = \begin{bmatrix} v^* & \theta^* & p^* & q^* \end{bmatrix}$ subject to

- **soft constraints** $x^* = \textbf{argmin}_x\, J(x)$
- **hard constraints** $x \in \mathcal{X}$
- **chance constraints** $\mathbb{P}\left[x \notin \mathcal{X}\right] < \epsilon$

## Feedforward control

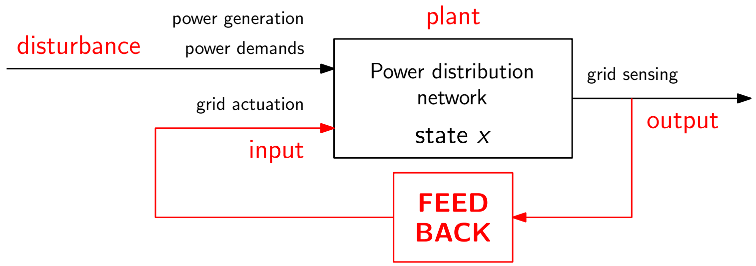



**Conventional approach: Optimal Power Flow**

- Similar to power transmission grid OPF
- Motivated by encouraging results on **OPF convexification** (Lavaei (2012), Farivar (2013), ...)
- Requires **full disturbance knowledge - full communication**
- Heavily **model based**
- Requires **co-design** of grid control and users' behavior



## Feedback control

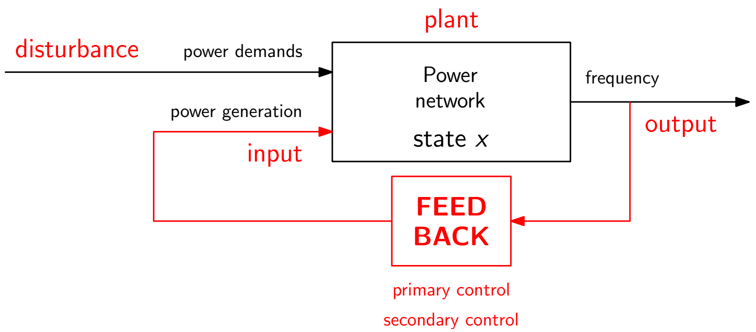



**Control theory answer**

- Robustness against **parametric uncertainty/unmodeled disturbance**
- Time varying demand/generation becomes **disturbance**
- **Model-free** design
- Explored so far only for limited cases (e.g. purely local VAR control)
- Allows **modular design** of grid control and users' behavior



## A similar scenario: frequency control




In the **transmission grid**, feedback is used for **frequency regulation**

- Frequency deviation as a **implicit signal** for power unbalance
- Purely local proportional control: **primary droop control**
- Integral control: **secondary frequency regulation**



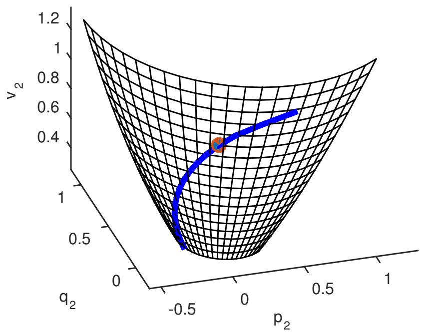
## A TRACTABLE MODEL FOR CONTROL DESIGN

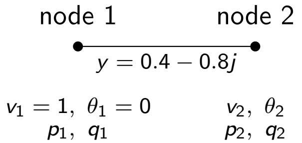
# Power flow manifold

- **Grid state** $x = \begin{bmatrix} v & \theta & p & q \end{bmatrix}$
- Set of all states that satisfy the **grid equations** $\text{diag}(u)\overline{Yu} = s$

$\rightarrow$ **power flow manifold** $\mathcal{M} := \{x \,|\, F(x) = 0\}$

- Regular submanifold of dimension $2n$ ($6n$ if three-phase)

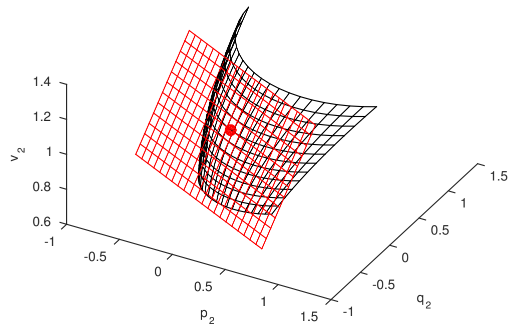



$$v_1^2 g - v_1 v_2 \cos(\theta_1 - \theta_2) g - v_1 v_2 \sin(\theta_1 - \theta_2) b = p_1$$
$$-v_1^2 b + v_1 v_2 \cos(\theta_1 - \theta_2) b - v_1 v_2 \sin(\theta_1 - \theta_2) g = q_1$$
$$v_2^2 g - v_1 v_2 \cos(\theta_2 - \theta_1) g - v_1 v_2 \sin(\theta_2 - \theta_1) b = p_2$$
$$-v_2^2 b + v_1 v_2 \cos(\theta_2 - \theta_1) b - v_1 v_2 \sin(\theta_2 - \theta_1) g = q_2$$



# Power flow manifold approximation




**Best linear approximant**

$$A_{x^*}(x - x^*) = 0$$
$$A_{x^*} := \left.\frac{\partial F(x)}{\partial x}\right|_{x=x^*}$$

Tangent plane at a nominal power flow solution $x^* \in \mathcal{M}$

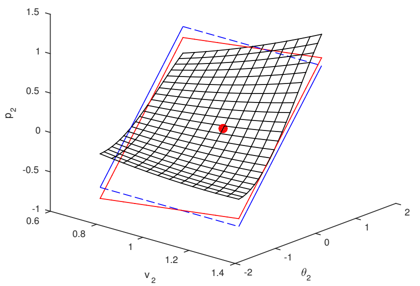
Example $x^*$: no-load solution

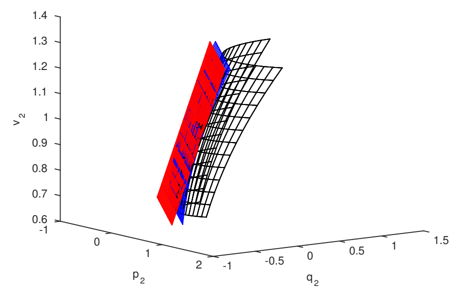
- **Implicit** – No input/outputs (not a disadvantage)
- **Sparse** – The matrix $A_{x^*}$ has the sparsity pattern of the grid graph
- **Structure preserving** – Elements of $A_{x^*}$ depend on local parameters

$\rightarrow$ Bolognani & Dörfler, Allerton (2015)
$\rightarrow$ Source code on `github`



# Power flow manifold approximation




**Standard models**

Adding assumption one obtains

- linear coupled power flow
- DC power flow
- rectangular DC flow

**Nonlinear coordinate transf.**

$$\tilde{x}_h = \tilde{x}_h(x_h), \quad \frac{\partial \tilde{x}_h}{\partial x_h} = 1$$

Different manifold curvature!

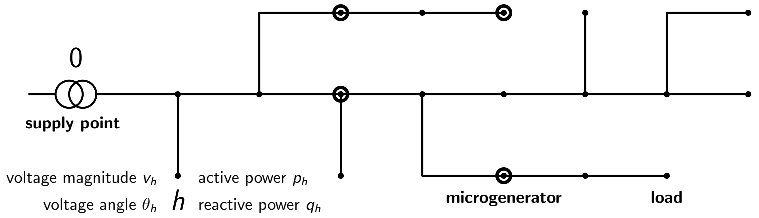
- $v_h \rightarrow v_h^2$ : LinDistFlow



## CONTROL DESIGN EXAMPLES

## REACTIVE POWER CONTROL FOR VOLTAGE REGULATION

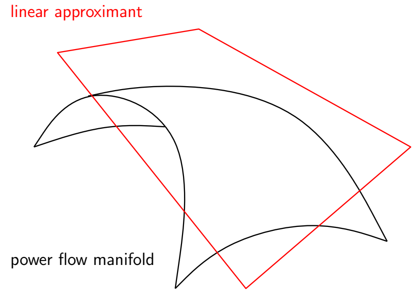
# Problem statement




- **Inputs:** reactive power $q_h$ of microgenerators
- **Outputs:** voltage measurement $v_h$ at the microgenerators
- **Control objective:**
  - **Soft constraints:** minimize $J(x) = v^T L v$ (voltage drops)
  - **Hard constraints:** guarantee $V^{\min} \leq v_h \leq V^{\max}$ at all sensors



# Control design for soft constraint




1. Modeling assumption

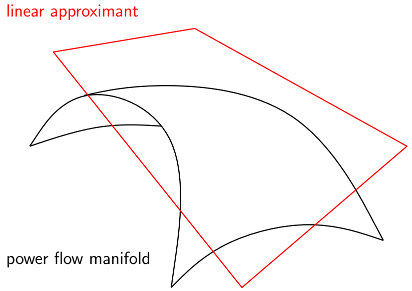
**Modeling assumption:** constant $R/X$ ratio $\rho$.

$A_{x^*}(x - x^*) = 0$ becomes (around the no-load state)

$$\left[\begin{array}{cc|cc} \rho L & -L & -I & 0 \\ -L & -\rho L & 0 & -I \end{array}\right] \begin{bmatrix} v \\ \theta \\ p \\ q \end{bmatrix} = \mathbb{0}$$



# Control design for soft constraint




1. Modeling assumption
2. Control specs

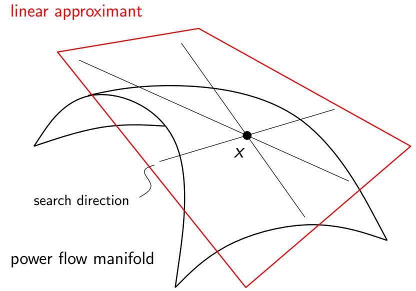
**Control specification:** Distributed and asynchronous.

$$\text{Minimal update } \delta q \qquad \begin{cases} q_h \leftarrow q_h + \delta \\ q_k \leftarrow q_k - \delta \end{cases}$$

$\rightarrow$ **Communication graph** $\mathcal{G}_{\text{comm}}$ describing all possible updates (pairs $h, k$).



# Control design for soft constraint

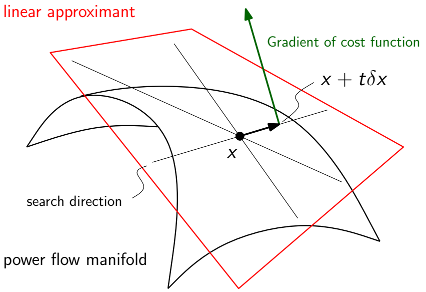



1. Modeling assumption
2. Control specs
3. Proj on linear manifold

**Search directions:** By projecting each possible direction $\delta q$ on the linear manifold ker $A_{x^*}$, we obtain feasible search directions in the state space.

$$\delta x = \begin{bmatrix} -\frac{1}{1+\rho^2} L^\dagger \delta q \\ -\frac{\rho}{1+\rho^2} L^\dagger \delta q \\ \mathbb{0} \\ \delta q \end{bmatrix}$$

## Control design for soft constraint




1. Modeling assumption
2. Control specs
3. Proj on linear manifold
4. Derive feedback law

**Optimal step length:** Given a search direction $\delta x$, we determine the step length that minimizes the cost function $J(x) = v^T L v$.

$$\nabla J(x) = \begin{bmatrix} 2Lv \\ \mathbb{0} \\ \mathbb{0} \\ \mathbb{0} \end{bmatrix} \qquad \nabla J(x + t\delta x)^T \delta x = 0 \quad \Rightarrow \quad t = (1+\rho^2)\frac{v^T \delta q}{\delta q^T L^\dagger \delta q}$$

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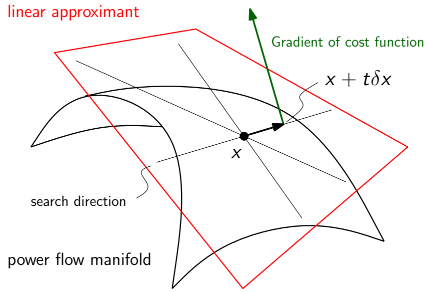
Control design examples
Reactive power control for voltage regulation

## Control design for soft constraint




1. Modeling assumption
2. Control specs
3. Proj on linear manifold
4. Derive feedback law

Because the model is sparse and structure preserving...

$$t = (1+\rho^2)\frac{v^T \delta q}{\delta q^T L^\dagger \delta q} = (1+\rho^2)\frac{v_h - v_k}{X_{hk}}$$

**Gossip-like feedback law** $\begin{cases} q_h \leftarrow q_h + (1+\rho^2)\frac{v_h - v_k}{X_{hk}} \\ q_k \leftarrow q_k - (1+\rho^2)\frac{v_h - v_k}{X_{hk}} \end{cases}$

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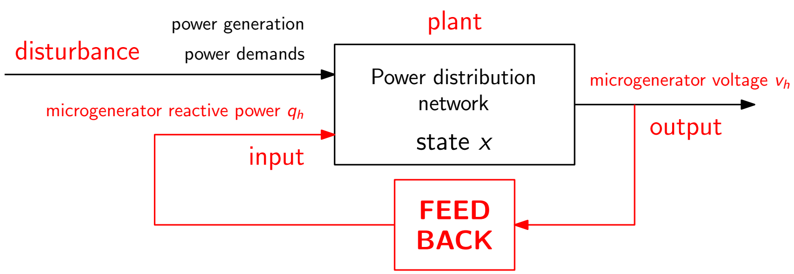
## Convergence and performance analysis




- **Asynchronous distributed feedback control**
  - no demand or generation measurement
  - limited model knowledge
  - no power flow solver
  - alternation of sensing and actuation.

$$\begin{cases} q_h \leftarrow q_h + (1+\rho^2)\frac{v_h - v_k}{X_{hk}} \\ q_k \leftarrow q_k - (1+\rho^2)\frac{v_h - v_k}{X_{hk}} \end{cases}$$

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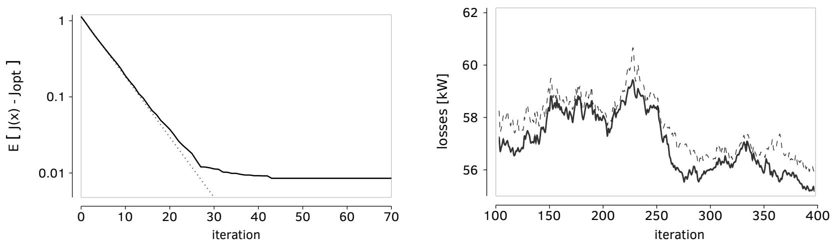
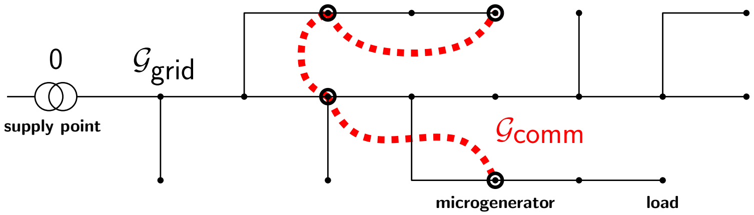
## Convergence and performance analysis




→ Bolognani & Zampieri, IEEE TAC (2013)

- Extension to $J(x) = \bar{u}^T L u$ **(power losses)**, if $\theta$ can be measured (PMUs).
- Proof of **mean square convergence** (with randomized async updates).
- Explicit bound on the **exponential rate of convergence**.
- Analysis of the **dynamic performance** (disturbance rejection).
- **Optimal communication graph:** $\mathcal{G}_{\text{comm}} \approx \mathcal{G}_{\text{grid}}$.

# Communication co-design




$\mathcal{G}_{\text{grid}}$
Sparsity of the power system

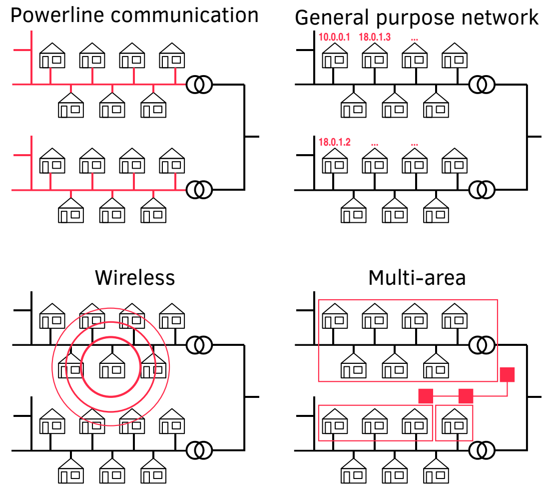
$\mathcal{G}_{\text{comm}}$
Sparsity of the communication graph

**Fundamental design problem:** implications of the **communication architecture** on the **control performance**.

Joint design vs. separation results

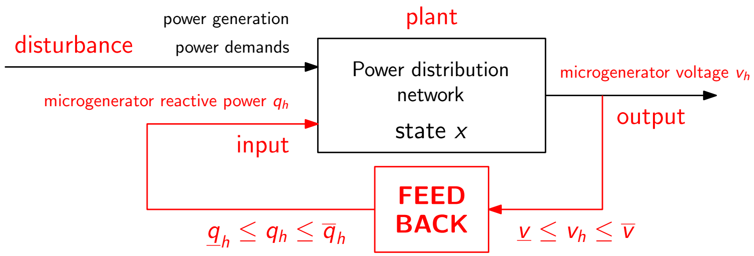


# Communication co-design






# Control design with hard constraints

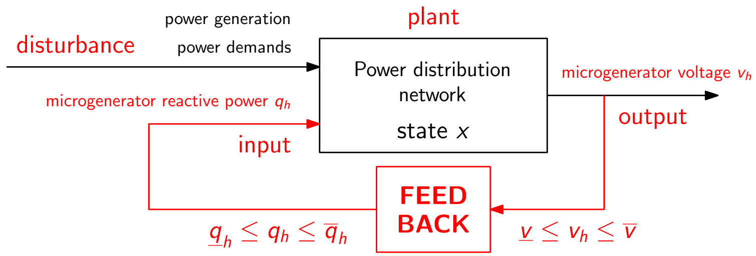



- **Power losses minimization**
- **Hard constraints** on inputs and outputs.
- Construct **Lagrangian** $\rightarrow$ **Saddle point algorithm**

$$\mathcal{L}(q, \lambda, \eta) = J(q) + \lambda^T(v - \overline{v}) + \eta^T(\underline{q}_h - q)$$



# Control design with hard constraints

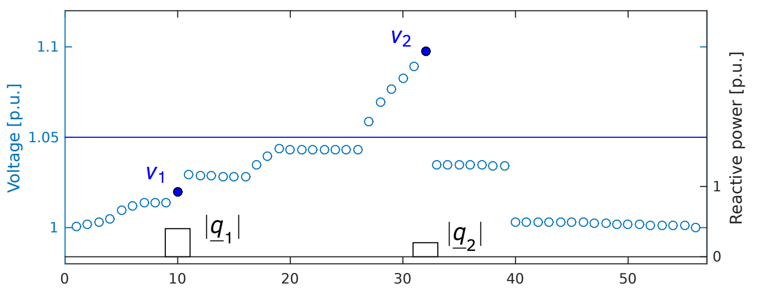



$$\lambda_h \leftarrow [\lambda_h + \alpha(v_h - \overline{v})]_{\geq 0}$$
$$\eta_h \leftarrow [\eta_h + \beta(\underline{q}_h - q_h)]_{\geq 0}$$
$$q \leftarrow q - \gamma \nabla J(q) - \lambda - \tilde{L}\eta$$

$-\tilde{L}\eta$ **Diffusion term** that requires **nearest-neighbor communication**.

$\rightarrow$ Bolognani, Carli, Cavraro & Zampieri, IEEE TAC (2015)

# Simulations and comparison

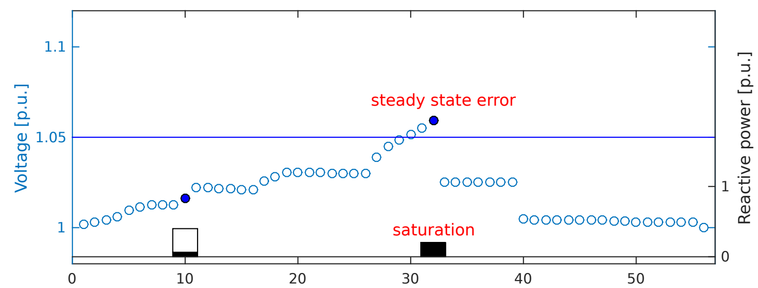


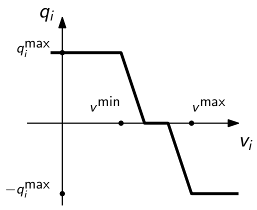

Modified IEEE 123 Distribution Test Feeder → `github`
Light load + 2 microgenerators → **overvoltage**

2 sets of constraints: **voltage limits** $v_h \leq \bar{v}$ ; **max reactive power** $\underline{q}_h \leq q_h$



# Simulations and comparison



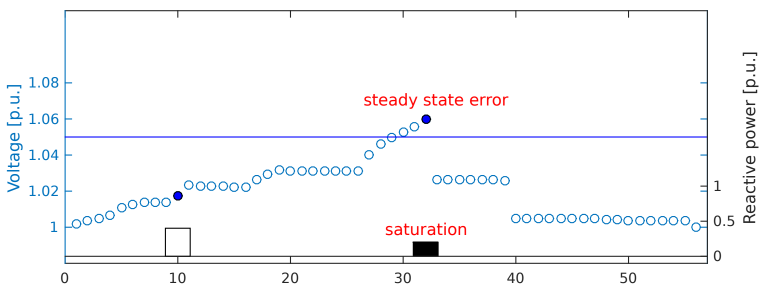

$$q_h(t) = -f(v_h(t))$$

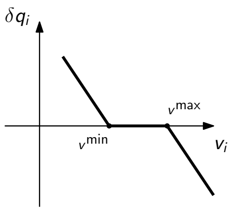
Fully decentralized, proportional controller.

*Latest grid code drafts, Vovos (2007), Turitsyn (2011), Aliprantis (2013), ...*



# Simulations and comparison



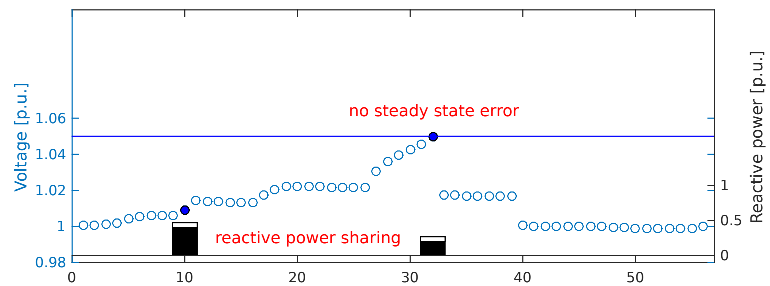

$$q_h(t+1) = q_h(t) - f(v_h(t))$$

Fully decentralized, integral controller.

*Zero steady error without saturation limits. Li (2014)*



# Simulations and comparison




$$\lambda_h \leftarrow [\lambda_h + \alpha(v_h - \bar{v})]_{\geq 0}$$
$$\eta_h \leftarrow [\eta_h + \beta(\underline{q}_h - q_h)]_{\geq 0}$$
$$q \leftarrow q - \gamma \nabla J(q) - \lambda - \tilde{L}\eta$$

Networked feedback control **(neighbor-to-neighbor async communication)**

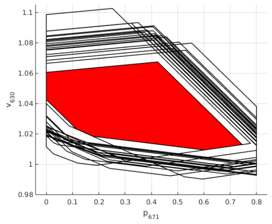
→ Cavraro, Bolognani, Carli & Zampieri, IEEE CDC (2016)

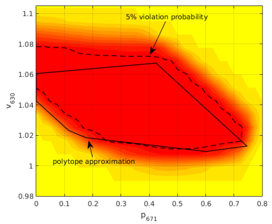
# Chance constraints on the state

**Chance-constrained decision**

$$\min_{\text{input } \delta} \quad J(\delta)$$
$$\text{subject to} \quad \text{Prob}\left[x \notin \mathcal{X}_c\right] < \epsilon$$

- $\mathcal{X}_c$ can encode
  - under/over voltage limits
  - power injection limits
  - voltage stability region
    → Bolognani & Zampieri, IEEE TPS (2015)
- A **stochastic model** for the disturbance is available
- Via **linear approximant** → **deterministic polytope constraints**




→ Bolognani & Dörfler, PSCC (2016)

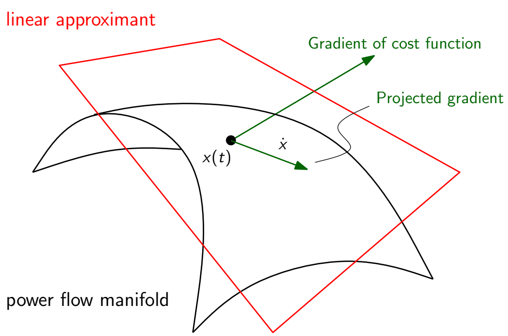


## NEXT STEP

## OPTIMIZATION ON THE POWER FLOW MANIFOLD



# Optimization on the power flow manifold

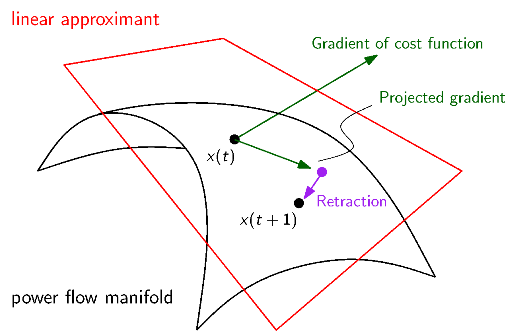



**Continuous time trajectory on the manifold:**

1. $\nabla J(x)$: gradient of the cost function **(soft constraints)** in ambient space
2. $\Pi_x \nabla J(x)$: projection of the gradient on the linear approximant in $x$
3. Evolve according to $\dot{x} = -\gamma \Pi_x \nabla J(x)$



# Staying on the power flow manifold




$$x = \begin{bmatrix} x_{\text{exo}} \\ x_{\text{endo}} \end{bmatrix}$$

**Exogenous variables**
Inputs/disturbances that are imposed on the model.
*Reactive power injection $q_i$*

**Endogenous variables**
Determined by the physics of the grid.
*Voltage $v_i$*

**Iterative algorithm:** at each step

1. Compute $\Pi_x \nabla J(x)$ (sparse $A_{x(t)}$ ⇒ distributed algorithm)
2. Actuate system based on $\delta x = -\gamma \Pi_x \nabla J$ (exogeneous variables / inputs)
3. Retraction step $x(t+1) = R_{x(t)}(\delta x)$ $\quad \Rightarrow x(t+1) \in \mathcal{M}$.

From **iterative optimization algorithm** to **feedback control on manifolds**.

# Hard constraints on exogenous variables

**Feasible input region**

- Can be enforced via **saturation** of the corresponding coordinates
- Primal feasibility at all times
- The resulting **feasible input region** is invariant with respect to the retraction.
  - We can saturate $\delta x = -\gamma \Pi_x \nabla J(x)$ because

$$x + \delta(x) \in \mathcal{F} \quad \Rightarrow \quad x(t+1) = R_{x(t)}(\delta x) \in \mathcal{F}$$

$\rightarrow$ **Geometric Projected Dynamical Systems**

- Extension of results on existence and uniqueness of executions for hybrid automata to manifolds
- Guarantees of no Zeno execution

Ongoing work with Adrian Hauswirth, Gabriela Hug, Florian Dörfler.



# Hard constraints on endogeneous variables

**Operational constraints**

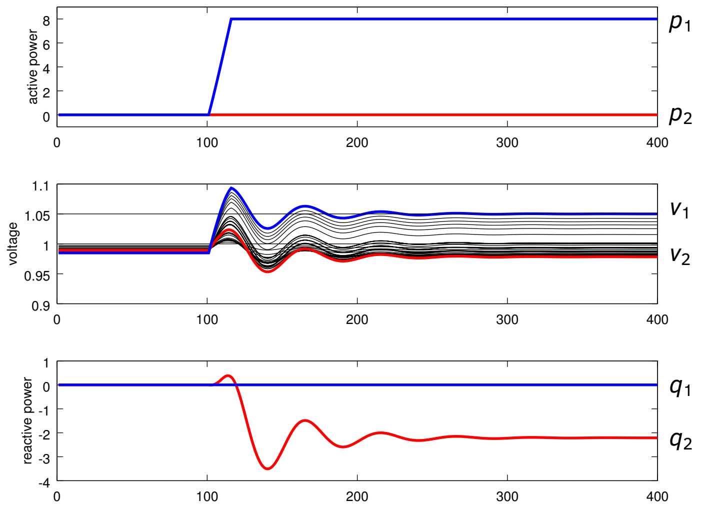
- Barrier functions not suitable:
  - Backtracking line search is not possible in closed loop
  - Primal feasibility cannot be guaranteed during tracking
- Time-varying penalty functions not suitable:
  - Persistent feedback control for tracking
- Can be tackled via **dualization** / Lagrangian approach.
- The corresponding **operational constraints** are satisfied at steady state, despite model uncertainty.

$\rightarrow$ **Saddle/primal-dual algorithm on manifolds**

Ongoing work with Adrian Hauswirth, Gabriela Hug, Florian Dörfler.



# Optimization on the power flow manifold



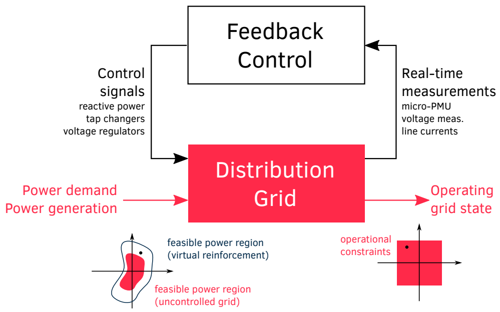



**CONCLUSIONS**

# Conclusions

A power system problem for control theory tools!

- **A tractable model**
  - implicit linear
  - sparse
  - structure preserving
- **Output feedback in power systems**
  - model-free
  - robust
  - limited measurement
- **Networked control**
  - co-design?
- **Feedback control on the power flow manifold**
  - exploit the physics of the system in the loop




An optimization-on-manifold approach to the design of distributed feedback control in smart grids — Saverio Bolognani
Conclusions

Thanks!

**Saverio Bolognani**
bsaverio@ethz.ch

ifa ETH zürich