

# Grid-forming Control for Power Converters based on Matching of Synchronous Machines <sup>★</sup>

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## Abstract

We consider the problem of grid-forming control of power converters in low-inertia power systems. Starting from an average-switch three-phase inverter model, we draw parallels to a synchronous machine (SM) model and propose a novel grid-forming converter control strategy which dwells upon the main characteristic of a SM: the presence of an internal rotating magnetic field. In particular, we augment the converter system with a virtual oscillator whose frequency is driven by the DC-side voltage measurement and which sets the converter pulse-width-modulation signal, thereby achieving exact matching between the converter in closed-loop and the SM dynamics. We then provide a sufficient condition assuring existence, uniqueness, and global asymptotic stability of equilibria in a coordinate frame attached to the virtual oscillator angle. By actuating the DC-side input of the converter we are able to enforce this sufficient condition. In the same setting, we highlight strict incremental passivity, droop, and power-sharing properties of the proposed framework, which are compatible with conventional requirements of power system operation. We subsequently adopt disturbance decoupling techniques to design additional control loops that regulate the DC-side voltage, as well as AC-side frequency and amplitude, while in the end validating them with numerical experiments.

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## 1 Introduction

The electrical power system is currently undergoing significant changes in its structure and mode of operation due to a major shift in generation technology from synchronous machines (SMs) to power electronics-based DC/AC converters, or simply *inverters*. As opposed to SMs, which store kinetic energy in their rotational inertia, these devices are on the one hand designed with little or no built-in energy storage capacity, while on the other hand actuated at much faster time scales. Large SMs with their rotational inertia, their self-synchronizing physics, and their controls act as safeguards against faults and disturbances – all of which are absent in *low-inertia systems* with a dominant share of distributed and variable renewable sources interfaced

through inverters. Hence, the proper control of inverters is regarded as one of the key challenges when massively integrating renewable energy sources (Kroposki et al., 2017; Taylor et al., 2016; Denis et al., 2015).

Converter control strategies are classified in two groups. While there is no universally accepted definition, converters are usually termed *grid-following* if their controls are designed for a stiff grid, and they deliver power at the stiff AC grid frequency usually measured through a phase-locked loop (PLL). Otherwise, converters are termed *grid-forming* if they are assigned to interact with a non-stiff grid similarly as SMs do by balancing kinetic and electrical energy such that a frequency consensus is achieved. A low-inertia system cannot be operated with only grid-following units. With this in mind, we review the literature on grid-forming control.

The inherent self-synchronizing behavior of SMs has inspired controllers such as *droop* and *virtual synchronous machines* (VSMs) (Torres and Lopes, 2013; Karapanos et al., 2011; Van Wesenbeeck et al., 2009; D’Arco and Suul, 2013; Chen et al., 2011; Zhong and Weiss, 2011). These controllers are designed to emulate the behavior of SM models of different degree of fidelity and are based on measurements of AC quantities such as injected power, frequency, and amplitude. For example, *inverse droop* and related VSM control strategies measure the AC frequency through a PLL and accordingly adapt the con-

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verter power injection based on a simple SM swing equation model. The latter is encoded in a micro-controller whose outputs are tracked by the converter modulation signal typically through a cascaded control architecture. For these and other VSM implementations, the time delays resulting from measuring and processing AC quantities render control often ineffective (ENTSO-E, 2016; Bevrani et al., 2014; Denis et al., 2015). Droop control can also be implemented by measuring the injected power and accordingly adapting the converter frequency (Guerrero et al., 2013), but its applicability is limited to inductive grids and with a possibly narrow region of attraction (Sinha et al., 2015; Dörfler et al., 2016; De Persis and Monshizadeh, 2015). Additionally, the inverter’s DC-side storage element is often not included in the model, nor in the control design, which, as far as we can tell, misses a key insight: namely, the DC bus voltage reflects the power imbalance and serves as valuable control signal. Finally, alternative control strategies employ *nonlinear virtual oscillators* fed by AC current measurements (Sinha et al., 2015; Colombino et al., 2017). For these strategies global stability certificates are known, but their design and analysis is quite involved (as a result, no controllers for regulation of amplitudes and frequency is known thus far) and their compatibility with SMs is unclear to this date.

Another set of literature relevant to our methodology is *passivity-based control* (PBC) (Van Der Schaft, 2000) and *interconnection and damping assignment* (IDA) (Ortega and Garca-Canseco, 2004). Their application to DC/DC converters (Escobar et al., 1999; Zonetti et al., 2014), AC/DC converters (Perez et al., 2004), and power systems in general (Caliskan and Tabuada, 2014; Fiaz et al., 2013) suggests a physically insightful analysis based on energy dissipation and shaping. As we will further see, our analysis relies also on a characterization of the power system steady-state specification (Groß et al., 2016; Groß and Dörfler, 2017) which restricts the class of admissible controllers.

Our main investigation is three-fold. First, we propose a novel grid-forming control strategy that matches the electromechanical energy exchange pattern in SMs. This is achieved by augmenting the converter dynamics with an internal model of a harmonic oscillator whose frequency takes the value of the DC-side voltage measurement. This voltage-driven oscillator is then assigned to drive the converter’s pulse-width-modulation cycle, thereby assuring that the closed-loop converter dynamics exactly *match* the SM dynamics, whereas the DC voltage serves as the key control and imbalance signal akin to the SM’s angular velocity. Based on a Lyapunov approach we provide a sufficient condition certifying existence, uniqueness, and global asymptotic stability of equilibria in a coordinate frame attached to the virtual oscillator angle. By actuating the DC-side input current we are able to satisfy this condition. We also demonstrate strict incremental passivity, droop,

and power-sharing properties of the closed-loop system. Our approach is grounded in foundational control methods, while being systematically extensible to PBC and IDA designs. Additionally, the key DC voltage signal is readily available while all other approaches rely on extensive processing of the AC measurements. Second, building on the proposed matching controller, we further design overarching control loops that regulate the DC voltage, AC frequency, and AC amplitude. This is done by pursuing an approach based on disturbance decoupling, which performs asymptotic output voltage amplitude tracking while rejecting the load current seen as a measurable disturbance. We then suggest extensions based on employing PBC and voltage-power droop control strategies, which have been previously investigated in various settings. Third and finally, we validate the performance and robustness of our designs by comparing them in numerical experiments of single and multi-converter scenarios.

The remainder of the paper is organized as follows. Section 2 introduces the models and the control objectives. Section 3 proposes the matching controller and derives its properties. Section 4 designs the regulation and disturbance rejection controllers. Section 5 presents our numerical case study, and Section 6 concludes the paper.

## 2 Three-Phase Converter Model, Synchronous Machine Model, & their Analogies

### 2.1 Preliminaries on AC and DC signal spaces

We consider three-phase AC quantities  $z = (z_a, z_b, z_c) \in \mathbb{R}^3$  that are assumed to be balanced:  $z_a + z_b + z_c = 0$ . We denote by  $z_{\alpha\beta} \in \mathbb{R}^2$  the restriction of  $z$  to the plane orthogonal to  $[1 \ 1 \ 1]$  and omit the third so-called zero sequence component, denoted by  $z_\gamma \in \mathbb{R}$ ; see Remark 1.

We define the desired power system steady state as in (Groß et al., 2016; Groß and Dörfler, 2017): a DC signal  $z_{dc}^* \in \mathbb{R}$  is said to be in steady-state when it satisfies  $\dot{z}_{dc}^* = 0$ ; an AC signal  $z_{\alpha\beta}^* \in \mathbb{R}^2$  is said to be in (synchronous and balanced) steady state when it satisfies  $\dot{z}_{\alpha\beta}^* = \omega^* \mathbf{J} z_{\alpha\beta}^*$  for some nonzero frequency  $\omega^* \in \mathbb{R}$ . Here  $\mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  is the identity and  $\mathbf{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$  is a rotation matrix analogous to the imaginary unit  $\sqrt{-1}$  when working in complex rather than  $\alpha\beta$ -coordinates. In this paper,  $\|\cdot\|$  denotes the standard Euclidean norm.

Given an AC signal  $z_{\alpha\beta} \in \mathbb{R}^2$  in  $\alpha\beta$ -frame and an angle  $\theta \in \mathbb{S}^1$ , we define  $z_{dq} \in \mathbb{R}^2$  in  $dq$ -frame as

$$z_{dq} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} z_{\alpha\beta}. \quad (1)$$

Observe that a steady-state AC signal  $\dot{z}_{\alpha\beta}^* = \omega^* \mathbf{J} z_{\alpha\beta}^*$  in  $\alpha\beta$ -frame with synchronous frequency  $\omega^*$  is mapped to an equilibrium  $\dot{z}_{dq}^* = 0$  in a  $dq$ -frame with appropriate transformation angle  $\theta^* = \omega^* t + \phi$  for some  $\phi \in \mathbb{S}^1$ .

Throughout this article, a signal  $z_{ref}$  denotes a user-defined set-point, whereas  $z_{dq}^*$  or  $z_{\alpha\beta}^*$  will denote a steady state induced by external inputs, e.g., unknown loads.

## 2.2 Three-Phase DC/AC Converter Model

We consider a standard average-switch model of the three-phase inverter, represented by a continuous-time system whose main nonlinearity is contained in the modulation block, as depicted in Figure 1. See (Tabesh and Iravani, 2009) for a more in-depth overview.

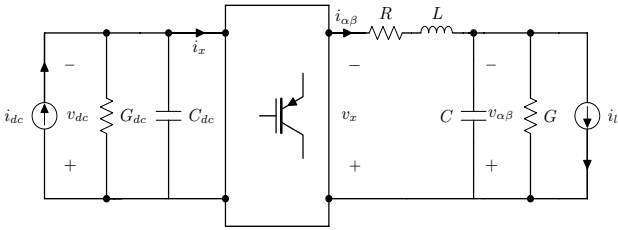


Fig. 1. Circuit diagram of a 3-phase DC/AC converter

The DC circuit consists of a constant current source  $i_{dc} > 0$  in parallel with a capacitance  $C_{dc} > 0$  and a parasitic conductance  $G_{dc} > 0$ . The DC current taken by the switching block is denoted by  $i_x \in \mathbb{R}$ , while  $v_{dc} \in \mathbb{R}$  represents the voltage across the DC capacitance.

The AC circuit contains at each phase an inductance  $L > 0$  in series with a resistance  $R > 0$  connected to a shunt capacitance  $C > 0$ . Here  $v_{\alpha\beta} \in \mathbb{R}^2$  denotes the AC voltage across the output capacitor. The load is represented by a shunt conductance  $G > 0$  at each phase together with the AC load current,  $i_l \in \mathbb{R}^2$ . Furthermore,  $i_{\alpha\beta} \in \mathbb{R}^2$  denotes the AC current in the inductors and  $v_x \in \mathbb{R}^2$  the average AC voltage at the switching node.

The switching block represents an averaged model of a 6-switch 2-level inverter which converts DC voltage into AC voltage according to a complementary pulse-width-modulation (PWM) carrier and a modulation signal  $m_{\alpha\beta} \in \mathbb{R}^2$ . For the time scales of interest, we assume a sufficiently high switching frequency which allows us to discard the PWM carrier harmonics. Due to the converter topology, the switching block defines the identities

$$i_x = \frac{1}{2} m_{\alpha\beta}^\top i_{\alpha\beta} ; v_x = \frac{1}{2} m_{\alpha\beta} v_{dc} ,$$

where  $m_{\alpha\beta} \in \mathbb{R}^2$  is the modulation signal in  $\alpha\beta$ -frame, corresponding to the average of the converter's PWM signals over one switching period and is defined such that its components take values inside the unit disk.

By putting it all together, our three-phase DC/AC converter model can be written as

$$C_{dc} \dot{v}_{dc} = -G_{dc} v_{dc} + i_{dc} - \frac{1}{2} i_{\alpha\beta}^\top m_{\alpha\beta} \quad (2a)$$

$$L \dot{i}_{\alpha\beta} = -R i_{\alpha\beta} - v_{\alpha\beta} + \frac{1}{2} v_{dc} m_{\alpha\beta} \quad (2b)$$

$$C \dot{v}_{\alpha\beta} = -G v_{\alpha\beta} - i_l + i_{\alpha\beta} . \quad (2c)$$

**Remark 1 (Zero sequence)** We construct the modulation signal such that  $m_\gamma = 0$  which implies that  $v_{x,\gamma} = 0$ . For a balanced load, it also holds that  $i_{l,\gamma} = 0$ . We are left with the following dynamics for the  $\gamma$ -subsystem:

$$L \dot{i}_\gamma = -R i_\gamma - v_\gamma \quad (3a)$$

$$C \dot{v}_\gamma = -G v_\gamma + i_\gamma . \quad (3b)$$

Since (3) is an asymptotically stable linear system, the omission of the  $\gamma$ -component is well justified.  $\square$

## 2.3 Control objectives

In this section, we formalize the control objectives to be achieved via the two main actuation inputs, the modulation signal  $m_{\alpha\beta}$  and the DC-side current injection  $i_{dc}$ . Broadly speaking we require the following:

(i) *Grid-forming*: The objective of *grid-forming control* is best defined by mimicking the electromechanical interaction of a SM with the grid rather than prescribing the converter modulation frequency to the grid frequency, e.g., via a PLL. The synchronization properties of SMs rely on a particular kinetic to electrical energy exchange pattern. This can be induced in the DC/AC converter by exactly matching the SM's dynamics.

(ii) *Stable steady states*: We intend to stabilize the DC signal  $v_{dc}$  and AC signals  $i_{\alpha\beta}$ ,  $v_{\alpha\beta}$  to synchronous steady states which may depend on the converter parameters, load demands, and control gains, as characterized in (Groß et al., 2016; Groß and Dörfler, 2017).

(iii) *Strict incremental passivity*: We aim to induce strict incremental passivity (Van Der Schaft, 2000) with respect to the AC and DC ports,  $u = (i_{dc}, i_l)$  and  $y = (v_{dc}, v_{dq})$ , and with respect to a desired steady-state solution  $z^* = (v_{dc}^*, i_{dq}^*, v_{dq}^*)$ . More precisely, we seek a positive definite and continuously differentiable storage function  $\mathcal{V}(z - z^*)$ , in appropriate coordinates, whose derivative along trajectories of the closed-loop system satisfies

$$\dot{\mathcal{V}}(z - z^*) \leq -W(z - z^*) + (u - u^*)^\top (y - y^*) ,$$

where  $W$  is positive definite. This passivity property is considered a key local certificate to ensure stability in

an interconnected power grid (Fiaz et al., 2013; Caliskan and Tabuada, 2014; De Persis and Monshizadeh, 2015).

(iv) *Regulation and disturbance rejection*: Independently of the load parameters  $i_l$  and  $G$ , we aim to regulate the steady-state frequency  $\omega^*$  of all AC quantities as well as the steady-state amplitudes of DC and AC voltages  $v_{dc}^*$  and  $\|v_{\alpha\beta}^*\|$  to reference values  $\omega_{ref} > 0$ ,  $v_{dc,ref} > 0$ , and  $r_{ref} > 0$ .

(v) *Droop compatibility*: In a neighbourhood of the desired steady-state solution  $z^*$ , we aim to achieve a linear *droop* characteristic between the converter modulation frequency and its power output as:  $\omega - \omega_{ref} = d(P - P_{ref})$ , where  $d > 0$  is the droop gain to be specified,  $P_{ref} > 0$  is a power injection set-point and  $P$  is the delivered active power. Such a local droop behavior is known to guarantee power sharing and compatibility with other droop-like controllers in a power system (Sinha et al., 2015; Dörfler et al., 2016).

In the sequel, we will further specify these objectives (in more suitable coordinates) and also consider alternative objectives such as voltage amplitude droop control.

#### 2.4 The Synchronous Machine Model

In what follows, we consider a SM model which lends itself useful in designing the matching controller. We consider a single-pole-pair, non-salient rotor, SM under constant excitation, defined in  $\alpha\beta$ -frame as in (Caliskan and Tabuada, 2014), together with a capacitor at its AC terminal, and described by the state-space model

$$\dot{\theta} = \omega \quad (4a)$$

$$M\dot{\omega} = -D\omega + \tau_m + i_{\alpha\beta}^\top L_m i_f \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \end{bmatrix} \quad (4b)$$

$$L_s \dot{i}_{\alpha\beta} = -Ri_{\alpha\beta} - v_{\alpha\beta} - \omega_v L_m i_f \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \end{bmatrix} \quad (4c)$$

$$C\dot{v}_{\alpha\beta} = -Gv_{\alpha\beta} - i_l + i_{\alpha\beta}. \quad (4d)$$

Here,  $M > 0$  and  $D > 0$  are the rotor inertia and damping coefficients,  $\tau_m$  is the driving mechanical torque,  $L_m > 0$  is the stator-to-rotor mutual inductance,  $L_s > 0$  the stator inductance. We denote the rotor angle by  $\theta \in \mathbb{S}^1$ , its angular velocity by  $\omega \in \mathbb{R}$ , the magnetic flux in the stator winding by  $\lambda_{\alpha\beta} \in \mathbb{R}^2$ , and the stator resistance by  $R > 0$ . At its terminals the SM is interfaced to the grid through a shunt capacitor with capacitance  $C > 0$  and capacitor voltage  $v_{\alpha\beta} \in \mathbb{R}^2$ , a constant load conductance  $G > 0$ , and the load current extraction denoted by  $i_l \in \mathbb{R}^2$ . The strength of the rotating magnetic flux inside the SM (4) is given by the rotor current  $i_f$  which is assumed to be regulated to a constant value, as in (Caliskan and Tabuada, 2014; Groß et al., 2016).

Observe the similarities between the inverter model (2) and the SM model (4). The DC capacitor is analogous to the rotor moment of inertia, while the electrical torque and the electromotive force (EMF) (the rightmost terms in (4b) and (4c)) play the same role as  $i_x$  and  $v_x$ . The self-synchronizing properties of a multi-machine power system are attributed to exchange of kinetic and electrical energy through the electrical torque and the EMF. In the following section, we will assign this very mechanism for the inverter dynamics (2).

### 3 Grid-Forming SM Matching Control

From Groß and Dörfler (2017), we know that every converter modulation controller inducing a synchronous, balanced, and sinusoidal steady state must necessarily include an *internal model* of an oscillator of the form  $\dot{m}_{\alpha\beta}^* = \omega^* \mathbf{J} m_{\alpha\beta}^*$ . Thus, the first step in our design is to assign a sinusoidal modulation scheme parameterized in polar coordinates by means of a virtual angle  $\theta_v \in \mathbb{S}^1$  as

$$\dot{\theta}_v = \omega_v ; \quad m_{\alpha\beta} = \mu \begin{bmatrix} -\sin(\theta_v) \\ \cos(\theta_v) \end{bmatrix}, \quad (5a)$$

where  $\omega_v \in \mathbb{R}$  and  $\mu \in ]0, 1]$  are the assignable modulation frequency and amplitude to be specified later. In the next step, we design a grid-forming modulation controller by *matching* the converter model (2) augmented with the internal model control loop (5a) to the SM model (4). By visual inspection we observe that model matching is achieved by dynamic feedback

$$\omega_v = \eta v_{dc} \quad (5b)$$

where the constant  $\eta = \omega_{ref}/v_{dc,ref} > 0$  encodes the ratio between the nominal AC frequency  $\omega_{ref}$  and the DC voltage reference  $v_{dc,ref}$ . All subsequent developments will be based on the matching control (5).

**Remark 2 (Equivalent SM interpretation)** *In the following, we highlight the similarities between the generator model (4) and the converter model (2) under the control scheme (5). By writing  $i_x$  and  $v_x$  as*

$$i_x = \frac{\mu}{2} i_{\alpha\beta}^\top \begin{bmatrix} -\sin(\theta_v) \\ \cos(\theta_v) \end{bmatrix} ; \quad v_x = \frac{\mu}{2} v_{dc} \begin{bmatrix} -\sin(\theta_v) \\ \cos(\theta_v) \end{bmatrix} \quad (6)$$

*we identify the AC-side switch voltage  $v_x$  with an equivalent EMF by choosing the modulation amplitude as  $\mu = -2\eta L_m i_f$ . We also identify the DC-side switch current  $i_x$  with the equivalent torque*

$$\tau_e = \frac{1}{2\eta} i_{\alpha\beta}^\top \mu \begin{bmatrix} -\sin(\theta_v) \\ \cos(\theta_v) \end{bmatrix} = \frac{1}{\eta} i_x. \quad (7)$$

Finally, by defining the equivalent angular velocity as  $\omega_v = \eta v_{dc}$  we rewrite the closed loop (2), (5) as the equivalent SM

$$\dot{\theta}_v = \omega_v \quad (8a)$$

$$\frac{C_{dc}}{\eta^2} \dot{\omega}_v = -\frac{G_{dc}}{\eta^2} \omega_v + \frac{i_{dc}}{\eta} - \frac{1}{\eta} i_x \quad (8b)$$

$$L \dot{i}_{\alpha\beta} = -R i_{\alpha\beta} - v_{\alpha\beta} + \frac{1}{2\eta} \omega_v m_{\alpha\beta} \quad (8c)$$

$$C \dot{v}_{\alpha\beta} = -G l v_{\alpha\beta} + i_{\alpha\beta} - i_l, \quad (8d)$$

where we identify  $C_{dc}/\eta^2$ ,  $G_{dc}/\eta^2$ , and  $i_{dc}/\eta$  with the equivalent mechanical inertia damping, and mechanical driving torque.  $\square$

### 3.1 Closed-Loop Incremental Stability & Passivity

In this section, we show how the matching controller (5) induces desirable stability and passivity properties in an appropriate  $dq$ -frame and in incremental coordinates formulated with respect to an induced steady state. Consider now the closed-loop inverter dynamics (2), (5). By applying the  $dq$ -coordinate transformation (1) with angle  $\theta_v$  to  $i_{\alpha\beta}$  and  $v_{\alpha\beta}$ , we arrive at the following subsystem, which is independent of the angle state variable:

$$C_{dc} \dot{v}_{dc} = -G_{dc} v_{dc} + i_{dc} - \frac{\mu}{2} \begin{bmatrix} 0 \\ 1 \end{bmatrix}^\top i_{dq} \quad (9a)$$

$$L \dot{i}_{dq} = -(R\mathbf{I} + \omega_v L\mathbf{J}) i_{dq} + \frac{\mu}{2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_{dc} - v_{dq} \quad (9b)$$

$$C \dot{v}_{dq} = -(G\mathbf{I} + \omega_v C\mathbf{J}) v_{dq} - i_{l,dq} + i_{dq}. \quad (9c)$$

The following result characterizes the strict incremental passivity and stability properties of the  $dq$ -frame inverter model (9), with respect to a steady state solution.

**Theorem 3 (Closed-loop stability & strict passivity in  $dq$ -frame)** Consider the closed-loop inverter model (9) and assume that there exists steady state  $x^* = (v_{dc}^*, i_{dq}^*, v_{dq}^*)$  that satisfies

$$\frac{C^2 \|v_{dq}^*\|^2}{4G} + \frac{L^2 \|i_{dq}^*\|^2}{4R} < \frac{G_{dc}}{\eta^2}. \quad (10)$$

Then, the following statements hold:

- (1) the inverter model (9) is strictly incrementally passive with respect to the input  $(i_{dc} - i_{dc}^*, i_{l,dq} - i_{l,dq}^*)$  and the output  $(v_{dc} - v_{dc}^*, v_{dq} - v_{dq}^*)$ ;
- (2)  $x^*$  is the unique real-valued equilibrium; and
- (3) for stationary inputs  $(i_{dc}, i_{l,dq}) = (i_{dc}^*, i_{l,dq}^*)$ , the steady state  $x^*$  is globally asymptotically stable.

**PROOF.** Our proof is inspired by Caliskan and Tabuada (2014). Starting from the assumptions of the theorem, we define the error coordinates  $\tilde{v}_{dc} = v_{dc} - v_{dc}^*$ ,  $\tilde{\omega}_v = \eta v_{dc} - \eta v_{dc}^*$ ,  $\tilde{i}_{dq} = i_{dq} - i_{dq}^*$ ,  $\tilde{v}_{dq} = v_{dq} - v_{dq}^*$ ,  $\tilde{i}_{l,dq} = i_{l,dq} - i_{l,dq}^*$ ,  $\tilde{i}_{dc} = i_{dc} - i_{dc}^*$  such that the associated transient dynamics are expressed as

$$C_{dc} \dot{\tilde{v}}_{dc} = -G_{dc} \tilde{v}_{dc} + \tilde{i}_{dc} - \frac{\mu}{2} \begin{bmatrix} 0 \\ 1 \end{bmatrix}^\top \tilde{i}_{dq} \quad (11)$$

$$L \dot{\tilde{i}}_{dq} = -(R\mathbf{I} + \omega_v^* L\mathbf{J} + \tilde{\omega}_v L\mathbf{J}) \tilde{i}_{dq} - \tilde{\omega}_v L\mathbf{J} i_{dq}^* + \frac{\mu}{2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tilde{v}_{dc} - \tilde{v}_{dq}$$

$$C \dot{\tilde{v}}_{dq} = -(G\mathbf{I} + \omega_v^* C\mathbf{J} + \tilde{\omega}_v C\mathbf{J}) \tilde{v}_{dq} - \tilde{\omega}_v C\mathbf{J} v_{dq}^* + \tilde{i}_{dq} - \tilde{i}_{l,dq}$$

By considering the physical storage of the circuit elements, we define the incremental positive definite and differentiable storage function  $\mathcal{V}_1 : \mathbb{R}^5 \rightarrow \mathbb{R}_{>0}$  by

$$\mathcal{V}_1 = \frac{1}{2} C_{dc} \tilde{v}_{dc}^2 + \frac{1}{2} L \tilde{i}_{dq}^\top \tilde{i}_{dq} + \frac{1}{2} C \tilde{v}_{dq}^\top \tilde{v}_{dq}. \quad (12)$$

Due to skew symmetry of  $\mathbf{J}$ , the derivative of  $\mathcal{V}_1$  along the trajectories of the error system (11) evaluates to

$$\dot{\mathcal{V}}_1 = - \begin{bmatrix} \tilde{v}_{dc} & \tilde{i}_{dq}^\top & \tilde{v}_{dq}^\top \end{bmatrix} \mathcal{P} \begin{bmatrix} \tilde{v}_{dc} & \tilde{i}_{dq}^\top & \tilde{v}_{dq}^\top \end{bmatrix}^\top - \tilde{v}_{dq}^\top \tilde{i}_{l,dq} + \tilde{i}_{dc} \tilde{v}_{dc},$$

where the symmetric matrix  $\mathcal{P} \in \mathbb{R}^{5 \times 5}$  is given by

$$\mathcal{P} = \begin{bmatrix} G_{dc} & \frac{\eta L}{2} (\mathbf{J} i_{dq}^*)^\top & \frac{\eta C}{2} (\mathbf{J} v_{dq}^*)^\top \\ \frac{\eta L}{2} (\mathbf{J} i_{dq}^*) & R\mathbf{I} & 0 \\ \frac{\eta C}{2} (\mathbf{J} v_{dq}^*) & 0 & G\mathbf{I} \end{bmatrix} \quad (13)$$

By evaluating all leading principal minors of  $\mathcal{P}$  we see that under condition (10),  $\mathcal{P}$  becomes positive definite. Hence, system (11) is strictly passive with input  $(\tilde{i}_{dc}, -\tilde{i}_{l,dq})$  and output  $(\tilde{v}_{dc}, \tilde{v}_{dq})$ . Moreover, by Lyapunov's direct method, the origin of (11) is asymptotically stable if we assume zero incremental input signals. Since  $\mathcal{V}_1$  is radially unbounded, we obtain global asymptotic stability as well as the absence of any other equilibrium.  $\square$

The stability condition (10) requires sufficiently large damping in the AC and DC components of the converter. However, the parasitic resistances  $R$  and  $G_{dc}$  can be arbitrarily small in practice. To alleviate this shortage of stabilizing dissipation, we implement a DC-side actuation akin to governor speed droop control for generators to enforce condition (10). We design the current source  $i_{dc}$  as the proportional (P) controller

$$i_{dc} = i_{dc,ref} - K_p (v_{dc} - v_{dc,ref}), \quad (14)$$

where  $K_p > 0$  is a gain, while  $i_{dc,ref} > 0$  and  $v_{dc,ref} > 0$  are set-points for the DC current source and DC voltage, respectively. By the analogous reasoning leading to Theorem 3, we arrive at the following corollary.

**Corollary 4 (Closed-loop stability with DC-side P-control)** *Consider the inverter model (9) with P-controller (14) on the DC-side input. Assume there is a steady state  $x^* = (v_{dc}^*, i_{dq}^*, v_{dq}^*, i_{dc}^*, i_{l,dq}^*)$  satisfying*

$$\frac{C^2 \|v_{dq}^*\|^2}{4G} + \frac{L^2 \|i_{dq}^*\|^2}{4R} < \frac{G_{dc} + K_p}{\eta^2}. \quad (15)$$

*Then the augmented system (9), (14), with input  $i_{l,dq}$  and output  $v_{dq}$  is strictly incrementally passive. Moreover, for stationary inputs  $i_{l,dq} = i_{l,dq}^*$ , the steady state  $x^*$  is unique and globally asymptotically stable.*

We underscore that condition (15) can be met by suitable choice of gain  $K_p$ . We conclude this subsection with the following remarks. Since we have only performed P-control thus far, we cannot necessarily guarantee asymptotic regulation of  $v_{dc}$  to a particular  $v_{dc,ref}$ . Later in Section 4, we will specify under which conditions a desirable closed-loop steady state exists, as assumed earlier, when it is unique, and how to regulate  $v_{dc}$  to  $v_{dc,ref}$ .

The incremental passivity property highlighted in Theorem 3 and Corollary 4 is regarded as a key requirement for stability under interconnection, see (Fiaz et al., 2013; Caliskan and Tabuada, 2014), however this requires a single coordinate frame analysis for the networked scenario. Since in our work we use a  $dq$ -coordinate frame attached to a particular converter angle, the analysis does not pertain to a multi-inverter setup. Nevertheless, this property is preserved in all our subsequent developments. In what follows, we investigate the steady-state droop behavior of the closed loop (9), (14).

### 3.2 Droop properties of matching control

A key requirement for *plug-and-play* operation in power systems is power sharing amongst multiple inverters by means of a droop characteristic trading off power injection with the voltage amplitude and frequency (Dörfler et al., 2016). In this section, we investigate these properties for the closed-loop system (2), (5), (14).

Let  $r_x^* > 0$  and  $\omega_x^* > 0$  be the steady-state amplitude and frequency of the switching voltage. Following (Akagi et al., 1983), we define the active and reactive power at the load and switching node as

$$\begin{bmatrix} P_l^* \\ Q_l^* \end{bmatrix} = \begin{bmatrix} v_d^* & v_q^* \\ v_q^* & -v_d^* \end{bmatrix} \begin{bmatrix} i_{l,d}^* \\ i_{l,q}^* \end{bmatrix}, \quad \begin{bmatrix} P_x^* \\ Q_x^* \end{bmatrix} = \begin{bmatrix} i_d^* & i_q^* \\ i_q^* & -i_d^* \end{bmatrix} \begin{bmatrix} v_{x,d}^* \\ v_{x,q}^* \end{bmatrix}.$$

Two converters  $i, j$  are said to achieve *proportional power sharing* at a pre-defined ratio  $\rho > 0$  if  $P_{x,i}^*/P_{x,j}^* = \rho$ .

Finally, the linear sensitivity factors relating steady-state active power injection  $P_x^*$  to voltage amplitude  $r_x^*$  and frequency  $\omega_x^*$ , are represented here by the *droop coefficients*  $d_{r_x} = \partial P_x / \partial r_x$  and  $d_{\omega_x} = \partial P_x / \partial \omega_x$ . The following proposition puts these quantities into relation.

**Proposition 5 (Active/ Reactive Power Injections)** *Consider the inverter model (2) together with the matching controller (5) and the DC-side P-control (14). Assume that all DC and AC signals are in steady state with synchronous frequency  $\omega_x^* = \omega^*$ , and define  $i_0 = i_{dc,ref} + K_p v_{dc,ref}$ . The following statements hold:*

- (1) *Nose curves: the switching voltage amplitude and its frequency have the following expression as a function of exogenous inputs*

$$r_x^* = \frac{\mu}{4(G_{dc} + K_p)} \left( i_0 \pm \sqrt{i_0^2 - 4(G_{dc} + K_p)P_x^*} \right),$$

*and accordingly  $\omega_x^* = 2\eta r_x^* / \mu$ . Moreover, the reactive power  $Q_x^*$  is independent of  $(r_x^*, \omega_x^*)$ .*

- (2) *Droop behavior: Around an operating point  $(r_x^*, \omega_x^*)$ , the expression for the droop coefficients is given by:*

$$d_{r_x} = -\frac{8(G_{dc} + K_p)}{\mu^2} r_x^* + \frac{2i_0}{\mu}, \quad (16a)$$

$$d_{\omega_x} = -\frac{2(G_{dc} + K_p)}{\eta^2} \omega_x^* + \frac{i_0}{\eta}. \quad (16b)$$

- (3) *Power sharing: Consider any pair of converters  $i$  and  $j$ ,  $\{i, j\} \in \mathbb{N}$  with identical values of DC conductance  $G_{dc} = 0$ , DC voltage reference  $v_{dc,ref} > 0$  and control gain  $\eta > 0$ , the converters achieve proportional power sharing ratio  $\rho = P_{x,i}^*/P_{x,j}^*$  if*

$$K_{p,i} = \rho K_{p,j}, \quad i_{dc,ref,i} = \rho i_{dc,ref,j}, \quad (17)$$

*or equivalently if  $d_{\omega,i} = \rho d_{\omega,j}$  and  $P_{dc,i} = \rho P_{dc,j}$ .*

**PROOF.** To prove statement (1), consider the DC circuit dynamics (2a) at steady state, i.e., when  $\dot{v}_{dc} = 0$ :

$$0 = -(G_{dc} + K_p)v_{dc}^* + i_0 - i_x^*. \quad (18)$$

We multiply (18) by  $v_{dc}$  to obtain quadratic expression relating  $P_x = i_x^T v_{dc}$  and  $v_{dc}$ , at steady state:

$$v_{dc}^* = \frac{i_0 \pm \sqrt{i_0^2 - 4(G_{dc} + K_p)P_x^*}}{2(G_{dc} + K_p)}. \quad (19)$$

Note that the amplitude  $r_x^*$  and frequency  $\omega_x^*$  at the switching node can be expressed as

$$r_x^* = \frac{1}{2}\mu v_{dc}^* ; \omega_x^* = \eta v_{dc}^*. \quad (20)$$

The claimed nose curves follow directly from (19), (20). We then use (20) in (19) to obtain the steady-state active power  $P_x^*$  as

$$\begin{aligned} P_x^* &= \frac{-4(G_{dc} + K_p)}{\mu^2} r_x^{*2} + \frac{2i_0 r_x^*}{\mu} \\ &= \frac{-(G_{dc} + K_p)}{\eta^2} \omega_x^{*2} + \frac{i_0}{\eta} \omega_x^*. \end{aligned} \quad (21)$$

By linearizing the above equations around the operating point  $(r_x^*, \omega_x^*)$ , we find the droop slopes in (16). Finally, the proportional power sharing ratio  $\rho > 0$  between two converters  $i$  and  $j$  is given by

$$\rho = \frac{P_{x,i}^*}{P_{x,j}^*} = \frac{\frac{-K_{p,i}}{\eta^2} \omega_x^* + \frac{i_{0,i}}{\eta}}{\frac{-K_{p,j}}{\eta^2} \omega_x^* + \frac{i_{0,j}}{\eta}}, \quad (22)$$

where we replaced  $P_{x,i}^*$  and  $P_{x,j}^*$  by the expressions in (21) with  $G_{dc} = 0$  and identical values for  $v_{dc,ref} > 0$  and  $\eta > 0$ . The latter equality is satisfied if (17) holds.  $\square$

The following remarks are in order: Statement (1) gives two solutions for the voltage amplitude  $r_x^*$ . Among these two, the so-called *high-voltage solution* (with the plus sign) is the practically relevant operating point. From statement (1), we can also deduce that the maximal active power  $\bar{P}_x$  that can be delivered at the switching node,  $\bar{P}_x = i_0^2 / (4(G_{dc} + K_p))$ , is naturally constrained by the maximal DC power. Finally, the power sharing conditions (17) are perfectly analogous to the ones in conventional droop control (Dörfler et al., 2016): the droop slopes and the power set-points must respect the same ratio  $\rho$ . We remark that similar expressions can be obtained for a non-zero conductance.

### 3.3 Relation to Other Converter Control Strategies

Our matching control can be understood from the viewpoint of PBC by writing the inverter (2) as the Port-Hamiltonian system (Van Der Schaft, 2000)

$$\dot{z} = [\mathcal{J}(m) - \mathcal{D}] \nabla H(z) + \mathcal{G}w,$$

where  $z = (z_1, z_2, z_3) = (C_{dc}v_{dc}, Li_{\alpha\beta}, Cv_{\alpha\beta})$  is the state,  $m$  is the modulation,  $w = (i_{dc}, -i)$  is an exogenous input,  $H(z) = \frac{1}{2}C_{dc}^{-1}z_1^2 + \frac{1}{2}z_2^\top L^{-1}z_2 + \frac{1}{2}z_3^\top C^{-1}z_3$  is the physical energy, as in (12), and the skew-symmetric

interconnection matrix, positive definite damping matrix, and input matrix are given, respectively, by

$$\mathcal{J}(m) = \begin{bmatrix} 0 & -\frac{1}{2}m^\top & 0 \\ \frac{1}{2}m & 0 & -\mathbf{I} \\ 0 & \mathbf{I} & 0 \end{bmatrix}, \mathcal{D} = \begin{bmatrix} G_{dc} & 0 & 0 \\ 0 & R\mathbf{I} & 0 \\ 0 & 0 & G\mathbf{I} \end{bmatrix}, \mathcal{G} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & \mathbf{I} \end{bmatrix}.$$

The Port-Hamiltonian structure is preserved upon augmenting the inverter with the internal model (5a). On this ground, we can link our approach to that of PBC and IDA-based matching control (Ortega and Garcia-Canese, 2004). In particular, matching controller (5) together with P-controller (14) can be understood as IDA reshaping the  $\mathcal{J}$  and  $\mathcal{D}$  matrices.

Our control strategy is also reminiscent of oscillator-based controller methods. By defining  $m \in \mathbb{R}^2$  as the controller state, we can rewrite (5) as

$$\dot{m} = \omega_v \mathbf{J}m,$$

i.e, the matching control (5) is an oscillator with constant amplitude  $\|m(0)\| = \mu$  and state-dependent frequency  $\omega_v = \eta v_{dc}$  as feedback for the converter dynamics (2). This control strategy resembles the classic *proportional resonant control* (Teodorescu et al., 2006) with the difference that the frequency of the oscillator (3.3) adapts to the DC voltage which again reflects the grid state. Another related control strategy is *virtual oscillator control* encoding the inverter terminal dynamics as a nonlinear limit cycle oscillator adapting to the grid state (Johnson et al., 2014; Sinha et al., 2015; Colombino et al., 2017).

## 4 Voltage and Frequency Regulation

Starting from the model-matching controller (5), we now look to design outer control loops for the current source  $i_{dc}$  as well as the modulation amplitude  $\mu$  with the aim of tracking a given constant reference initially for the DC capacitor voltage and then also for the AC capacitor voltage amplitude. In what follows, we consider the closed-loop model (9) in  $dq$ -frame, and design controllers for a setup which has as load model a constant conductance  $G > 0$  as well as a constant (in  $dq$ -frame) load current  $i_{l,dq} \in \mathbb{R}^2$ . Arguably, a constant load current, as it appears here, is a compromised load model for many reasons, nevertheless, it can still be used to account for disturbances beyond linear conductance loads or uncertainties. Finally, since the current drawn by the load  $i_{l,dq}$  is actually measurable (ditto in the non-constant case), it is a useful signal for disturbance rejection, see Corollary 8.

### 4.1 Exact Frequency Regulation via Integral Control

In some scenarios, e.g., in islanded microgrids, it is desirable that inverters also contribute to frequency regulation (usually called secondary control) rather than mere

droop control. Inspired by frequency regulation of SMS via governor control, i.e., controlling the torque in (4) as a function of the frequency, we propose a frequency regulation strategy by pairing the passive inputs and outputs,  $\tilde{i}_{dc} = i_{dc} - i_{dc,ref}$  and  $\tilde{v}_{dc} = v_{dc} - v_{dc,ref}$ , respectively, in the inverter model (9) to track a reference frequency  $\omega_{ref} = \eta v_{dc,ref}$ . We adopt the PID controller

$$i_{dc} = i_{dc,ref} - K_p \tilde{v}_{dc} - K_i \int_0^t \tilde{v}_{dc}(\tau) d\tau - K_d \dot{\tilde{v}}_{dc}, \quad (23)$$

where  $i_{dc,ref} > 0$  and  $K_p, K_i, K_d > 0$  are control references and gains. For a constant current load  $i_{l,dq} = i_{l,dq}^*$ , the converter model (9) together with the control (23) can be expressed in  $dq$ -frame error coordinates, after introducing the integral term  $\xi = \int_0^t \tilde{v}_{dc}(\tau) d\tau$  with  $\xi(0) = 0$  and steady-state value  $\xi^*$ , as

$$\begin{aligned} \dot{\xi} &= \tilde{v}_{dc} \\ (C_{dc} + K_p) \dot{\tilde{v}}_{dc} &= -(G_{dc} + K_p) \tilde{v}_{dc} - K_{i,dc} \xi - \frac{\mu}{2} \begin{bmatrix} 0 \\ 1 \end{bmatrix}^\top \tilde{i}_{dq} \\ L \dot{\tilde{i}}_{dq} &= -(R\mathbf{I} + \omega_v^* L\mathbf{J} + \tilde{\omega}_v L\mathbf{J}) \tilde{i}_{dq} - \tilde{\omega}_v L\mathbf{J} i_{dq}^* \\ &\quad + \frac{\mu}{2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tilde{v}_{dc} - \tilde{v}_{dq} \\ C \dot{\tilde{v}}_{dq} &= -(G\mathbf{I} + \omega_v^* C\mathbf{J} + \tilde{\omega}_v C\mathbf{J}) \tilde{v}_{dq} - \tilde{\omega}_v C\mathbf{J} v_{dq}^* + \tilde{i}_{dq}. \end{aligned} \quad (24)$$

Since PID control of the DC-voltage (23) is common practice in DC/AC converters, pairing it with the matching control (5) also allows for AC frequency regulation.

The following theorem concerns existence, uniqueness, and stability of a desired steady state of the closed-loop system (24) satisfying  $v_{dc}^* = v_{dc,ref}$  and  $\omega_{ref} = \eta v_{dc,ref}$ . Typically, in such systems,  $\omega_{ref}$  can be seen as the grid reference frequency, while  $v_{dc,ref}$  the rated voltage of the converter's DC-link capacitor. By appropriately choosing the gain  $\eta$ , we are able to achieve both specifications.

**Theorem 6 (Exact frequency regulation)** *Consider the closed-loop inverter model (24). The following statements hold:*

- (1) *There exists a unique steady state at the origin with synchronous AC frequency given by  $\omega_{ref} > 0$ .*
- (2) *If the sufficient condition (15) is satisfied, this unique steady state is globally asymptotically stable.*

**PROOF.** A steady-state of the closed loop (24) is characterized by  $\tilde{v}_{dc} = 0$  and a linear set of equations  $A [\tilde{\xi} \ \tilde{i}_{dq}^\top \ \tilde{v}_{dq}^\top]^\top = 0$ ,  $A \in \mathbb{R}^{5 \times 5}$ . It can be shown that  $A$  is nonsingular and hence  $[\tilde{\xi} \ \tilde{i}_{dq}^\top \ \tilde{v}_{dq}^\top]^\top = 0$ . Thus, there is a unique zero steady-state for the error system. The

stability proof of this steady state is analogous to the proof of Theorem 3 and Corollary 4 after augmenting the storage function  $\mathcal{V}_1$  as  $\mathcal{V}_2 = \mathcal{V}_1 + \frac{1}{2} K_i \tilde{\xi}^2 + \frac{1}{2} K_d \tilde{v}_{dc}^2$ , accounting for the state  $\tilde{\xi} = \xi - \xi^*$ . With these modifications the derivative of the storage function  $\mathcal{V}_2$  becomes

$$\dot{\mathcal{V}}_2 = - \begin{bmatrix} \tilde{v}_{dc} & \tilde{i}_{dq}^\top & \tilde{v}_{dq}^\top \end{bmatrix} \mathcal{P} \begin{bmatrix} \tilde{v}_{dc} & \tilde{i}_{dq}^\top & \tilde{v}_{dq}^\top \end{bmatrix}^\top \leq 0,$$

where  $\mathcal{P}$  is as in (13) with  $G_{dc}$  replaced by  $G_{dc} + K_p$ . Finally a LaSalle argument accounting for the state  $\tilde{\xi}$  guarantees global asymptotic stability.  $\square$

In what follows, we highlight the benefits offered by PID control (23). First, the P-control on the DC voltage enhances the overall system stability, as discussed before. Second, by comparing the open and closed-loop systems (9) and (24), we observe that the effect of the PID gains is to provide additional DC conductance, inductance and capacitance. Lastly, from a conventional power system perspective, it is instructive to write the frequency error dynamics (normalized by  $\eta^2$ ) as

$$\frac{(C_{dc} + K_d)}{\eta^2} \dot{\tilde{\omega}}_v = - \frac{(G_{dc} + K_p)}{\eta^2} \tilde{\omega}_v - \frac{K_i}{\eta^2} \int \tilde{\omega}_v - \tau_e,$$

which for  $K_i = 0$  resemble the classic swing equation with synthetic droop and inertia induced by  $K_p$  and  $K_d$ .

We conclude that for secondary frequency regulation – independent of the particular control strategy – a sufficiently large DC energy storage is required to cope with any imbalance. If the task of frequency regulation is to be shouldered by multiple inverters, then the decentralized integral control in (23) can be easily adapted to broadcast AGC-like or consensus-based distributed integral control schemes (De Persis and Monshizadeh, 2015; Dörfler et al., 2016; Dörfler and Grammatico, 2017), which are robust and assure power sharing.

#### 4.2 Amplitude Regulation by Disturbance Feedback

This section investigates three controllers designed to regulate the voltage amplitude at the output capacitor  $\|v_{\alpha\beta}\|$  to a desired amplitude  $r_{ref} > 0$ , independently of the load current  $i_{l,dq}$ . The latter is assumed to be constant during the time scales of interest. We consider as actuation input the amplitude of the modulation  $\mu = \|m_{\alpha\beta}\|$ , analogously to synchronous machine excitation control. We first characterize the feasibility of this task as a function of the system parameters.

##### 4.2.1 Feasibility and feedforward control

Before we derive the first controller, let us characterize the feasibility of our control objective, namely  $\|v_{\alpha\beta}\| - r_{ref} = 0$  when at steady-state.



**Theorem 7 (Existence of Constraint Equilibria)**

Consider the closed-loop inverter model (24). For given set-points  $r_{ref} > 0$ ,  $v_{dc,ref} > 0$  and constant load current  $i_{l,dq} \in \mathbb{R}^2$ , define the quantity

$$p = (L^2\omega_{ref}^2 + R^2)\|i_{l,dq}\|^2 - r_{ref}^2(C^2R^2\omega_{ref}^2 + L^2G^2\omega_{ref}^2 + (CL\omega_{ref}^2 - 1)^2 + (RG + 1)^2 - 1), \quad (25)$$

where  $\omega_{ref} = \eta v_{dc,ref}$ . Then, the following statements are equivalent:

- (1) There exists a unique steady state  $(\xi^*, v_{dc}^*, i_{dq}^*, v_{dq}^*)$  that satisfies  $\|v_{dq}^*\| = r_{ref}$  and  $\mu > 0$ ; and
- (2)  $p < 0$ .

**PROOF.** We formulate the equilibria of system (24) as

$$0 = v_{dc}^* - v_{dc,ref} \quad (26a)$$

$$0 = -(G_{dc} + K_p)v_{dc}^* - K_{i,dc}\xi^* - \frac{\mu}{2} \begin{bmatrix} 0 \\ 1 \end{bmatrix}^\top i_{dq}^* \quad (26b)$$

$$0 = -(RI + \omega_v^*LJ)i_{dq}^* + \frac{\mu}{2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_{dc}^* - v_{dq}^* \quad (26c)$$

$$0 = -(GI + \omega_v^*CJ)v_{dq}^* - i_{l,dq} + i_{dq}^* \quad (26d)$$

$$0 = v_{dq}^{*\top} v_{dq}^* - r_{ref}^2, \quad (26e)$$

where equation (26e) encodes the constraint accounting for the regulated amplitude. We solve these equations for  $\mu$  as an explicit function of  $i_{l,dq}$  and otherwise constant parameters. By subsequently eliminating the variables  $(\xi^*, v_{dc}^*, i_{dq}^*, v_{dq}^*)$ , we arrive at the quadratic equation

$$0 = \mu^2 - s\mu + \frac{4}{v_{dc,ref}^2}p,$$

where  $4p/v_{dc,ref}^2$ , with  $p < 0$  from (25), is the product and  $s = \frac{4}{v_{dc,ref}}(Ri_{l,q} + \omega_{ref}Li_{l,d})$  the sum of the two solutions  $\mu_{\pm}$  of the quadratic equation. These solutions

$$\mu_{\pm} = \frac{Ri_{l,q} + \omega_{ref}Li_{l,d} \pm \sqrt{(Ri_{l,q} + \omega_{ref}Li_{l,d})^2 - p}}{v_{dc,ref}/2} \quad (28)$$

are real and have opposite signs  $\mu_+ > 0$ ,  $\mu_- < 0$  if and only if  $p < 0$ . In what follows, we restrict ourselves to the unique positive solution  $\mu_+ > 0$ . Notice from (26a) that  $v_{dc}^* = v_{dc,ref}$ . After replacing  $\mu_+$  into (26b)-(26d), the remaining equations are linear  $A[\xi \ i_{dq}^\top \ v_{dq}^\top]^\top = 0$  with  $A \in \mathbb{R}^{5 \times 5}$  nonsingular as in the proof of Theorem 6. These equations can be solved uniquely for  $(\xi^*, i_{dq}^*, v_{dq}^*)$  which is consistent with (26e) by choice of  $\mu_+ > 0$ .  $\square$

The condition  $p < 0$  can be interpreted as a lower bound for the desired amplitude  $r_{ref}$  as a function of the load,  $r_{ref} > \frac{1}{\sqrt{\alpha}}\|i_{l,dq}^*\|$ , where we used the shorthand  $\alpha = (C^2R^2\omega_{ref}^2 + L^2G^2\omega_{ref}^2 + (CL\omega_{ref}^2 - 1)^2 + (RG + 1)^2 - 1)/(L^2\omega_{ref}^2 + R^2)$  as an equivalent circuit impedance. However, when the load is modeled solely as a conductance, the condition  $p < 0$  is satisfied for all  $r_{ref} > 0$ .

**Feedforward control:** Starting from the insights given by Theorem 7, we are able to construct a disturbance-feedforward, asymptotic output tracking controller which relies on measurement of the load current  $i_{l,dq}$  to produce the modulation input  $\mu$  according to (28). This approach can be regarded as a system inversion of the transfer path from  $i_{l,dq}$  to the regulated voltage output  $\|v_{dq}\|$ , a standard procedure in measurable disturbance decoupling. Furthermore, in the next subsection, we will discuss two extensions to this control strategy commending PBC and droop control specifications.

**Corollary 8 (Feedforward Control)** Consider the closed-loop inverter model (24). Assume that the load disturbance  $i_{l,dq} \in \mathbb{R}^2$  is a constant and measurable signal and that  $p$  defined in (25) is negative. Given a reference AC voltage  $r_{ref} > 0$ , consider the following modulation amplitude  $\mu > 0$  as feedforward control  $\mu = \mu_+$ , where  $\mu_+$  is as in (28). Further assume that the stability condition (15) holds. Then the unique equilibrium characterized by  $v_{dc}^* = v_{dc,ref}$  and  $\|v_{dq}^*\| = r_{ref}$  is globally asymptotically stable for the closed loop.

**PROOF.** For any constant  $\mu > 0$  the desired closed-loop equilibria are described in (26). The existence of such equilibria is guaranteed under the condition  $p < 0$  and for  $\mu_{\pm}$  as in (28). By assigning the positive solution  $\mu_+$  in (28), the amplitude  $\mu_+$  is constant for a given constant load  $i_{l,dq}$ . The stability claim now follows from the same reasoning as in the proof of Theorem 6.  $\square$

#### 4.2.2 Compatibility with existing control techniques

While very effective in achieving the prescribed steady-state specification, we find that the feedforward control in Corollary 8 leaves room to more robust extensions. We next investigate two additional controllers and show how they are compatible with what we have done so far.

**PI-PBC:** We start by deriving a PBC feedback, inspired by Zonetti et al. (2014), which amounts to identifying a new passive output. Consider again system (24)

but with input  $\mu(\xi, v_{dc}, i_{dq}, v_{dq})$  yet to be designed

$$\begin{aligned}
\dot{\tilde{\xi}} &= \tilde{v}_{dc} \\
(C_{dc} + K_p)\dot{\tilde{v}}_{dc} &= -(G_{dc} + K_p)\tilde{v}_{dc} - K_i\tilde{\xi} \\
&\quad - \frac{\mu^* + \tilde{\mu}}{2} \begin{bmatrix} 0 \\ 1 \end{bmatrix}^\top \tilde{i}_{dq} - \frac{\tilde{\mu}}{2} \begin{bmatrix} 0 \\ 1 \end{bmatrix}^\top i_{dq}^* \\
L\dot{\tilde{i}}_{dq} &= -(RI + \omega_v^*LJ + \tilde{\omega}_vLJ)\tilde{i}_{dq} - \tilde{\omega}LJi_{dq}^* \\
&\quad + \frac{\mu^* + \tilde{\mu}}{2} \begin{bmatrix} 0 \\ 1 \end{bmatrix}^\top \tilde{v}_{dc} + \frac{\tilde{\mu}}{2} \begin{bmatrix} 0 \\ 1 \end{bmatrix}^\top v_{dc}^* - \tilde{v}_{dq} \\
C\dot{\tilde{v}}_{dq} &= -(GI + \omega_v^*CJ + \tilde{\omega}_vCJ)\tilde{v}_{dq} - \tilde{\omega}CJv_{dq}^* + \tilde{i}_{dq},
\end{aligned} \tag{29}$$

where  $\tilde{\mu} = \mu - \mu^*$  with  $\mu^* = \mu_+$  from (28) assuming a constant measurable  $i_{l,dq}$  and that the prescribed equilibrium of (29) satisfies (26), or equivalently  $p < 0$ . Observe that system (29) is passive with respect to input  $\tilde{\mu} = \mu - \mu^*$ , output  $y = \tilde{i}_q v_{dc}^* - i_q^* \tilde{v}_{dc}$ , and storage function  $\mathcal{V}_2$  from before, such that

$$\dot{\mathcal{V}}_2 = - \begin{bmatrix} \tilde{v}_{dc} & \tilde{i}_{dq}^\top & \tilde{v}_{dq}^\top \end{bmatrix} \mathcal{P} \begin{bmatrix} \tilde{v}_{dc} & \tilde{i}_{dq}^\top & \tilde{v}_{dq}^\top \end{bmatrix}^\top + \tilde{\mu}^\top y.$$

This last observation motivates the PI-PBC law

$$\tilde{\mu} = -\kappa_p y - \kappa_i \tilde{v}, \dot{\tilde{v}} = y, \tag{30}$$

where  $y = \tilde{i}_q v_{dc}^* - i_q^* \tilde{v}_{dc}$  and  $\kappa_p, \kappa_i > 0$ .

**Proposition 9 (PI-PBC)** *Consider system (29) with the PI-PBC feedback (30). Assume that the load disturbance  $i_{l,dq} \in \mathbb{R}^2$  is a constant measurable signal and that  $p$  defined in (25) is negative. Further assume that the stability condition (15) holds. Then the unique equilibrium characterized by  $v_{dc}^* = v_{dc,ref}$  and  $\|v_{dq}^*\| = r_{ref}$  is globally asymptotically stable for the closed loop.*

**PROOF.** Consider the radially unbounded Lyapunov function  $\mathcal{V}_3 = \mathcal{V}_2 + \frac{\kappa_i}{2} \tilde{v}^2$  and its derivative along trajectories of (29), (30)

$$\dot{\mathcal{V}}_3 = - \begin{bmatrix} \tilde{v}_{dc} & \tilde{i}_{dq}^\top & \tilde{v}_{dq}^\top \end{bmatrix} \mathcal{P} \begin{bmatrix} \tilde{v}_{dc} & \tilde{i}_{dq}^\top & \tilde{v}_{dq}^\top \end{bmatrix}^\top - \kappa_p y^2 \leq 0,$$

where  $\mathcal{P}$  is as in (13) with  $G_{dc}$  replaced by  $G_{dc} + K_p$ . Assuming condition (15) is met, a LaSalle argument accounting for the evolution of  $\tilde{\xi}$  and  $\tilde{v}$  guarantees global asymptotic stability.  $\square$

Observe that the PI-PBC law (30) implicitly assumes a constant and measurable load current  $i_{l,dq}$  to define the steady-state values  $\mu^*$  and  $i_{dq}^*$ . Hence, provided that this

quantity is measurable and all system parameters are known both the feedforward control (28), as well as PI-PBC (30) endow the closed-loop system with the ability of rejecting any constant disturbance  $i_{l,dq} \in \mathbb{R}^2$ .

**Droop control:** Finally, another approach in the power systems literature is the voltage-power droop control, from which we derive

$$\mu = \mu_{ref} + d(P_l - P_{ref}), \tag{31}$$

where  $\mu_{ref}, P_{ref}$  are set-points for the modulation amplitude and load power, respectively,  $d > 0$  is the droop coefficient, and  $P_l = i_{l,dq}^\top v_{dq}$  is the measured load power. Droop trades off the modulation amplitude  $\mu$  and the power  $P_l$  drawn by the load, and it induces a steady-state amplitude  $\|v_{dq}^*\|$  that is not necessarily equal to the prescribed  $r_{ref}$ . In spite of this, it is well known that droop control is agnostic to system parameters. The following result shows that droop is compatible with our framework.

**Proposition 10 (Droop control)** *Consider inverter model (24) with droop control feedback (31). For an unknown constant disturbance  $i_{l,dq} \in \mathbb{R}^2$ , assume that the closed loop (24), (31) admits a steady state  $(\xi^*, v_{dc}^*, i_{dq}^*, v_{dq}^*)$ . Further assume that condition (15) holds. Then for a sufficiently small droop coefficient  $d > 0$ , this steady state is globally asymptotically stable for the closed loop.*

**PROOF.** We rewrite the closed-loop DC/AC converter in error coordinates with  $\mu = \mu_{ref} + d(P_l - P_{ref})$  as

$$\begin{aligned}
\dot{\tilde{\xi}} &= \tilde{v}_{dc} \\
(C_{dc} + K_p)\dot{\tilde{v}}_{dc} &= -(G_{dc} + K_p)\tilde{v}_{dc} - K_i\tilde{\xi} - \frac{\mu_{ref}}{2} \begin{bmatrix} 0 \\ 1 \end{bmatrix}^\top \tilde{i}_{dq} \\
&\quad - \frac{d(P_l - P_{ref})}{2} \begin{bmatrix} 0 \\ 1 \end{bmatrix}^\top \tilde{i}_{dq} - \frac{d\tilde{P}_l}{2} \begin{bmatrix} 0 \\ 1 \end{bmatrix}^\top i_{dq}^* \\
L\dot{\tilde{i}}_{dq} &= -(RI + \omega_v^*LJ + \tilde{\omega}LJ)\tilde{i}_{dq} - \tilde{\omega}LJi_{dq}^* \\
&\quad + \frac{\mu_{ref} + d(P_l - P_{ref})}{2} \begin{bmatrix} 0 \\ 1 \end{bmatrix}^\top \tilde{v}_{dc} + \frac{d\tilde{P}_l}{2} \begin{bmatrix} 0 \\ 1 \end{bmatrix}^\top v_{dc}^* - \tilde{v}_{dq} \\
C\dot{\tilde{v}}_{dq} &= -(GI + \omega_v^*CJ + \tilde{\omega}CJ)\tilde{v}_{dq} - \tilde{\omega}CJv_{dq}^* + \tilde{i}_{dq}
\end{aligned}$$

with  $\tilde{P}_l = P_l - P_l^*$  and  $P_l^*$  as the value of the load power at steady-state. The derivative of  $\mathcal{V}_2$  can be calculated analogously to the proof of Theorem 6, as  $\dot{\mathcal{V}}_2 = - \begin{bmatrix} \tilde{v}_{dc} & \tilde{i}_{dq}^\top & \tilde{v}_{dq}^\top \end{bmatrix} (\mathcal{P} + d\mathcal{M}) \begin{bmatrix} \tilde{v}_{dc} & \tilde{i}_{dq}^\top & \tilde{v}_{dq}^\top \end{bmatrix}^\top \leq 0$ , where  $\mathcal{P}$  is as in (13) and  $\mathcal{M}$  is a constant matrix, i.e., its entries

do not depend on the droop coefficient  $d$ . Since  $\mathcal{P}$  is positive definite under condition (15), there exists  $d > 0$  sufficiently small such that  $\mathcal{P} + d\mathcal{M}$  is positive definite. A LaSalle argument accounting for the evolution of  $\tilde{\xi}$  then shows global asymptotic stability of the equilibrium.  $\square$

## 5 Numerical Case Study

We validate and test the proposed controllers in a numerical case study. Consider for this purpose an inverter designed for  $10^4$  W power output in S.I. units:  $G_{dc} = 0.1$ ,  $C_{dc} = 0.001$ ,  $R = 0.1$ ,  $L = 5 \cdot 10^{-4}$ ,  $C = 10^{-5}$ , and nominal DC voltage of  $v_{dc,ref} = v_{dc}(0) = 1000$ . In order to obtain the desired open-circuit (no load) values  $r_{x,ref} = r_{ref} = 165$  and  $\omega_{x,ref} = \omega_{ref} = 2\pi 50$ , we choose the constant gains  $\eta = \frac{\omega_{ref}}{v_{dc,ref}} = 0.3142$ ,  $\mu = \frac{2r_{ref}}{v_{dc,ref}} = 0.33$ .

### 5.1 Voltage and Frequency Regulation – single inverter

To validate our results for frequency and amplitude regulation, we present three scenarios implementing the matching control (5) and the frequency regulation (23), together with the three different amplitude controllers. We consider a load step of 55% at  $t = 0.5$ s. The resulting amplitudes and power signals are shown in Figure 3, whereas Figure 5 shows a time-domain electromagnetic transient (EMT) simulation of the output capacitor voltage.

The parameters for our frequency controller (23) were selected as  $i_{dc,ref} = 100$ ,  $K_p = 1$ ,  $K_i = 10$ ,  $K_d = 0$  and  $\xi(0) = 0$ . For voltage control we consider the feedforward control (28), PI-PBC (30) with (in S.I  $\kappa_p = 0.1$ ,  $\kappa_i = 10$ ,  $\nu(0) = 0$ ), as well as droop control (31) (in S.I  $\mu_{ref} = 0.33$ ,  $d = 10^{-5}$  and  $P_{ref} = 10^4$ ) plotted as red, green and blue signals, respectively. For all considered controllers, the DC voltage exactly tracks the reference voltage  $v_{dc,ref} = 1000$ V. The feedforward and PI-PBC designs indeed also track the desired amplitude  $r_{ref} = 165$ V. Observe that the constant amplitude objective of these controllers requires higher steady-state current amplitudes after the load step. The droop controller on the other hand ensures a trade-off between the power load and AC voltage amplitude. We observe that all controllers yield well-behaved transient response to the step in disturbance.

### 5.2 Multi-Converter Case Study

Next we consider a network of two inverters connected in parallel to a conductance load via a  $\Pi$ -transmission line model, as described in Figure 2. The line inductor and lumped capacitor dynamics are considered as

$$C\dot{v}_{\alpha\beta,i} = -Gv_{\alpha\beta,i} + i_{\alpha\beta,i} - i_{net,i}$$

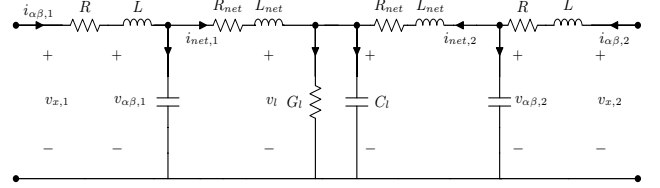


Fig. 2. Two inverters connected in parallel to a conductance load  $G_l > 0$  via a  $\Pi$ -line model. The  $\Pi$ -line parameters are (in S.I.)  $R_{net} = 0.5$ ,  $L_{net} = 2.5 \cdot 10^{-5}$ , and  $C_l = 2 \cdot 10^{-7}$ , where the capacitors account for filter and line charge capacitance.

$$\begin{aligned} L_{net}\dot{i}_{net,i} &= -R_{net}i_{net,i} + v_{\alpha\beta,i} - v_l \\ C_l\dot{v}_l &= -G_lv_l + i_{net,1} + i_{net,2}. \end{aligned}$$

We implemented the matching control (5) with appropriate gains (17) to demonstrate the proportional power sharing with ratio  $\rho = 3$ . We chose the current control parameters for the inverters as  $i_{dc,ref,1} = 100$ ,  $K_{p1} = 2$  in (S.I), neglected internal losses  $G_{dc,1} = G_{dc,2} = 0$ , fixed the modulation amplitude at  $\mu_1 = \mu_2 = 0.33$ , removed the integral action  $K_{i,1} = K_{i,2} = 0$ , and set all other parameters as before. Our simulation in Figure 4 displays a prescribed power sharing ratio of 3:1 under resistive load steps at times  $t = 0.3$ s and  $t = 0.7$ s.

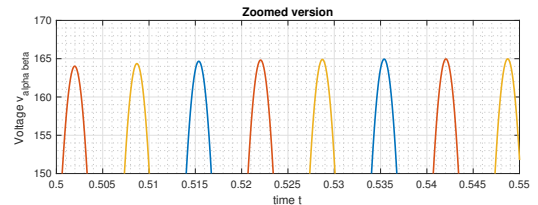


Fig. 5. EMT time-domain simulations of the AC bus voltage.

## 6 Conclusions

This paper addresses the problem of designing grid-forming inverter control strategies. Based on the idea of matching the dynamics of a SM, we enable by feedback the essential coupling between the inverter DC-side voltage and the AC-side frequency. In this manner we obviate the use of AC grid frequency measurement as well as the separate regulation of DC and AC-side circuits. Our matching controller provides droop behaviour, allows for proportional power sharing, and preserves passivity-based characteristics of the inverter. Moreover, the addition of synthetic damping and inertia is straightforward. We further pair the proposed controller with additional outer control loops for regulation of the AC frequency and amplitude in presence of disturbances. These outer controllers are designed based on passivity-based and disturbance decoupling methods and achieve regulation of two quantities of interest: output voltage frequency and its amplitude. In the light of our analysis, a natural counterpart is to investigate the compatibility of these objectives and to design suitable controllers that encompass a network of converters, as explored in the above numerical case study.

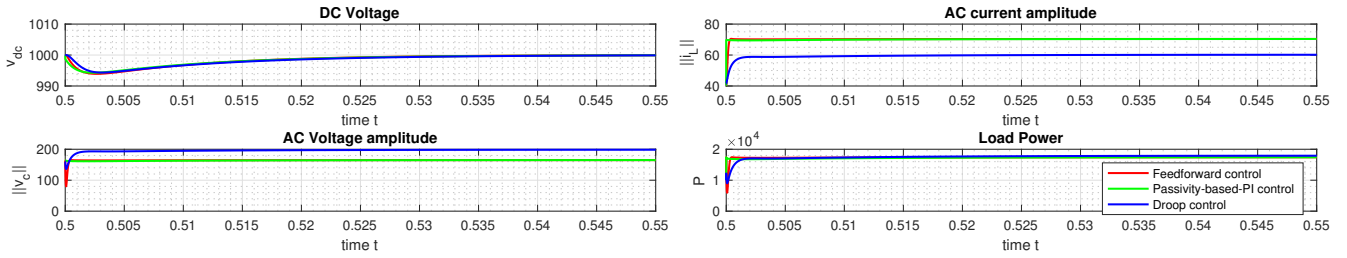


Fig. 3. A magnified version of time-domain simulations of a single inverter over a step change in the load.

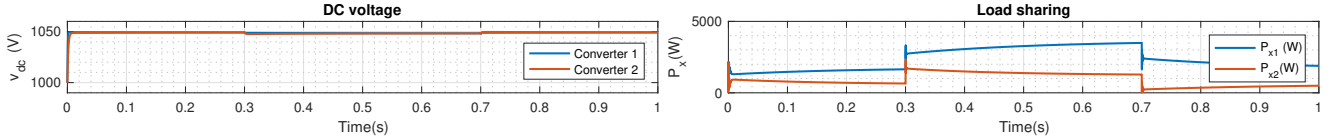


Fig. 4. Time-domain simulations of the parallel converter scenario in Figure 2 after a step in the load conductance  $G_L$ .

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