TV: From the Studio to the Living Room

PPS Seminar

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Abstract

From the PPS Seminar description: "The antenna tower on top of Uetliberg serves for the transmission of television programs for the region of Zürich. It is approximately 187 meters tall, and was built in 1990, for a cost of 11 Million Swiss Franks.

In this PPS, you will have an opportunity to learn about the technical details of the Uetliberg tower. We will take a field trip to the tower, where Swisscom will give us a tour and some insight into the technical aspects of supplying broadcasts to Zürich.

After the tour, you will investigate various aspects of the Uetliberg tower, such as the types of antenna arrays present, the direction characteristics of those arrays and how they are designed for the surrounding communities, as well some of the technical difficulties which must be overcome for signal transmission."

The tour to the Uetliberg tower gave us a slight but interesting insight on signal transmission. After the tour we had to make a small presentation about a subject concerning antennas or any kind of signal transmission. This report is the hard copy version of that presentation. The title of this work is the topic of a wide subject. It covers things such as the functionality of a video camera or internet TV-streams. We decided to pick 3 subtopics and cover them in detail. These areas are: Optical fibers by Pirmin Vogel, DVB-T by Benjamin Weber and DVB-T antenna design by Marco Karch.

Marco Karch
Pirmin Vogel
Benjamin Weber
1 Optical Fibers

1.1 Introduction

Optical fibers are used to transmit data over long distances with high bandwidths and they consist of glass extracted from glass sand/silica. There are different types of optical fibers with their own purposes but they are all based on the same principle: Light rays are guided through the fiber by total internal reflection.

1.2 Total Internal Reflection

As one can derive from the Snell’s law

\[
\frac{\sin \phi_1}{\sin \phi_2} = \frac{n_2}{n_1}
\]

an electromagnetic wave traveling from a medium with lower refraction index into a second medium with higher refraction index gets refracted towards the perpendicular of the interface. Therefore the angle of refraction \( \phi_2 \) is smaller than the angle of incidence \( \phi_1 \) in this case (figure 1). According to this a wave traveling in the opposite direction gets refracted away from the perpendicular of the interface, \( \phi_1 > \phi_2 \). If one now wants to achieve total reflection \( \phi_2 = 90^\circ \) is needed. Considering the equations for the index of reflection of a transversal parallel or vertical polarized electromagnetic wave and inserting \( \cos \phi_2 = 0 \) we obtain that the magnitude of the index of reflection is actually 1.

\[
|\Gamma_\parallel| = \left| \frac{Z_{w2} \cos \phi_2 - Z_{w1} \cos \phi_1}{Z_{w2} \cos \phi_2 + Z_{w1} \cos \phi_1} \right|_{\cos \phi_2 = 0} = 1
\]

\[
|\Gamma_\perp| = \left| \frac{Z_{w2} \cos \phi_1 - Z_{w1} \cos \phi_2}{Z_{w2} \cos \phi_1 + Z_{w1} \cos \phi_2} \right|_{\cos \phi_2 = 0} = 1
\]

1.3 Structure

To ensure the total internal reflection and so the wave guiding characteristics of optical fibers a special structure is needed. In figure 2 one can see the structure of a typical mono- or singlemode fiber. The wave is guided through the very tiny core with a common diameter of 8 to 10 \( \mu m \). The core consists of glass with a high refraction index (\( \approx 1.48 \)) and is surrounded by the cladding which consists of glass, too, but has a lower refraction index (\( \approx 1.46 \)). Therefore waves traveling through the core get reflected and not transmitted into the cladding if they strike onto the interface between core and cladding. The structure of core and cladding is surrounded by a buffer layer and a jacket to give more mechanical stability to the fiber and to assure the glass against physical damage (water, compression etc.). Usually the buffer layer is made from resin and the jacket consists of plastic.
1.4 Modal Dispersion

Due to the finite diameter of the core, rays with a low angle of incidence get reflected more often and cover a longer distance while passing the fiber than rays with a high angle of incidence (figure 3).

Therefore rays with different angles of incidence have different signal propagation delays. If a steep pulse consisting of many rays with different angles of incidence for example enters the fiber some rays take more time to travel through the fiber than others. The result is a stretched pulse at the output. The signal gets blurred (figure 4). This effect is called modal dispersion and limits the bandwidth and maximum transmission distance as it is proportional to the length of the fiber. To reduce the modal dispersion one can minimize the core diameter to ensure that only rays with a high angle of incidence can pass the fiber. This is called light of low mode in contrary to light of high mode which consists of rays with low angle of incidence and gets reflected more often and therefore covers a longer distance. Fibers with a core diameter of about 50 to 200 µm transmit light of low as well as light of high mode at the same time. That’s why they are called multimode fibers. If the core diameter is reduced to about 8 to 10 µm only light of one single mode can pass the fiber and will traverse exactly the same path almost parallel to the axis of the fiber and exit at the same time. Now we have a single- or monomode fiber with a much higher bandwidth than...
1.5 Different Types of Fibers

![Diagram of optical fibers](image)

Figure 5: The three different types of glass fibers

In figure 5 one can see a comparison of the three types of optical fibers. Multimode step index fibers have constant refractive indexes within the core and the cladding. They are used for short distances because of high dispersion due to the fact that a lot of different modes are transmitted at the same time. Multimode graded index fibers are characterized by non-constant refractive index which means that the refractive index of the core decreases as the radius increases. Observing the equation $v = \frac{c}{n}$ which relates the velocity of an electromagnetic wave in a medium to the ratio of the speed of light in vacuum to the refractive index of the medium we obtain that rays which travel particularly through the center of the core (light of low mode, high angle of incidence) are retarded more than rays of high mode which travel more through regions of low refractive indices. The rays bend smoothly as they approach the cladding. This results in some kind of signal propagation delay synchronization and therefore lower dispersion. Mono- or singlemode fibers have a very small core diameter and transmit light of only one mode almost parallel to their own axis. Therefore extremely precise equipment needs to be used. For example laser diodes which emit monochromatic light are used as light sources for monomode fibers instead of LEDs. Because of the higher output power of laser diodes and the very weak dispersion due to the fact that only light of one mode is transmitted, monomode fibers are used for very long distances and transfer rates.

1.6 The Bandwidth Length Product

The characteristical quantity for optical fibers is the bandwidth length product which is simply computed by multiplying the maximum length by the bandwidth. In table 1 there are some typical values. Compare them with the bandwidth length product of a standard 1000Base-T Gigabit ethernet cable (maximum length = 100 m, bandwidth = 62.5 MHz).
### Table 1: Typical values for the bandwidth length product

<table>
<thead>
<tr>
<th>type of fiber</th>
<th>bandwidth length product</th>
</tr>
</thead>
<tbody>
<tr>
<td>multimode step index</td>
<td>$100 \text{ MHz} \cdot \text{km}$</td>
</tr>
<tr>
<td>multimode graded index</td>
<td>$1 \text{ GHz} \cdot \text{km}$</td>
</tr>
<tr>
<td>monomode</td>
<td>$10 \text{ GHz} \cdot \text{km}$</td>
</tr>
<tr>
<td>gigabit ethernet</td>
<td>$6.25 \text{ MHz} \cdot \text{km}$</td>
</tr>
</tbody>
</table>

#### 1.7 Fields of Application

The characteristical attributes of the different types of fibers lead to different fields of application. As one can see in figure 6, only monomode fibers with laser diodes as light sources are able to transfer 1000 Mbit/s over distances of about 10 km. For low transfer rates and distances even fibers consisting of plastic instead of glass are used.

#### 1.8 Comparison with Copper Cables

There are several advantages over copper cables:

- Immunity to electromagnetic interference
- Very high transfer bandwidth
- Small damping even at long ranges
- High thermic and chemical stability
- Almost unlimited and cheap resources (glass sand/silica)
- Less material necessary for shielding, optical fibers are much lighter

But there are also some disadvantages of course. For example the installation and the equipment are more expensive and therefore results a low cost-effectiveness at short ranges for optical fibers.

#### 1.9 Damping

Of course there are damping losses in optical fibers, too. But as written above they are much smaller than die damping losses in copper cables. Damping in fibers is the result of splices, connectors, the absorption by the core and cladding caused by impurities and the fact that always a small fraction of the light is transmitted into the cladding. As the damping and dispersion depend on the used wavelength there are different wavelengths used...
for transmission depending on the type of the fiber. The common wavelengths are 850 nm, 1310 nm and 1550 nm. In table 2 there are some typical values.

<table>
<thead>
<tr>
<th>type of fiber</th>
<th>damping [dB/km]</th>
<th>wavelength [nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>multimode step index</td>
<td>5 – 6</td>
<td>850</td>
</tr>
<tr>
<td>multimode graded index</td>
<td>2.5 – 3.5</td>
<td>850</td>
</tr>
<tr>
<td>monomode</td>
<td>0.36</td>
<td>1310</td>
</tr>
</tbody>
</table>

Table 2: Typical values for damping

2 DVB-T

2.1 What is DVB?

DVB stands for Digital Video Broadcasting. It is a standardization of several methods to broadcast digital information. Mostly video signals are broadcasted using DVB standards, even though any digital data transmission is possible using the same method. There are several DVB standards, the most significant ones are listed here:

DVB-S is the standard for satellite video transmission.

DVB-T is the standard for terrestrial video transmission. It will be discussed further in the following chapters.

DVB-C is the standard for cable video transmission.

DVB-H is the standard for handheld devices. This is a low resolution standard, as handhelds usually are equipped with small screens.

2.2 What is DVB-T?

DVB-T, as mentioned above, is the transmission standard for terrestrial video transmission. It is mostly used for transmitting TV-signals. It uses the same frequencies as analogue TV and thus also the same bandwidths, which are:

- 7 MHz using VHF (47 – 68 and 174 – 230 MHz). These are the analogue channels 1 through 13.
- 8 MHz using UHF (470 – 862 MHz). These are the analogue channels 14 through 83.

One significant difference to analogue television is that one channel usually holds 4 digital television programs (The following explanations presume 4 programs per channel as the Uetliberg station transmits 4 programs per channel.) Therefore a much bigger variety of programs is possible. With analogue television there were at most 83 different programs possible, with DVB-T there are now more than 332 different programs possible. The bandwidth of 7 MHz and 8 MHz, respectively, is used more efficiently through COFDM (Coded Orthogonal Frequency Division Multiplex). This implies that the whole bandwidth is divided into thousands of subcarriers. These subcarriers are orthogonal to each other. In time domain these means that the scalar product of 2 subcarriers \( s_i(t) \) and \( s_j(t) \) equals zero.

\[
\int_{-\infty}^{\infty} s_i(\tau)^* \cdot s_j(\tau) d\tau = 0 \quad \forall \quad i \neq j
\]

In frequency domain at the actual carrier frequency all other subcarriers have zero frequency portion. This will be discussed in more detail in section 2.5. Furthermore each subcarrier is modulated with QAM, see section 2.4.

\(^{1}\)These specifications vary from country to country
The data stream is encoded in DVB-MPEG, which contains 4 MPEG 2 video streams and some redundant information. In the end, this leaves about 3 to 3.5 Mbits/s video stream per program. This data rate per program is variable, meaning depending on the desired video quality, e.g. sport shows require higher data rates, the channel data rate can be divided up differently to the 4 programs. The division information is sent along as part of the DVB-MPEG stream as redundant information, so that the receiver knows how to split the channel into its programs. A sampled analogue PAL television signal encoded as MPEG 2 video stream requires 3 - 5 Mbits/s, so quality of DVB-T is equal or less than analogue television. A DVD (also an MPEG 2 stream) has a 9.8 Mbits/s data rate.

2.3 Forward Error Correction (FEC)

Some of the redundant information in the DVB-MPEG stream is used for forward error correction. FEC is a highly sophisticated error correction method. It allows the receiver to detect and correct errors without query. Figure 7 illustrates in a simple manner how FEC works. During the encoding process, some redundant information is added to the actual message. That additional data holds information about the message in a very compressed way. During transmission errors occur. The receiver can due to the redundancy detect errors and correct them to a certain extent.

\[ \text{DVB-T code rate} = \frac{i}{i+1}, \quad i = 1, \ldots, 7 \]

<table>
<thead>
<tr>
<th>message</th>
<th>redundancy</th>
<th>sender</th>
<th>receiver</th>
<th>message</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>00</td>
<td>000</td>
<td>000</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>00</td>
<td>000</td>
<td>001</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>11</td>
<td>111</td>
<td>111</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>11</td>
<td>111</td>
<td>101</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3: Example for FEC
2.4 Quadrature Amplitude Modulation (QAM)

Each subcarrier of the 8 MHz and 7 MHz bandwidth, respectively, is modulated using QAM. This modulation method is a mixture between phase and amplitude modulation. Thus a higher data rate from 12 to 20 Mbits/s is achieved. The sender transmits a signal

\[ s(t) = I(t) \cos(2\pi f_0 t) + Q(t) \sin(2\pi f_0 t) \]

where \( f_0 \) is the carrier frequency and \( I(t), Q(t) \) are the modulating functions. The notation quadrature makes more sense now as the 2 functions sine and cosine are 90° off phase. If we write these 2 functions as complex exponential functions, then they both start in two different quadrants. What the sender unit has to do is summarized in figure 8. The system \( H_s(\omega) \) is a Fourier transformation from frequency to time domain.

![Diagram of QAM modulation on the sender side.](image)

**Figure 8: Quadrature amplitude modulation on the sender side.**

The receiving unit, needs to multiply the received signal \( s(t) \) with a cosine and sine, respectively. Ideally they must have the same phase as the cosine and sine the sender used to modulate the signal. In this case one can apply trigonometric identities and get the signals

\[
\begin{align*}
    r_I(t) &= s(t) \cdot \cos(2\pi f_0 t) = \frac{1}{2} I(t) [1 + \cos(4\pi f_0 t)] + \frac{1}{2} Q(t) \sin(4\pi f_0 t) \\
    r_Q(t) &= s(t) \cdot \sin(2\pi f_0 t) = \frac{1}{2} Q(t) [1 - \cos(4\pi f_0 t)] + \frac{1}{2} I(t) \sin(4\pi f_0 t)
\end{align*}
\]

One can see, that if we apply a low pass filter to \( r_I(t) \) and \( r_Q(t) \) the modulating functions \( I(t) \) and \( Q(t) \) are retrieved. This process is illustrated in figure 9. The system \( H_r(\omega) \) represents the mentioned low pass filter.
QAM is easily illustrated for digital signals in an I-Q-plane. The 2 modulating functions combined in a vector \( x(t) = (I(t), Q(t)) \) can technically point to any desired spot in the I-Q-plane. \( I(t) \) and \( Q(t) \) only take discrete values, though. Each point \((i, j)\) in the plane represents a symbol. The sender and receiver use the same symbolic table and in this way digital information is transmitted. It is not possible, however, for \( s(t) \) to be arbitrary big as there are magnitude restrictions on \( I(t) \) and \( Q(t) \) due to limited transmitting power caused by economic interests and legal restrictions. DVB-T uses QPSK (Quadrature phase-shift keying), which is similar to 4-QAM where \( I(t), Q(t) \in \{-1x, 1x\} \), \( (x \text{ depends on the over all transmitting power}) \) thus representing 4 points in the I-Q-plane, 16-QAM where \( I(t), Q(t) \in \{-3x, -1x, 1x, 3x\} \), thus representing 16 points. 64-QAM (64 represented points) is rarely used. It is important to understand that the values of \( I(t) \) and \( Q(t) \) mentioned for 4-QAM and 16-QAM are relative to each other. The higher the order of QAM the closer together points are and thus the signal is more susceptible to noise and other corruption.

The figure on the title page of this work is an actual recording of a DVB-S (satellite) signal using QPSK over a period of time. One can see clearly the 4 points even though there is some noise and distortion. Apparently there is no guard interval being used as there is no accumulation in the origin.

### 2.5 Advantages, Disadvantages

**Advantages** One very obvious advantage is the multitude of programs which can be transmitted. Another big advantage is the low transmitting power, due to strong error correction algorithms and powerful receivers than in analogue television. However, the most remarkable advantage is diversity reception, which means that a receiving unit can use the signal from several transmitting stations. Senders transmitting use the same frequency for a particular program and are synchronized by GPS. This synchronization assures that the functions used during the modulating process have the same phase and that symbols are sent simultaneously from each station. It is even possible to use reflected waves off hills and other objects for encoding, even though reflections arrive later at the receivers antenna then the direct wave. This even leads to an amplification of the signal. This is possible through a guard interval between two symbols. During the guard interval the modulating functions \( I(t) \) and \( Q(t) \) are set to zero. A symbol which arrives later will not interfere with the next symbol of the direct signal as the direct signal will be zero right after the symbol ends (see Figure[10]). Further advantages: DVB-T is cheaper in transmitting than analogue television and one can record the digital video stream without data loss.
Disadvantages  Fast moving receivers are very interference-prone due to the Doppler-Effect, mainly known with acoustic waves. A receiver moving towards a transmitting station will pick up a signal with a higher frequency than a receiver moving tangential to a transmitting station. At a speed of 60 km/h radial to the station this results in a frequency shift of 30 Hz. The Fourier transform of each subcarrier is of the form

\[ S(\omega) = \frac{\sin(a\omega)}{a\omega} \]  

with \( \frac{\pi}{a} = \Delta f \)

\( \Delta f \) is the distance between 2 subcarrier frequencies. This assures that at the actual carrier frequency of a subcarrier all other carriers have zero frequency portions. In case of a fast moving receiver and multiple reception this may change: If a receiver moves radial to a station A and tangential to a station B, the signal from station A shifts in frequency but the signal from station B doesn’t. Hence at the carrier frequency of each subcarrier the other subcarrier’s frequency portion is not zero any longer, which leads to strong interference. A smaller disadvantage is a long transmission delay. The encoding and modulating process on the sender side and the demodulating and decoding process on the receiver side can take up to 8 seconds. Especially during sport shows this can be irritating, as people watching the show on an analogue channel are informed earlier of results during the show.

2.6 Uetliberg Specifications

- 16-QAM modulation
- 896 µs symbol length
- 1/4 guard (fraction of symbol length)
- Only UHF channel 32 is in use, it contains the 4 programs SF 1, SF 2, TSR 1 and TSI 1
- The 8 MHz channel splits into 6817 subcarriers (8k COFDM).
- The distance between subcarriers is 1.116 kHz.
- actual bandwidth used: 7.608 MHz
- The max data rate is 16.59 Mbits/s.
- 5/6 code rate
- applied sampling rate: 9.143 MHz

3 DVB-T Antenna Design

3.1 Introduction

As DVB-T emitters try to radiate the most efficient in all directions there is one sort of antennas that is perfectly suited to achieve this aim - the dipole antennas. They first were developed in 1886 by Heinrich Hertz. Dipole antennas are used to transmit and receive radio frequency energy by sending out or receiving electromagnetic waves. In theory they are the simplest species of antennas.
3.2 Technical Specification (Half-Wave Antenna)

The most common sort of dipole used is the 'Lambda-half'-dipole (λ/2-dipole) which will be shown as example to explain the functionality of dipole antennas. In general, dipole is a typical LC-oscillator which is energized with a certain frequency at its feed line to provide a forced oscillation. These frequencies usually are HF (high frequency) or UHF (ultra high frequency) [several GHz] and contain the whole information of a video signal. If you take an LC-oscillator and use as inductivity only one coil and as capacity only the cables itself you get an LC-oscillator with special dimensions – a so called dipole oscillator (antenna). There are 2 different types of dipole-antennas used: the folded and the elongated dipole. The only difference between those 2 species is their impedance (75 Ohm the folded and 300 Ohm the elongated). We will now examine the folded dipole to understand how the radiation of electromagnetic waves is about to happen. A λ/2-dipole is formed by two quarter wavelength conductors or elements that are fed by a coaxial cable as energizing feed line. This configuration allows the transformation of HF AC (alternating current) into electromagnetic HF wave radiation.

![Figure 11: λ/2 dipole](image)

What actually happens is that due to the time delay between voltage and current (like in all LC-oscillators because of the imaginary part of the impedance of inductivity and capacity have a phase shift of ±ı = ±90°) leads to forced oscillations on the dipole conductors with AC-current nodes and AC-voltage anti-nodes at the ends of the conductors.

![Figure 12: AC-oscillations on conductors](image)

Figure 12 shows the external forced AC-oscillations on the conductors. The effect of those oscillations is that the conductors begin to send out electromagnetic energy in form of electromagnetic waves. This is the basic principle of a λ/2-dipole that transforms the current
flow into wave energy that can be easily transmitted nearly homogeneously in all directions.

3.3 Emission Diagrams

When we regard the electric and the magnetic HF-field of a $\lambda/2$-dipole we see that the $B$-field is orthogonal to the conductors, similar to the $B$-field resulting by a current running through a cable, and that the $E$-field again is orthogonal to the $B$-field according to the phase-relationship between $B$- and $E$-field in an electromagnetic wave (3-dimensional regarded orthogonal to each other).

![Figure 13: $\lambda/2$-dipole emission diagram](image)

Resulting of $B$- and $E$-field is a slightly flattened torus as radiation emission diagram. It is typical for a $\lambda/2$-dipole. If we compare this $\lambda/2$-dipole emission diagram to the diagrams of other dipoles that have a different length-definition for the conductors like $L = \lambda$ or $L = 2\lambda$ we can see that the $\lambda/2$-dipole is best suited to ensure an equal radiation in all directions:

![Figure 14: Emission diagrams of $\lambda/2$, $\lambda$, $2\lambda$-dipoles, respectively](image)

Figure 14 shows the radiation emission diagrams of 3 different dipoles ($L = \lambda/2$, $L = \lambda$, $L = 2\lambda$) plotted in a 2-dimensional view. One can easily find out that the red and the yellow circles show the intensity range of a $\lambda/2$ dipole whereas the green and the blue curves show a certain directed beam ratio with irregularly radiation intensities (maximums and minimums)
in several directions. In fact, this explains why the most common used antenna for a DVB-T signal design is the $\lambda/2$-dipole.

### 3.4 Antenna Gain

The antenna gain $G$ describes the ratio of surface power radiated by the antenna compared to the surface power radiated by a hypothetical isotropic antenna. $G$ is measured in decibel (dB). A hypothetical isotropic antenna is a theoretical point source of waves which exhibits the same magnitude measuring in all directions homogeneously. It is a reference radiator (that does not exist in reality) with which other sources are compared. Its gain $G$ is equal to 0 dB.

$$G = \frac{(\frac{P}{S})_{ant}}{(\frac{P}{S})_{iso}} \Rightarrow \frac{(\frac{P}{S})_{ant}}{1} = \frac{1}{2} c \varepsilon_{r} E_{\theta}^{2} \approx \frac{1}{120 \pi} E_{\theta}^{2}$$

If we want to calculate the gain of an antenna we first have to calculate the emission power emitted by the dipole and then divide it by the emission power of the isotropic radiator.

What we in fact have to do is to calculate the real part of the antenna’s impedance which is a little less than the 75 Ohm (for the isotropic radiator):

$$R_{\lambda} = 60 \ln(2\pi) = 60[\ln(2\pi r) - Ci(2\pi)] = 120 \int_{0}^{\pi} \frac{\cos(\frac{\pi}{2} \cos \theta)}{2 \sin \theta} d\theta$$

$$= 15 \left[ 2\pi^2 - \frac{1}{3} \pi^4 + \frac{4}{135} \pi^6 - \frac{1}{630} \pi^8 + \frac{4}{70875} \pi^{10} \ldots - \frac{1}{n} \frac{(2n)^{2n}}{n(2n)!} \right]$$

$$= 73.12960179171673235432131024310052433236972993 \ldots \Omega$$

The calculation of this leads to a total antenna gain of

$$G_{\lambda} = \frac{60^2}{30R_{\frac{\lambda}{2}}} = \frac{3600}{30R_{\frac{\lambda}{2}}} = \frac{120}{\frac{1}{2} \int_{0}^{\pi} \cos(\frac{\pi}{2} \cos \theta) d\theta}$$

$$\approx \frac{120}{73.1296} \approx 1.6409224 \approx 2.15088 \text{ dB}$$

after a quite complicated integration. (Those gains are only measured theoretically without regarding any losses!) Table 4 shows the (ideal) antenna gain for different dipole antennas that vary in their length from $L \ll \lambda$ to $L = 8\lambda$:

<table>
<thead>
<tr>
<th>Length L in $\lambda$</th>
<th>Gain</th>
<th>Gain (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ll 1$</td>
<td>1.50</td>
<td>1.76 dB</td>
</tr>
<tr>
<td>0.5</td>
<td>1.64</td>
<td>2.15 dB</td>
</tr>
<tr>
<td>1.0</td>
<td>1.80</td>
<td>2.55 dB</td>
</tr>
<tr>
<td>1.5</td>
<td>2.00</td>
<td>3.01 dB</td>
</tr>
<tr>
<td>2.0</td>
<td>2.30</td>
<td>3.62 dB</td>
</tr>
<tr>
<td>3.0</td>
<td>2.80</td>
<td>4.47 dB</td>
</tr>
<tr>
<td>4.0</td>
<td>3.50</td>
<td>5.44 dB</td>
</tr>
<tr>
<td>8.0</td>
<td>7.10</td>
<td>8.51 dB</td>
</tr>
</tbody>
</table>

Table 4: Gain of dipole antennas

What one can see is that the antenna gain increases the longer the conductors gets. This does not mean that a 5$\lambda$-dipole is even better than a $\lambda/2$-dipole. It describes only a certain
efficiency the antenna has but not how well-suited the antenna is for different tasks. The table does only give an ideal gain of those antenna-dipoles and in reality the gains differ from these calculations, they decrease more or less due to Ohm-losses in the conductors (and the AC-feed line). Furthermore the calculation only gives the maximum gain and as we can see in figure 15 that the gain is not the same in all directions. The more one moves parallel to the conductor axis the lower the gain gets.

![Figure 15: Gain of a λ/2-dipole](image)

The maximum gain of a λ/2-dipole is achieved orthogonal to the conductors and is about 3 $dBi$ in reality. Ideally the maximum gain would be those 2.15 $dBi$ calculated before.
References

[1] Information from the Uetliberg tour on March 11, 2008 by Swisscom