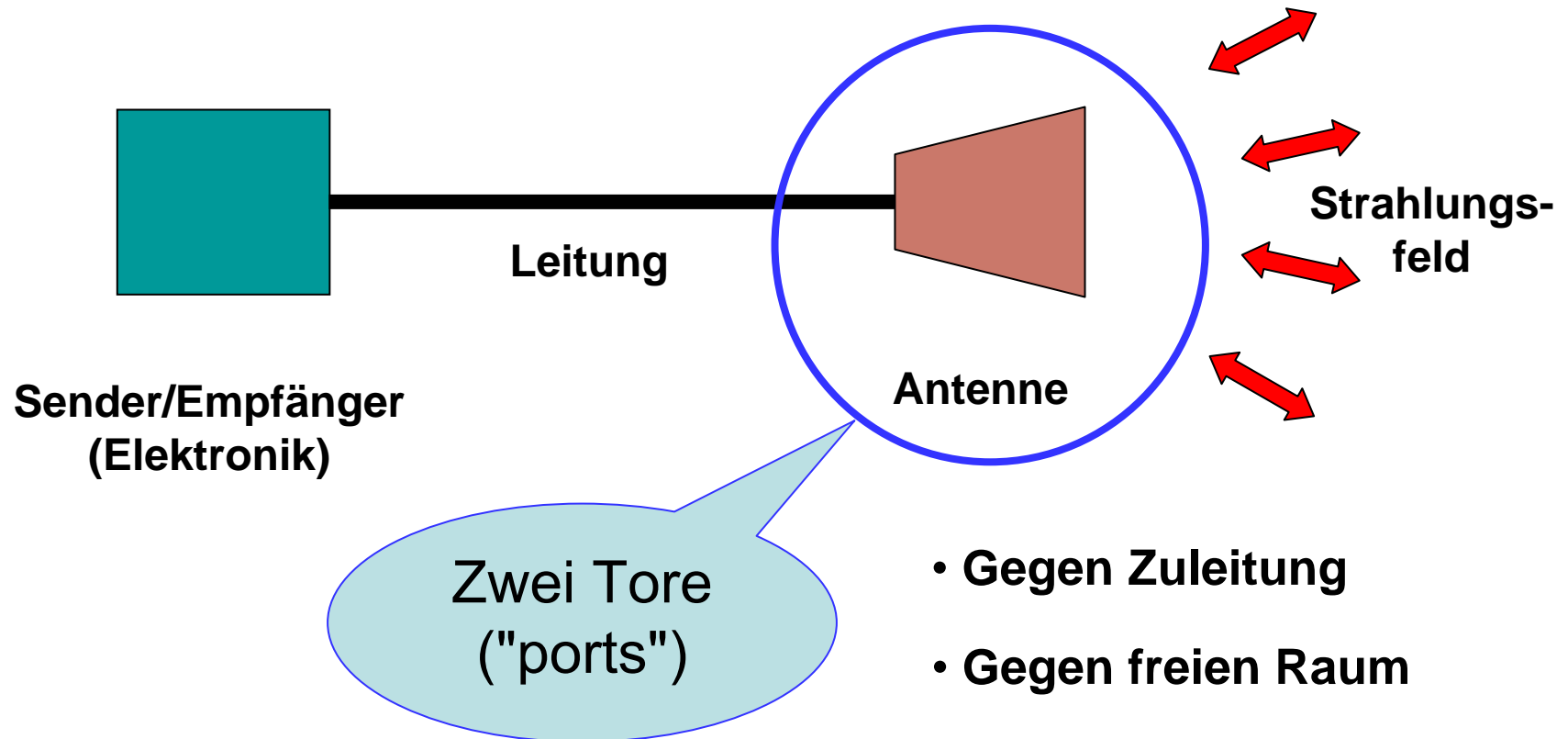


Übersicht

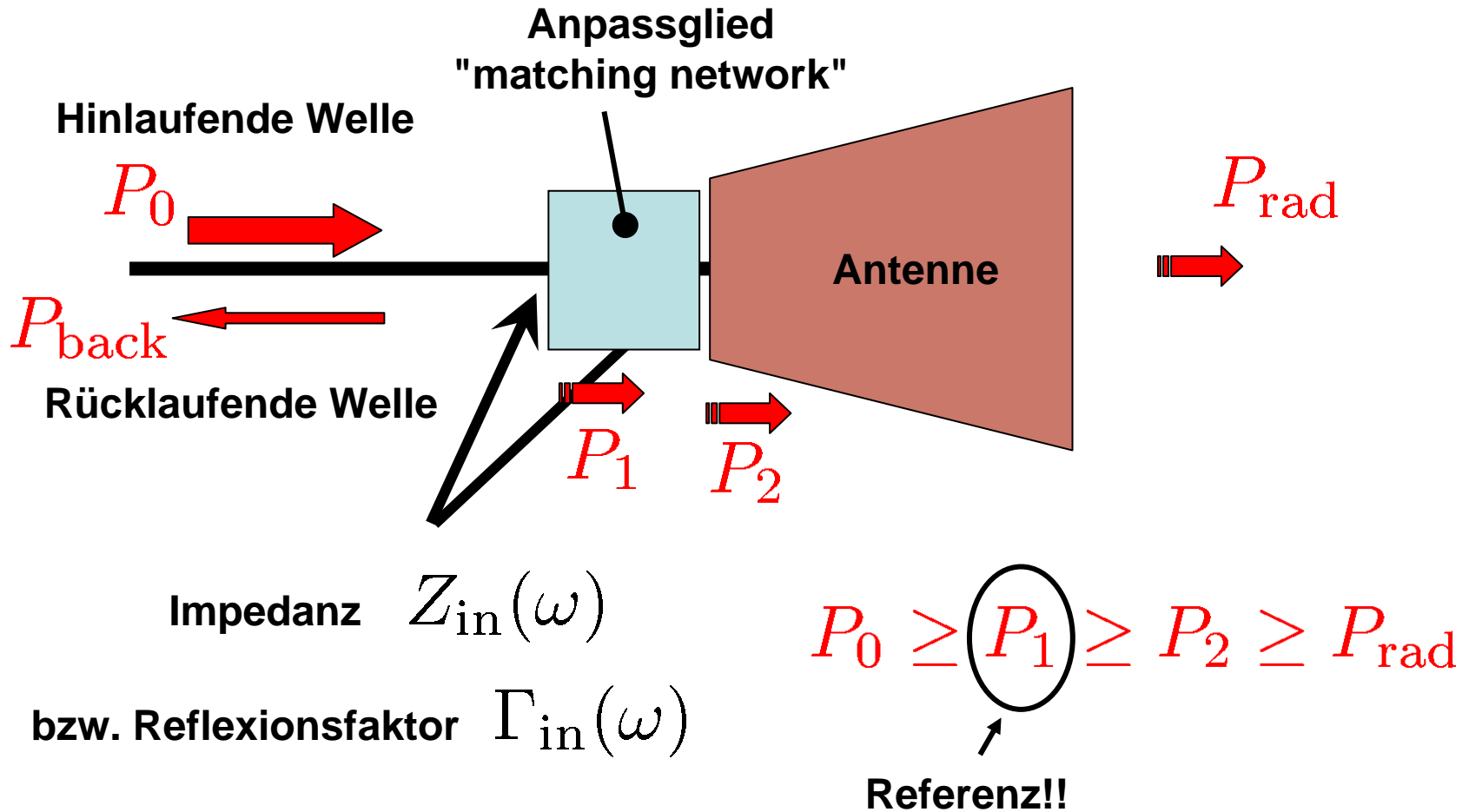
- **Antennen aus der Sicht der Speisung**
- **Antennen aus der Sicht des Fernfeldes**
- **Richtdiagramm und Strahldichte**
- **Wirkungsgrad, Richtfaktor, Gewinn, effektive Sendeleistung (ERP)**
- **Ersatzschaltung einer Antenne, Strahlungswiderstand**
- **Hertz'scher Dipol**

Antennen

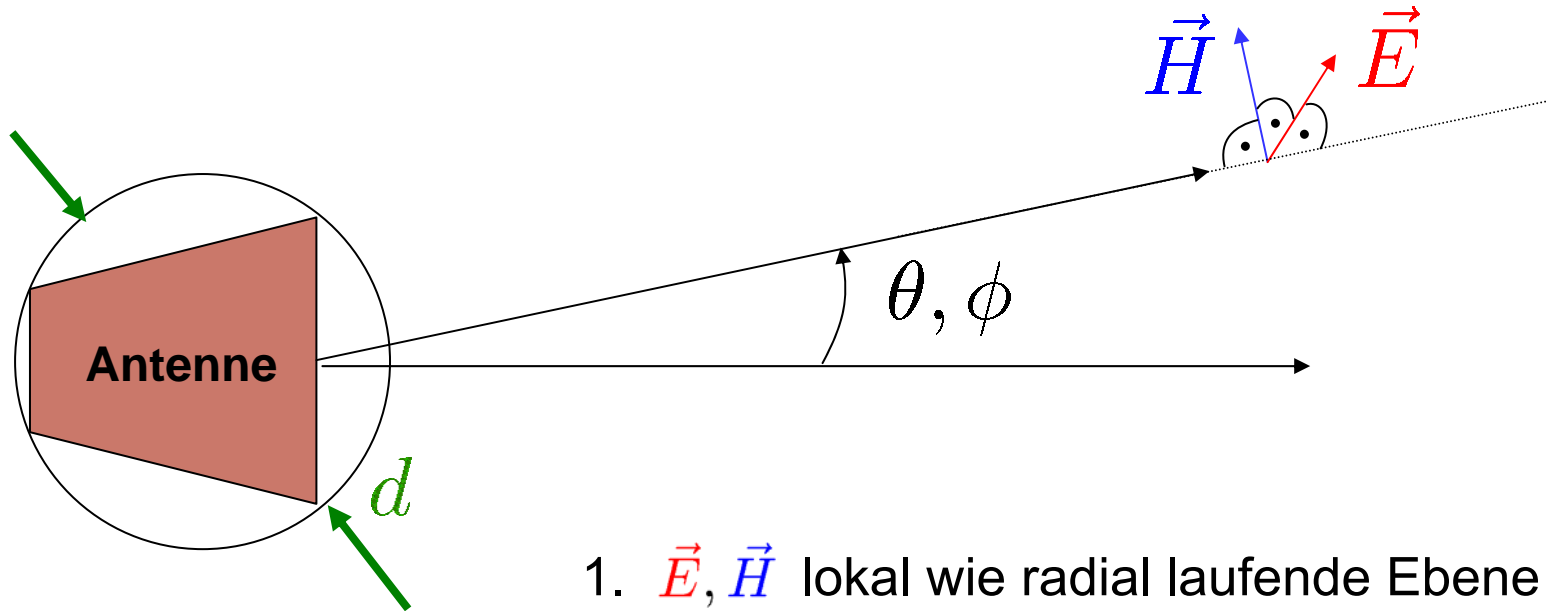
Es geht darum, elektromagnetische Energie in den freien Raum zu bringen bzw. von dort einzufangen.



Beschreibung des Antenneneingangs



Beschreibung des Fernfeldes



nur für
 $r \gg d$

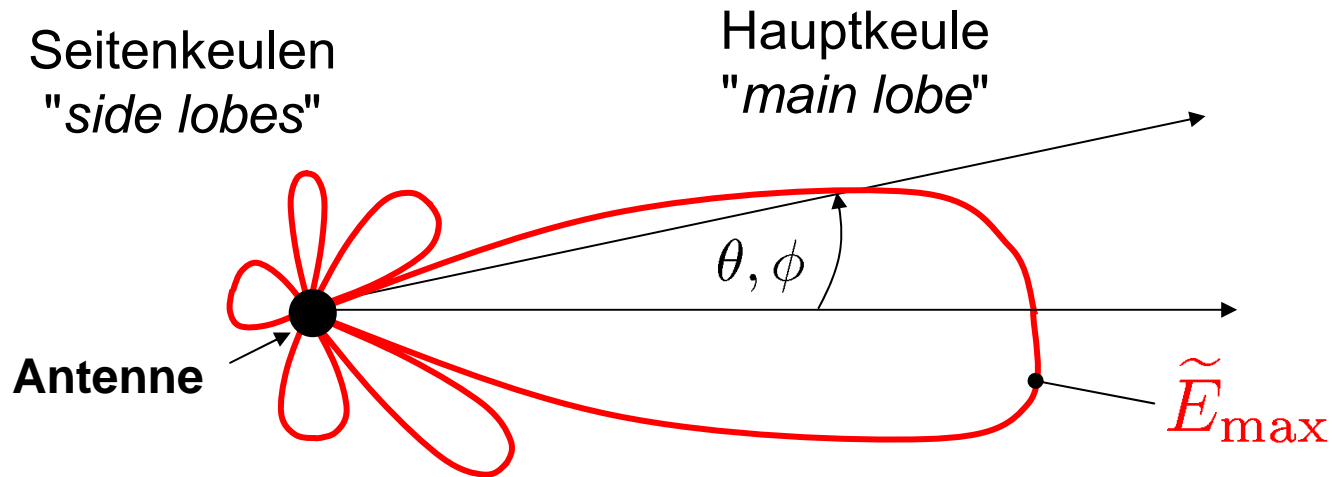
1. \vec{E}, \vec{H} lokal wie radial laufende Ebene Welle

2. $\vec{E}(r, \theta, \phi) = \frac{1}{r} \tilde{E}(\theta, \phi) \vec{e}_{\text{pol}}(\theta, \phi)$

Stärke
der Strahlung

Polarisation

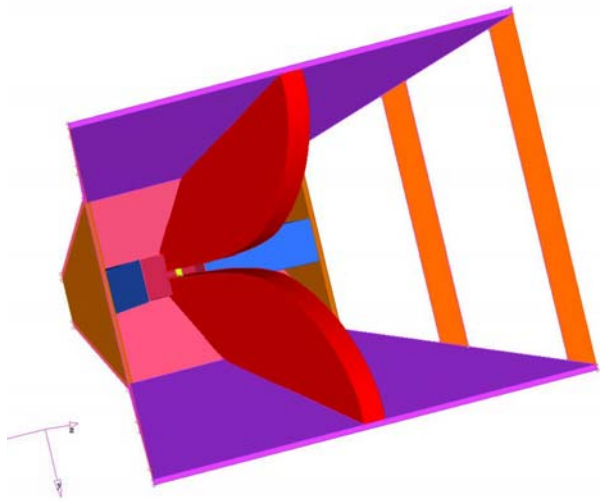
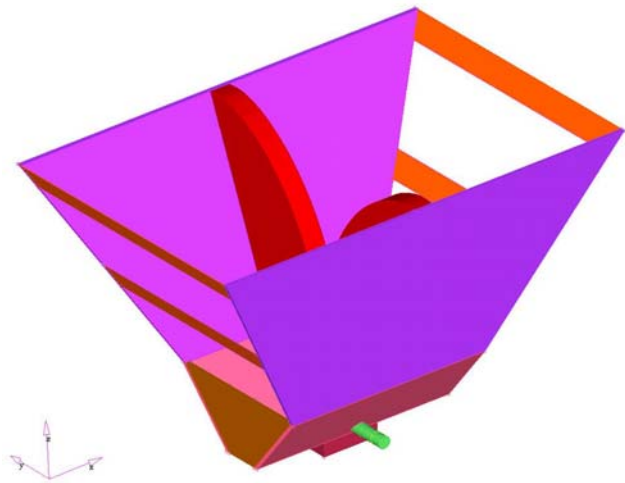
Beschreibung des Fernfeldes



$$\vec{E}(r, \theta, \phi) = \frac{1}{r} \tilde{E}(\theta, \phi) \vec{e}_{\text{pol}}(\theta, \phi)$$

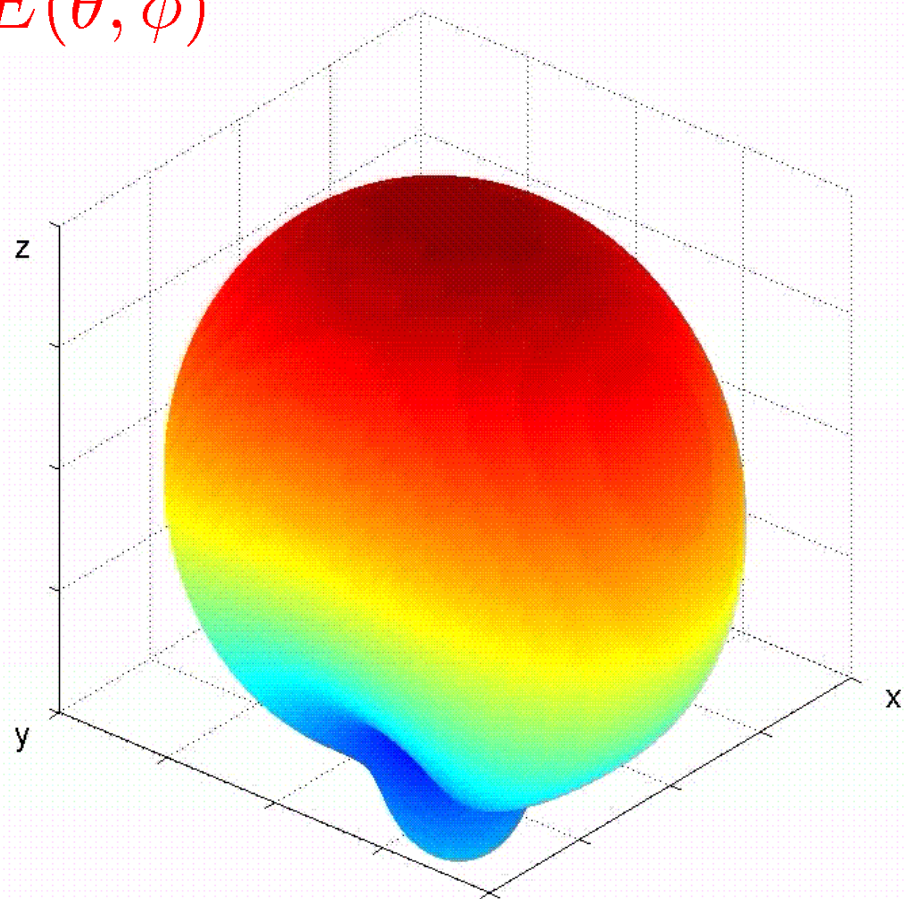
Strahlungsdiagramm
Richtdiagramm
Antennendiagramm
Richtcharakteristik
Strahlungscharakteristik
Empfangscharakteristik
"radiation pattern"

Beispiel ("double rigged horn antenna")



$$\tilde{E}(\theta, \phi)$$

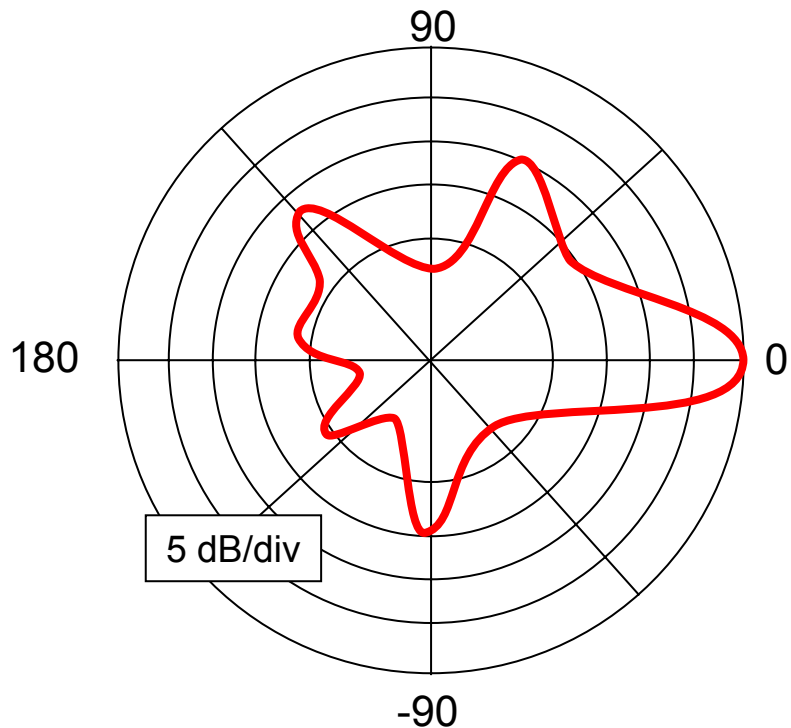
1 GHz



Felder & Komponenten II

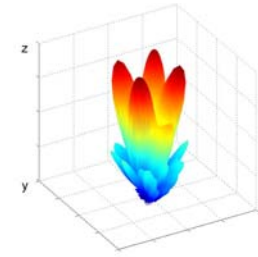
Weiteres zum Richtdiagramm

- Enthält (meist) **keine** Information zur Polarisation
- 3D-Darstellung gibt nur qualitativen Eindruck
- Darstellung in einzelnen Ebenen: $\tilde{E}(\theta) \Big|_{\phi=\text{const}}$

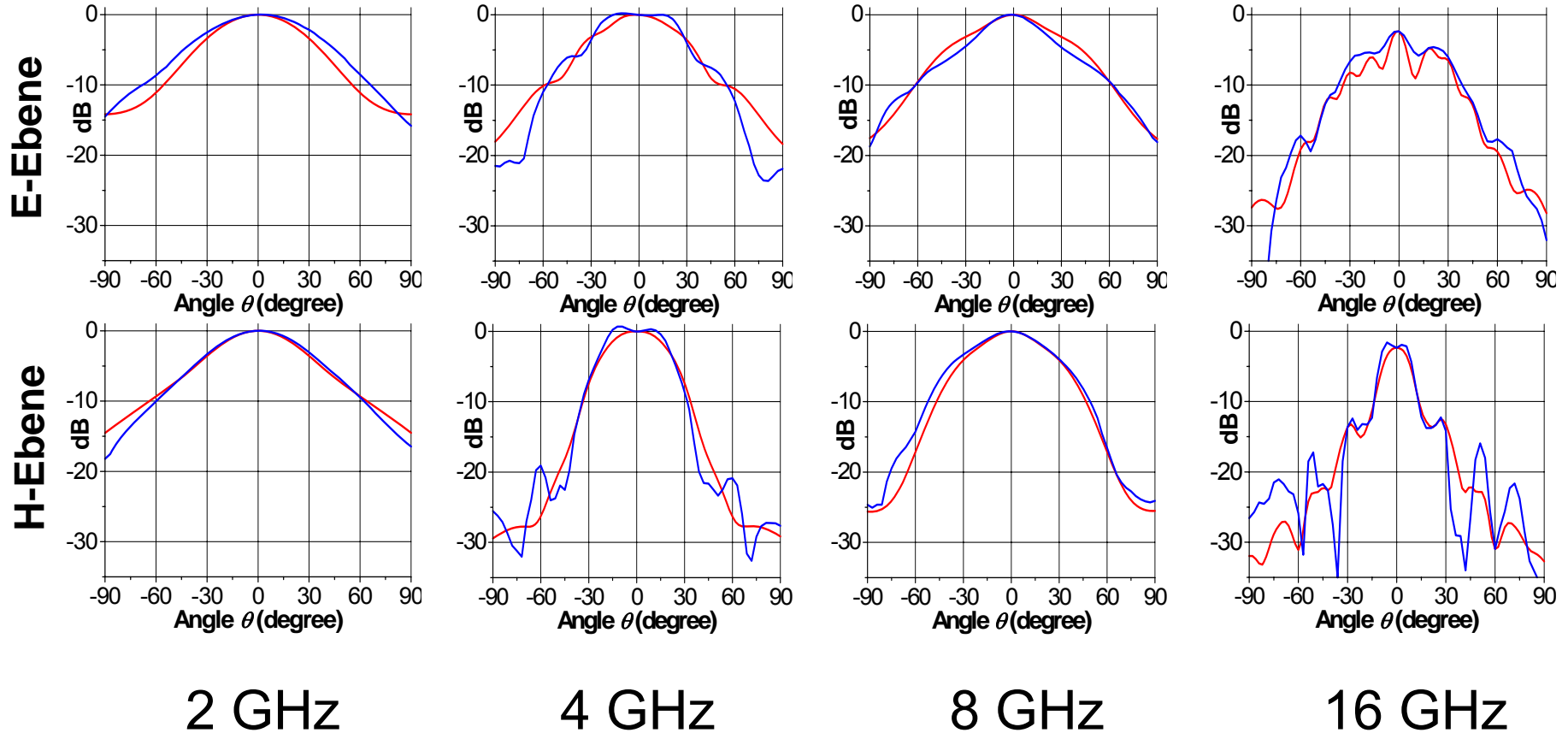


auch kartesische Darstellung!

kartesische Darstellung

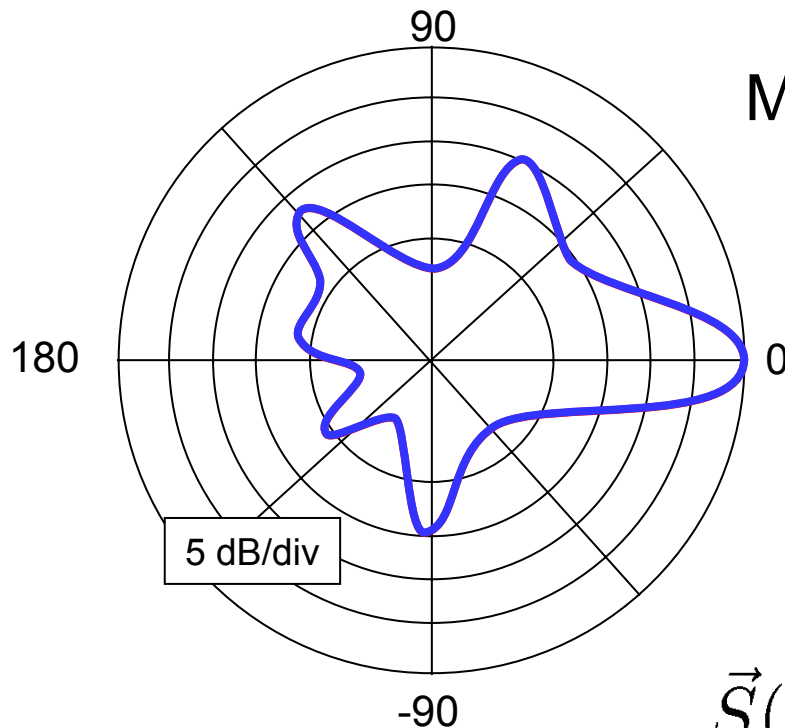


— Messung
— FVTD



Weiteres zum Richtdiagramm

Normalerweise bezogen auf $\tilde{E}(\theta, \phi)$ [V]



Manchmal auch auf **Strahldichte**

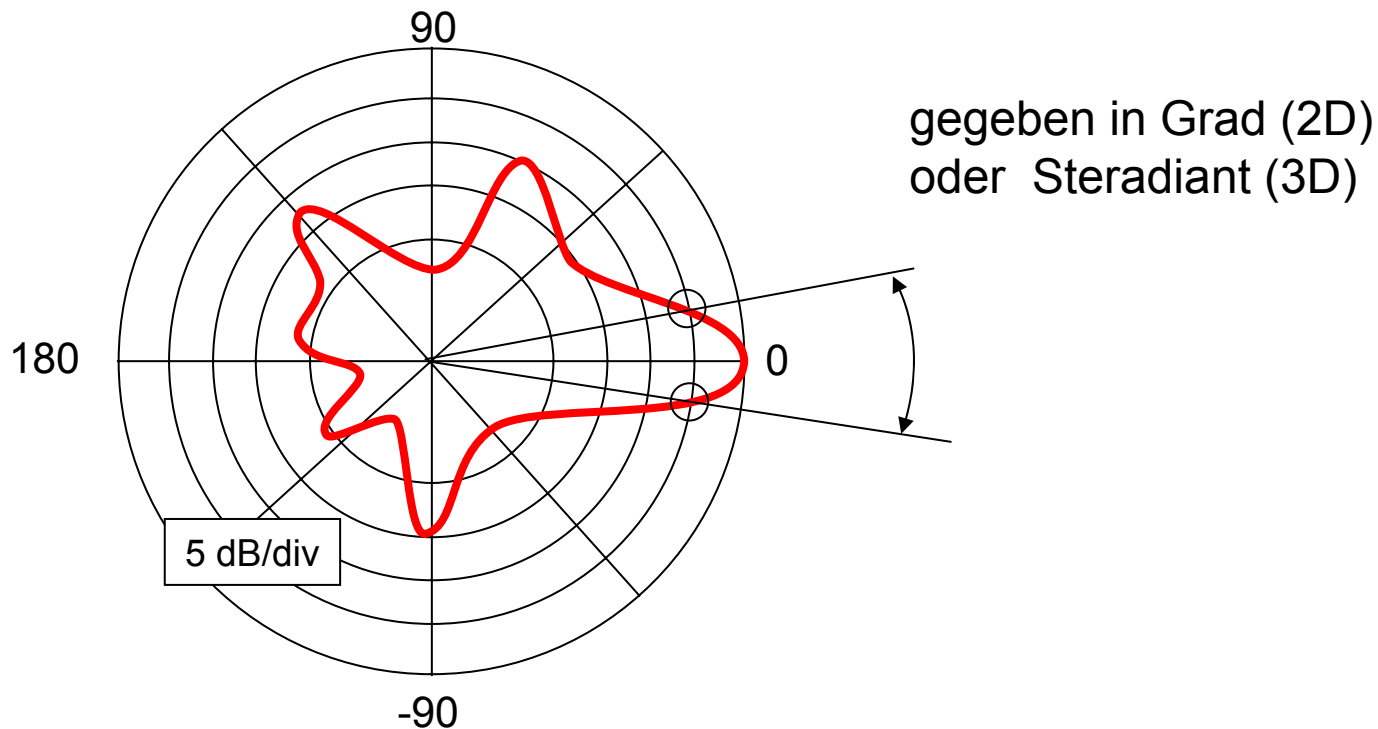
$$S(\theta, \phi) := \frac{\tilde{E}^2(\theta, \phi)}{Z_w} [\text{W}]$$

Achtung:
Nicht Poynting-Vektor!

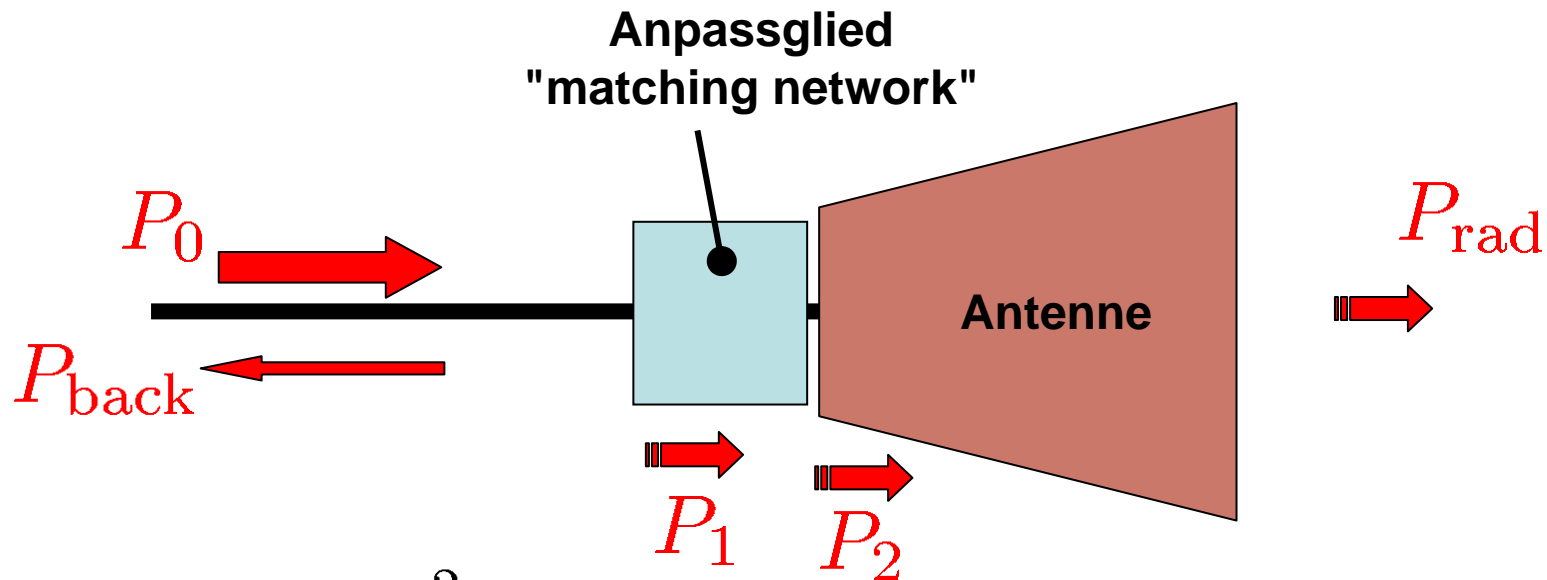
$$\vec{S}(r, \theta, \phi) \hat{=} \frac{S(\theta, \phi)}{r^2} \vec{e}_r \left[\frac{\text{W}}{\text{m}^2} \right]$$

Weiteres zum Richtdiagramm

- (x-dB-)Keulenzbreite ("*beam width*")



Strahlungsleistung und Wirkungsgrad

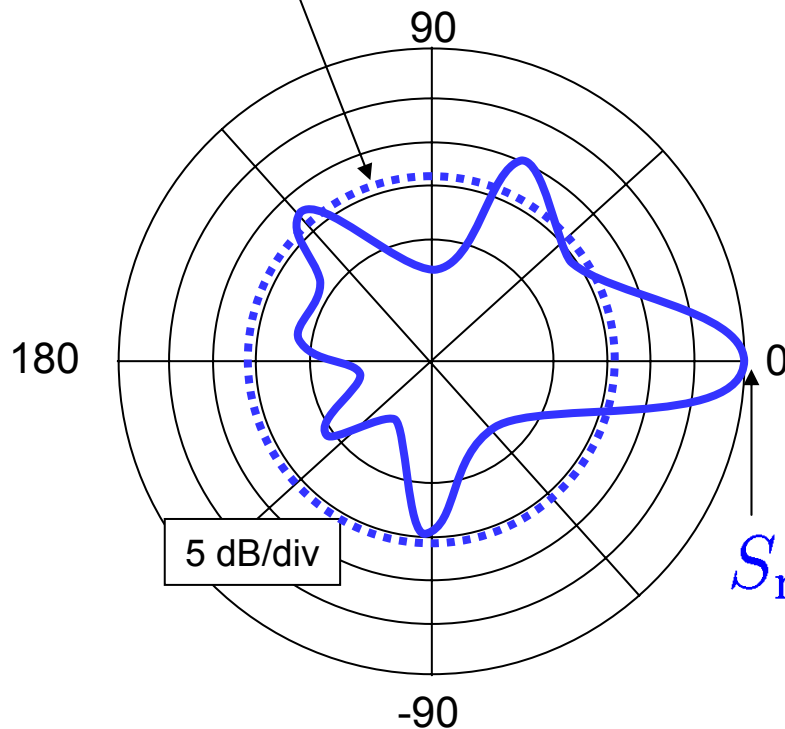


$$P_{\text{rad}} = \int_0^{\pi} \int_0^{2\pi} S(\theta, \phi) d\phi \sin \theta d\theta \quad [\text{W}]$$

Antennenwirkungsgrad $\eta = \frac{P_{\text{rad}}}{P_{1,2}}$

Richtfaktor D ("directivity")

Richtdiagramm eines isotropen Strahlers gleicher Leistung:



$$\cancel{S_{\text{iso}}(\theta, \phi)} = \frac{P_{\text{rad}}}{4\pi}$$

Richtfaktor:

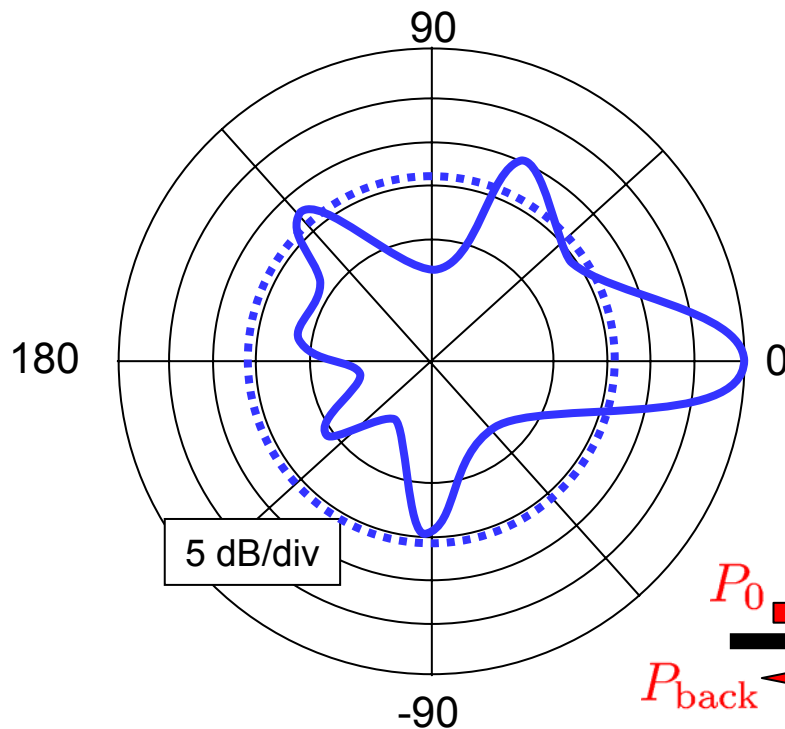
$$D = \frac{S_{\text{max}}}{S_{\text{iso}}} = 4\pi \frac{S_{\text{max}}}{P_{\text{rad}}}$$

$$S_{\text{max}} > S_{\text{iso}}$$

Antennen-Gewinn G ("gain")

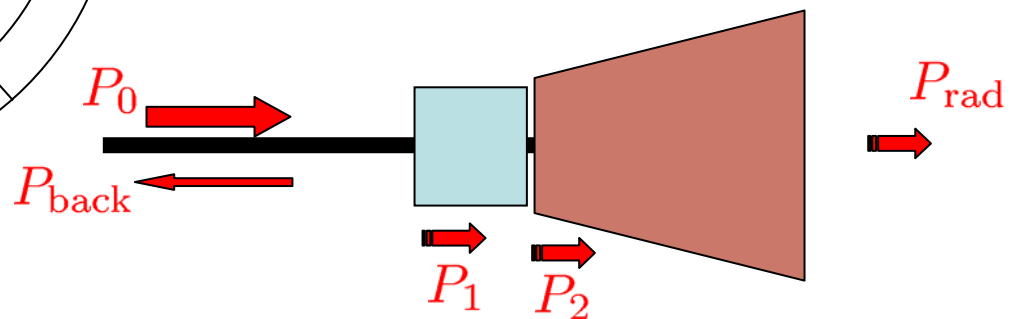
$$D = \frac{S_{\max}}{S_{\text{iso}}} = 4\pi \frac{S_{\max}}{P_{\text{rad}}}$$

$$\eta = \frac{P_{\text{rad}}}{P_{1,2}}$$



Antennen-Gewinn:

$$G = 4\pi \frac{S_{\max}}{P_{1,2}} = \eta \cdot D$$



Antennen-Gewinn: Beispiel 1

Horn mit Spiegel: gemäss Hersteller **G = 42 dB**

$$f = 1 \dots 18 \text{ GHz}$$

gemessen: **G = 32 dB**



Nur 10% der erwarteten Leistung
im Bereich der Hauptkeule!

Antennen-Gewinn: Beispiel 2

$$f = 380 \dots 400 \text{ MHz}$$

Undercover-Antenne: Bestehendes System **G = -26 dB**
(Draht-Dipol, fast auf der Haut)



Prototyp: **G = -12 dB**

Ideale Antenne hätte
G = 0 dB wegen
Rundstrahlcharakteristik!

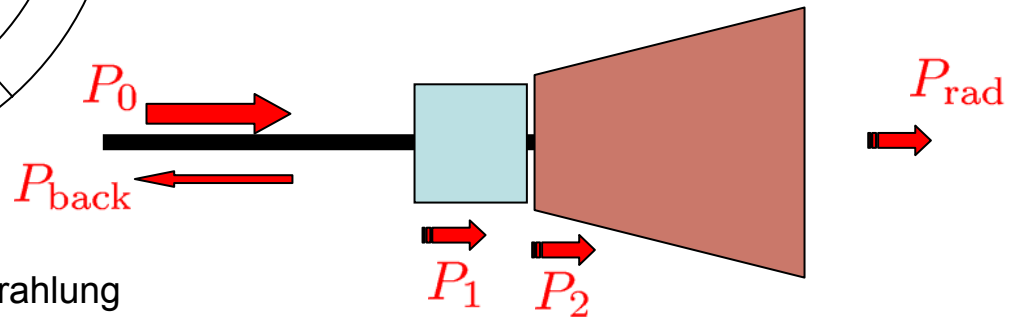
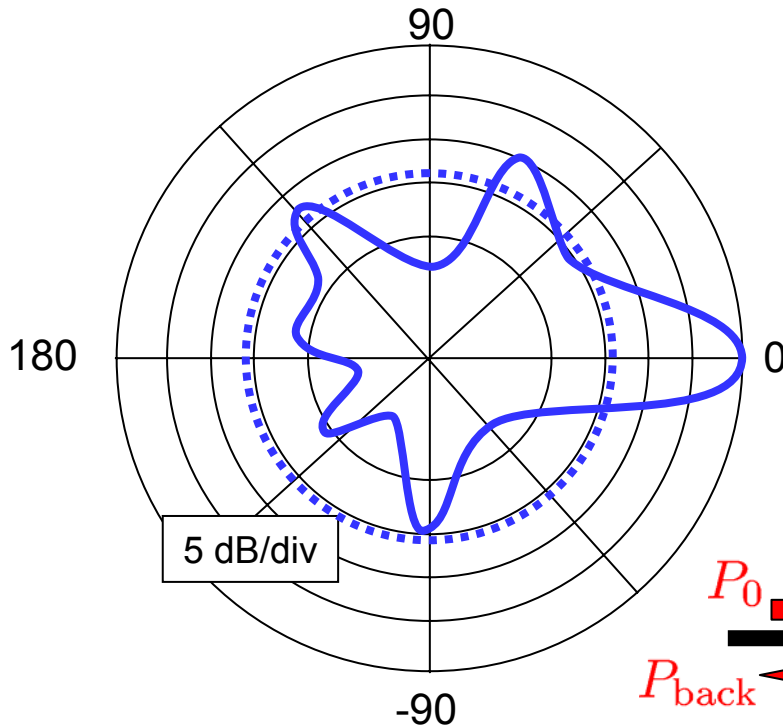
Effektive Sendeleistung ("effective radiated power", ERP)

Welche Leistung wäre nötig, wenn die tatsächliche Antenne ein **Kugelstrahler** wäre, der die gleiche Leistung P_{eff} in die gleiche Hauptstrahlrichtung abstrahlt?

$$G = 4\pi \frac{S_{\text{max}}}{P_{1,2}}$$

$$\frac{P_{\text{eff}}}{4\pi} = S_{\text{max}} = \frac{G P_{1,2}}{4\pi}$$

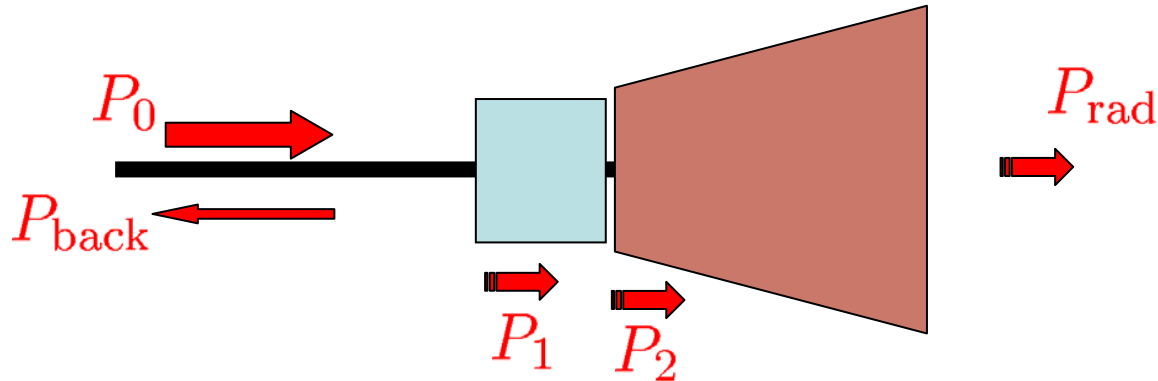
$$P_{\text{eff}} = G \cdot P_{1,2}$$



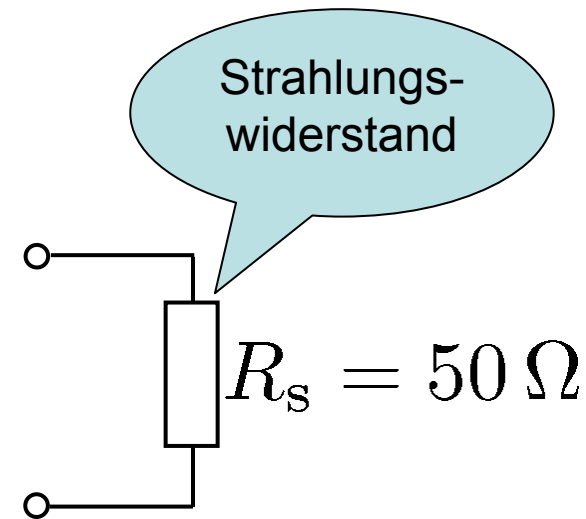
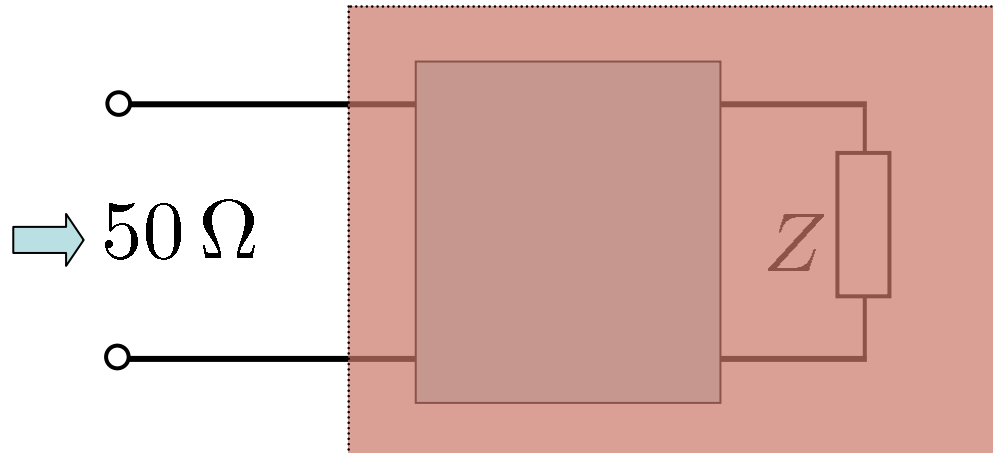
*Verordnung über die Nicht ionisierende Strahlung

$G = 20\text{dB} \rightarrow 10\text{ W reale Leistung ergeben } 1\text{ kW ERP}$

Ersatzschaltung einer Antenne

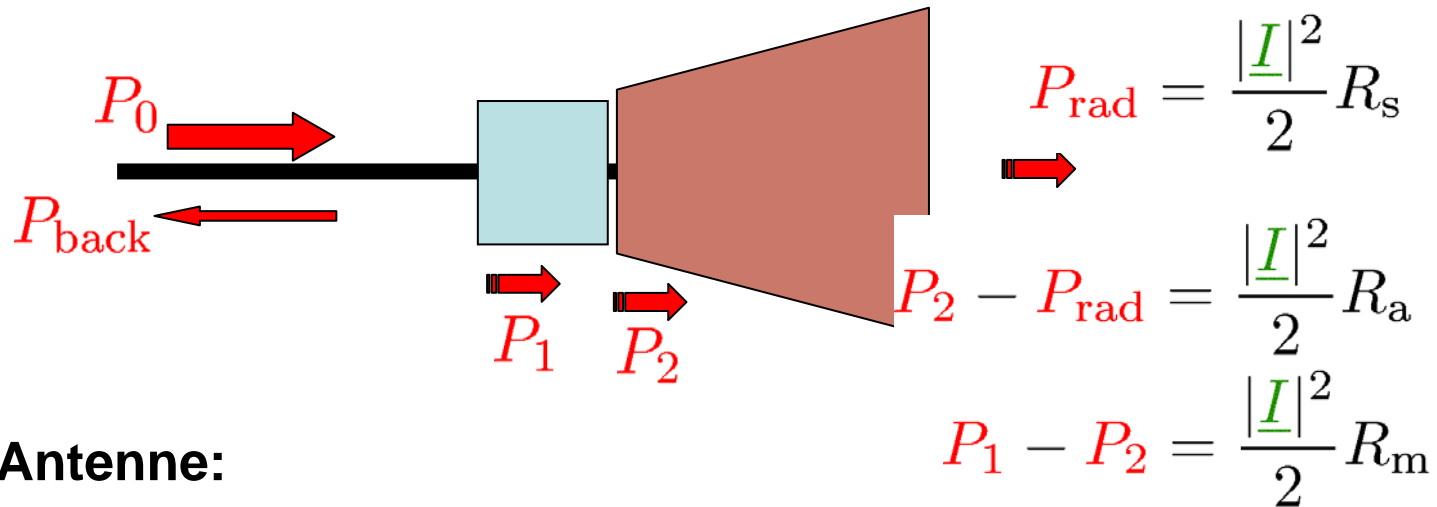


1. Ideale Antenne: $P_0 = P_1 = P_2 = P_{\text{rad}}$

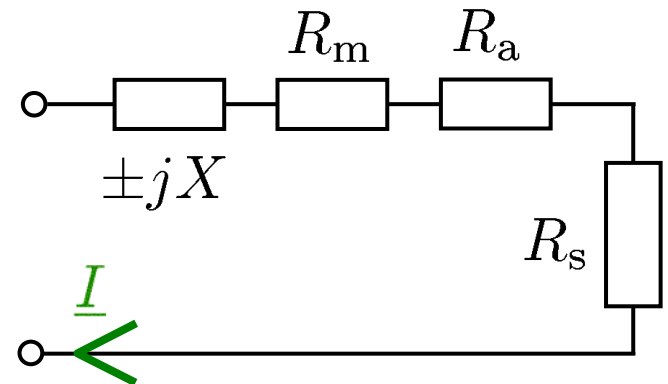
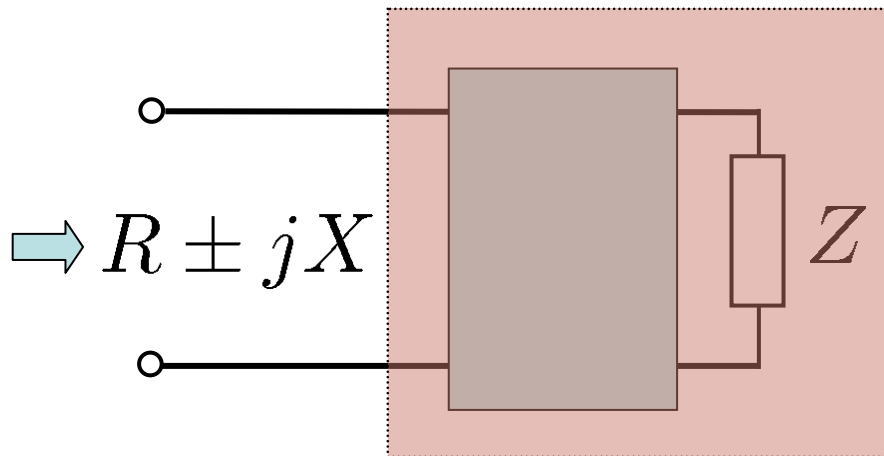


Felder & Komponenten II

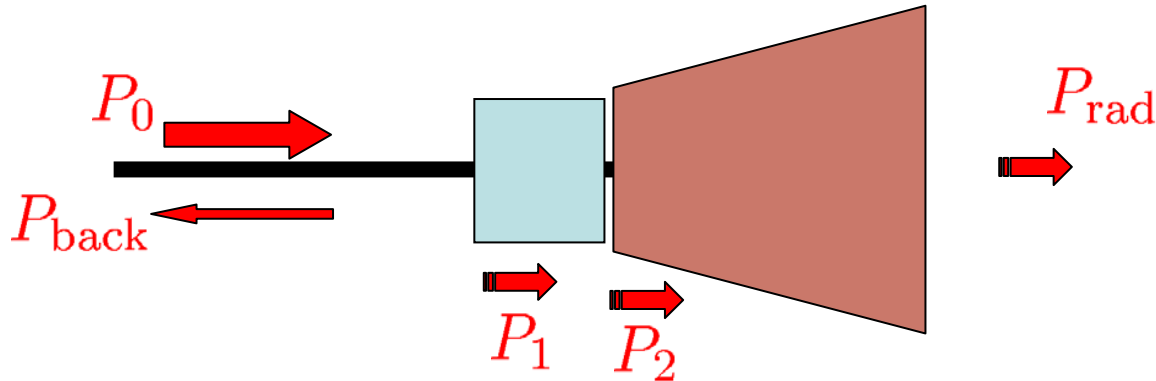
Ersatzschaltung einer Antenne



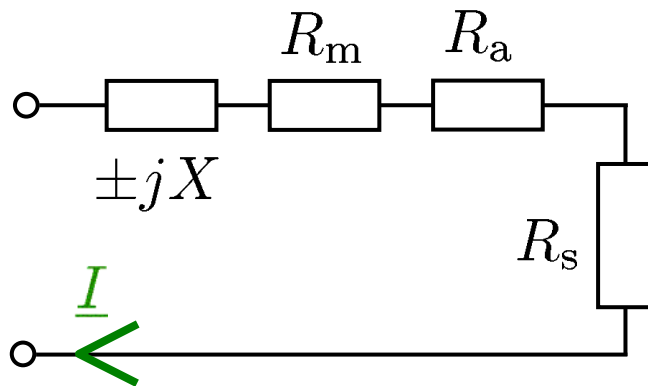
2. Reale Antenne:



Ersatzschaltung einer Antenne

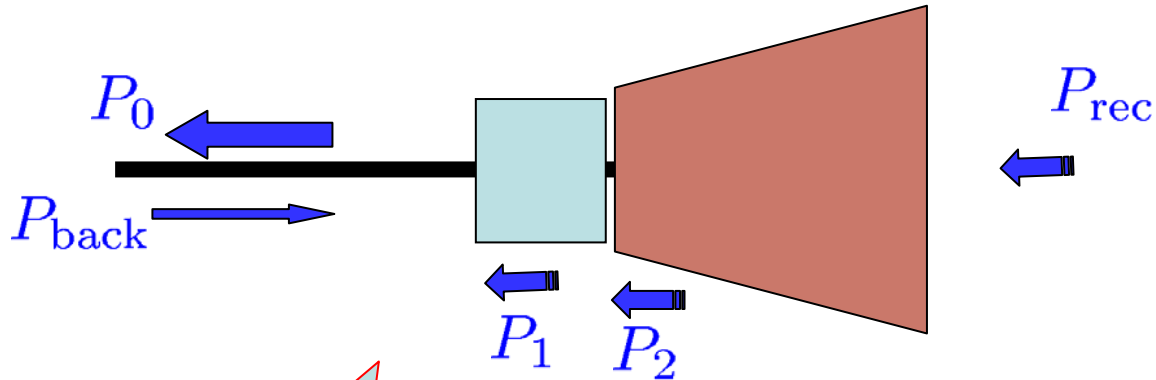


2. Reale Antenne:



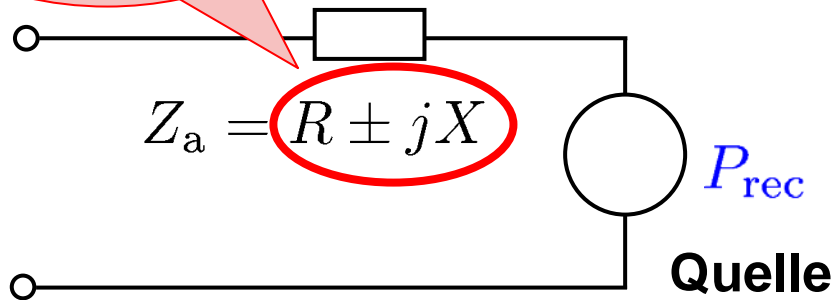
kompliziertere Ersatzschaltungen
sind möglich!
(z.B. um grösseren Frequenzbereich
abzudecken)

Antenne beim Empfangen



Wie beim Senden!

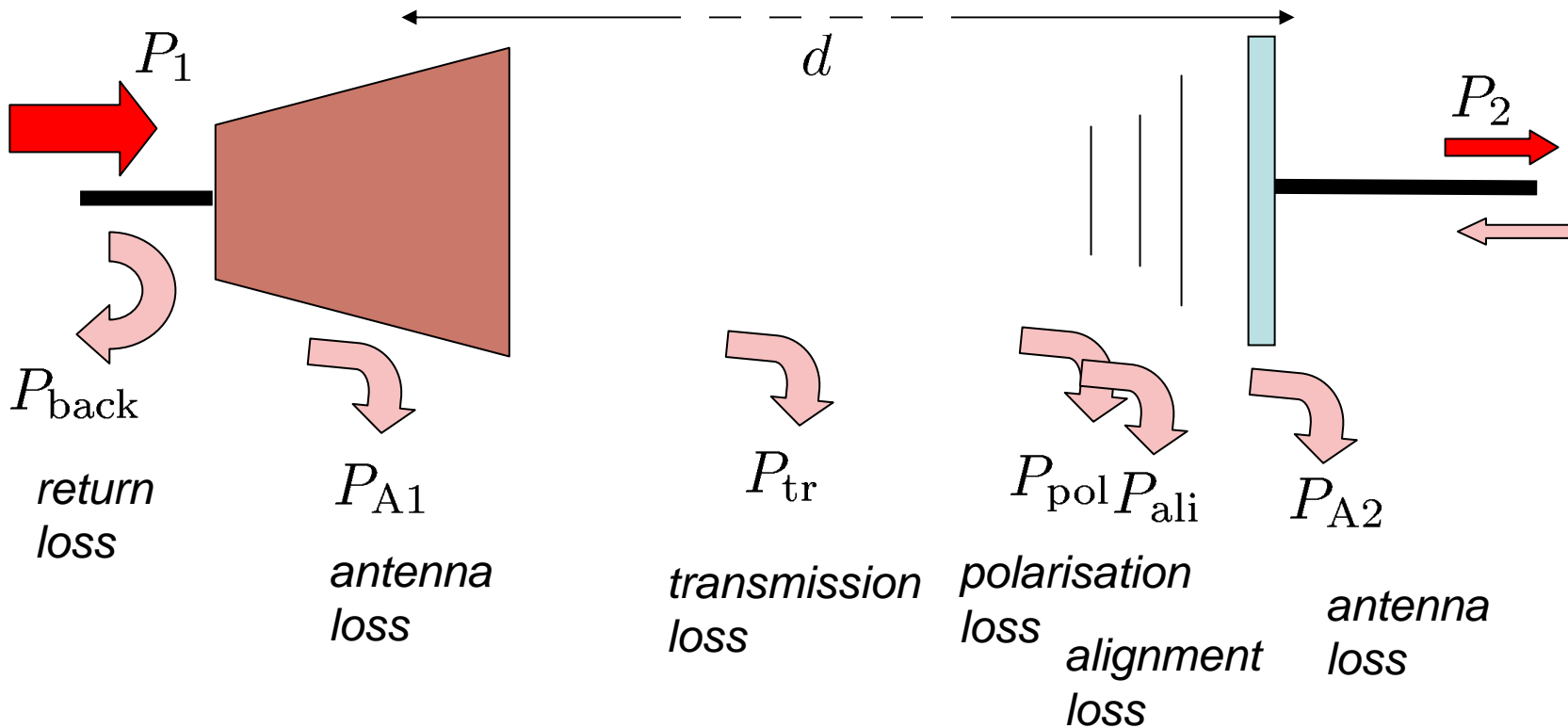
Reziprozität



Empfangscharakteristik
=
Sendecharakteristik

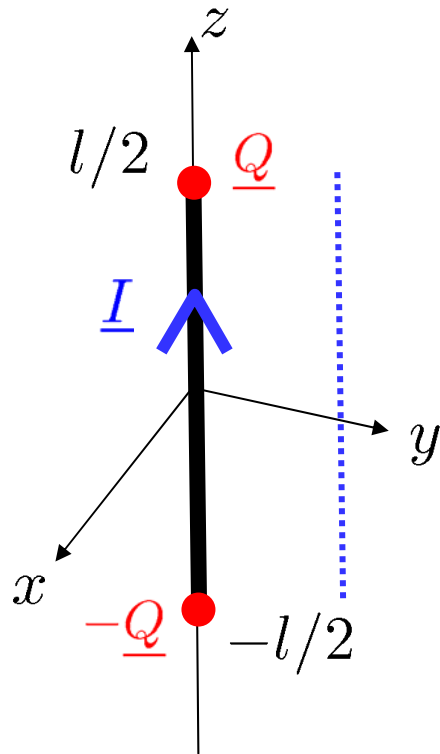
Gewinn G
Richtfaktor D

Drahtlose Übertragungsstrecke



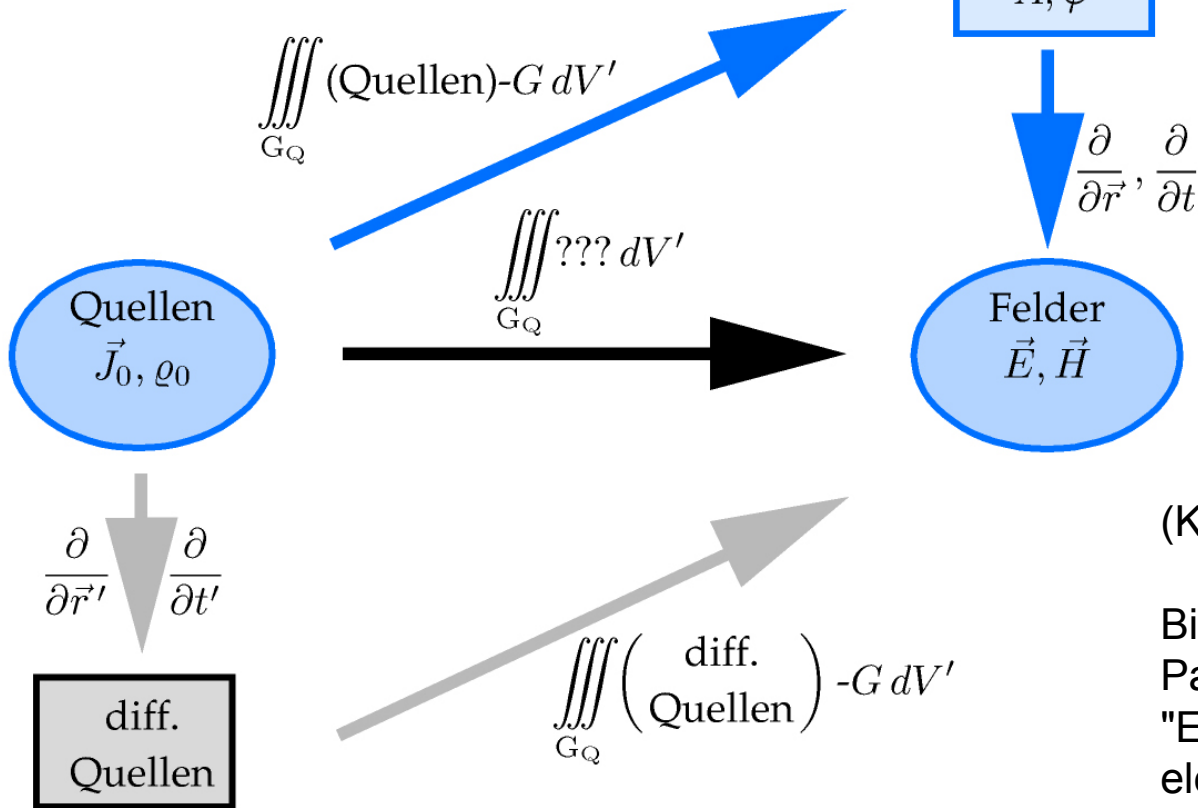
Leistungsanpassung wo immer möglich!!

Hertz'scher Dipol



Berechnung des Feldes
explizit und analytisch
möglich!

$$\underline{\vec{A}}(\vec{r}) = \frac{\mu}{4\pi} \iiint_{V'} \frac{\underline{\vec{J}} \cdot e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} dV'$$



$$\underline{\vec{E}} = -\text{grad } \underline{\varphi} - j\omega \underline{\vec{A}}$$

$$\underline{\vec{H}} = \frac{1}{\mu} \text{rot } \underline{\vec{A}}$$

(Kapitel 6, Fuk I)

Bild aus:
 Pascal Leuchtmann:
 "Einführung in die
 elektromagnetische Feldtheorie",
 Pearson, 2005.

Quellpunkt \vec{r}', t'

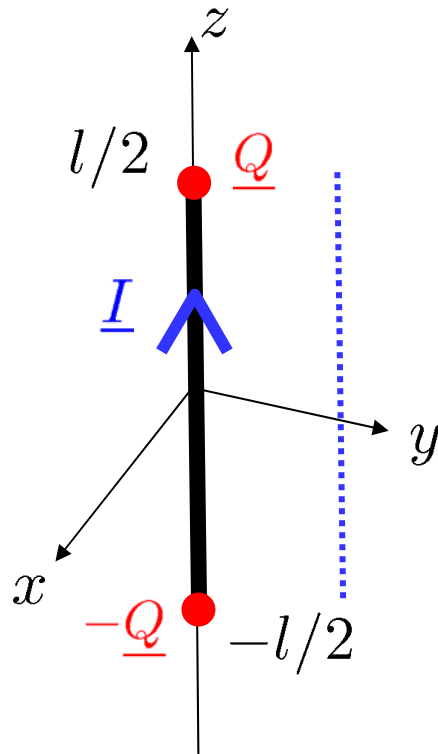
$$R := |\vec{r} - \vec{r}'|$$

$$t' := t - \frac{R}{v}$$

Aufpunkt \vec{r}, t

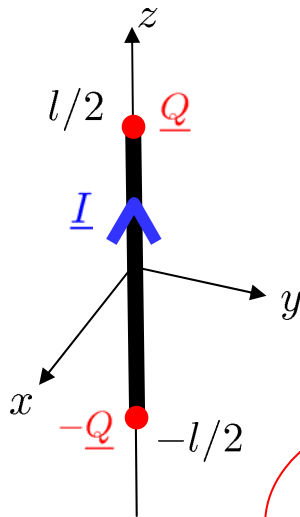
Felder & Komponenten II

Hertz'scher Dipol



1. Berechnung des Vektorpotentials $\underline{\vec{A}}(\vec{r})$ für alle "fernen" Punkte \vec{r}
2. Berechnung der Felder aus $\underline{\vec{A}}(\vec{r})$
3. Diskussion des Feldverlaufs
4. Berechnung der Antennenparameter (soweit sinnvoll)

Hertz'scher Dipol: Vektorpotential $\underline{\vec{A}}(\vec{r})$



$$\underline{\vec{A}}(\vec{r}) = \frac{\mu}{4\pi} \iiint_{V'} \frac{\underline{\vec{J}} \cdot e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} dV'$$

$$= \frac{\mu \underline{I}}{4\pi} \vec{e}_z \int_{-l/2}^{l/2} \frac{e^{-jk\sqrt{x^2+y^2+(z-z')^2}}}{\sqrt{x^2+y^2+(z-z')^2}} dz'$$

$$dV' = dF \cdot dz'$$

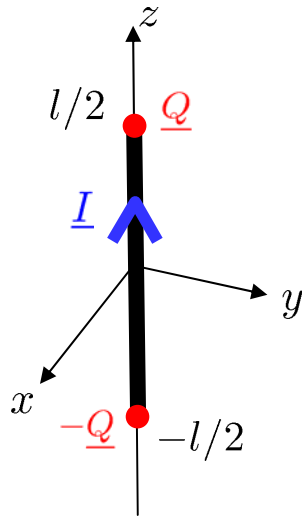
$$\iint \underline{\vec{J}} dF = \underline{I} \cdot \vec{e}_z$$

$$= \frac{\mu \underline{I} l}{4\pi} \vec{e}_z \frac{e^{-jk\sqrt{x^2+y^2+z^2}}}{\sqrt{x^2+y^2+z^2}} = \frac{\mu \underline{I} l}{4\pi} \vec{e}_z \frac{e^{-jkr}}{r}$$

$$x^2 + y^2 + z^2 \gg l^2$$

$$kl \ll 1$$

Hertz'scher Dipol: Felder aus $\underline{\vec{A}}(\vec{r})$



$$\underline{\vec{A}}(\vec{r}) = \frac{\mu \underline{I} l}{4\pi} \frac{e^{-jkr}}{r} \vec{e}_z$$

$$\vec{e}_z = \vec{e}_r \cos \theta - \vec{e}_\theta \sin \theta$$

$$\underline{\varphi} = \frac{\text{div } \underline{\vec{A}}}{-j\omega\mu\epsilon}$$

Lorentz-Eichung

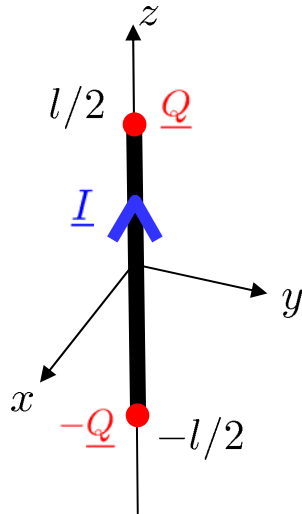
$$\underline{\vec{E}} = -\text{grad } \underline{\varphi} - j\omega \underline{\vec{A}} = \frac{\text{grad div } \underline{\vec{A}}}{j\omega\mu\epsilon} - j\omega \underline{\vec{A}}$$

$$\underline{\vec{H}} = \frac{1}{\mu} \text{rot } \underline{\vec{A}}$$

$$\underline{A}_r(r, \theta) = \underline{A}_z(r) \cos \theta = \frac{\mu \underline{I} l}{4\pi} \frac{e^{-jkr}}{r} \cos \theta$$

$$\underline{A}_\theta(r, \theta) = -\underline{A}_z(r) \sin \theta = -\frac{\mu \underline{I} l}{4\pi} \frac{e^{-jkr}}{r} \sin \theta$$

Hertz'scher Dipol: Felder aus $\underline{\vec{A}}(\vec{r})$



$$\underline{\vec{A}}(\vec{r}) = \frac{\mu \underline{I} l}{4\pi} \vec{e}_z \frac{e^{-jkr}}{r}$$

$$\underline{\vec{E}} = \frac{\text{grad div } \underline{\vec{A}}}{j\omega\mu\epsilon} - j\omega \underline{\vec{A}}$$

$$\underline{\vec{H}} = \frac{1}{\mu} \text{rot } \underline{\vec{A}}$$

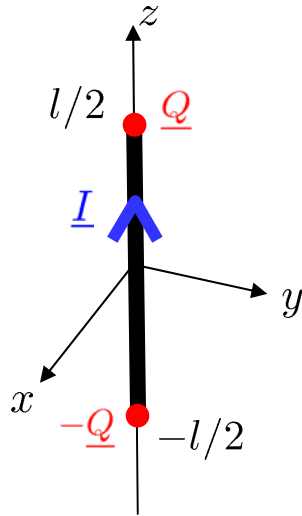
$$\underline{A}_r(r, \theta) = \frac{\mu \underline{I} l}{4\pi} \frac{e^{-jkr}}{r} \cos \theta$$

$$\underline{A}_\theta(r, \theta) = -\frac{\mu \underline{I} l}{4\pi} \frac{e^{-jkr}}{r} \sin \theta$$

$$\text{rot } \underline{\vec{A}} = \frac{\mu \underline{I} l}{4\pi} \left(jk + \frac{1}{r} \right) \vec{e}_\phi \frac{e^{-jkr}}{r} \sin \theta$$

$$\text{div } \underline{\vec{A}} = -\frac{\mu \underline{I} l}{4\pi} \left(jk + \frac{1}{r} \right) \frac{e^{-jkr}}{r} \cos \theta$$

Hertz'scher Dipol: Felder aus $\underline{\vec{A}}(\vec{r})$



$$\underline{\vec{A}}(\vec{r}) = \frac{\mu \underline{I} l}{4\pi} \vec{e}_z \frac{e^{-jkr}}{r}$$

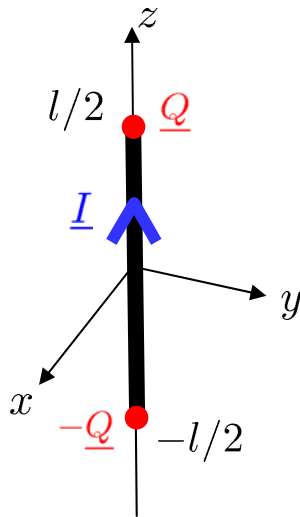
$$\underline{E}_r = \frac{\underline{I} l}{2\pi j \omega \epsilon} \frac{e^{-jkr}}{r^3} (1 + jkr) \cos \theta$$

$$\underline{E}_\theta = \frac{\underline{I} l}{4\pi j \omega \epsilon} \frac{e^{-jkr}}{r^3} (1 + jkr - k^2 r^2) \sin \theta$$

$$\underline{H}_\phi = \frac{\underline{I} l}{4\pi} \frac{e^{-jkr}}{r^2} (1 + jkr) \sin \theta$$

weitere Komponenten verschwinden!

Hertz'scher Dipol: Feldverlauf



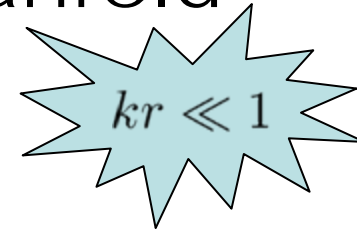
$$\text{Dipolmoment } \underline{p} := -\frac{\underline{I}l}{4\pi j\omega\varepsilon} = -\frac{\underline{I}lZ_w}{4\pi jk}$$

$$\underline{E}_r = -2\underline{p}\frac{e^{-jkr}}{r^3}(1 + jkr)\cos\theta$$

$$\underline{E}_\theta = -\underline{p}\frac{e^{-jkr}}{r^3}(1 + jkr - k^2r^2)\sin\theta$$

$$\underline{H}_\phi = -\underline{p}\frac{jk}{Z_w}\frac{e^{-jkr}}{r^2}(1 + jkr)\sin\theta$$

Hertz'scher Dipol: Nahfeld


$$kr \ll 1$$

$$\underline{E}_r = -2\underline{p} \frac{e^{-jkr}}{r^3} (1 + jkr) \cos \theta = -2 \frac{\underline{p}}{r^3} \cos \theta$$

$$\underline{E}_\theta = -\underline{p} \frac{e^{-jkr}}{r^3} (1 + jkr - k^2 r^2) \sin \theta = -\frac{\underline{p}}{r^3} \sin \theta$$

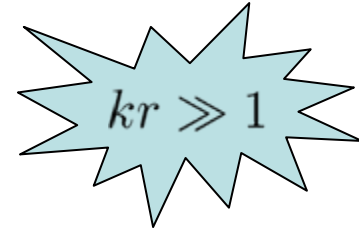
Elektrisches
Feld
dominiert!

$$\underline{H}_\phi = -\underline{p} \frac{jk}{Z_w} \frac{e^{-jkr}}{r^2} (1 + jkr) \sin \theta = -\underline{p} \frac{jk}{Z_w} \frac{1}{r^2} \sin \theta$$

90° phasenverschoben

"keine" radiale Leistung im Mittel?! $\Re \underline{\underline{S}} = \frac{1}{2} \Re(\underline{\underline{E}} \times \underline{\underline{H}}^*)$

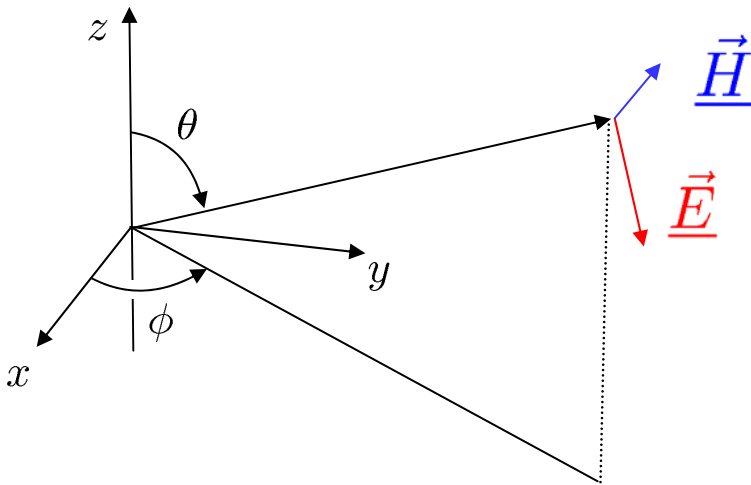
Hertz'scher Dipol: Fernfeld



$$\underline{E}_r = -2\underline{p} \frac{e^{-jkr}}{r^3} (1 + jkr) \cos \theta = \underline{0}$$

$$\underline{E}_\theta = -\underline{p} \frac{e^{-jkr}}{r^3} (1 + jkr - k^2 r^2) \sin \theta = \underline{p} k^2 \frac{e^{-jkr}}{r} \sin \theta$$

$$\underline{H}_\phi = -\underline{p} \frac{jk}{Z_w} \frac{e^{-jkr}}{r^2} (1 + jkr) \sin \theta = \underline{p} \frac{k^2}{Z_w} \frac{e^{-jkr}}{r} \sin \theta = \frac{1}{Z_w} \underline{E}_\theta$$



Ebene Welle in
radialer Richtung!

Hertz'scher Dipol: Strahldichte

$$\underline{E}_\theta = \underline{p} k^2 \frac{e^{-jkr}}{r} \sin \theta$$

$$\underline{p} := -\frac{\underline{I}l}{4\pi j\omega\epsilon} = -\frac{\underline{I}lZ_w}{4\pi jk}$$

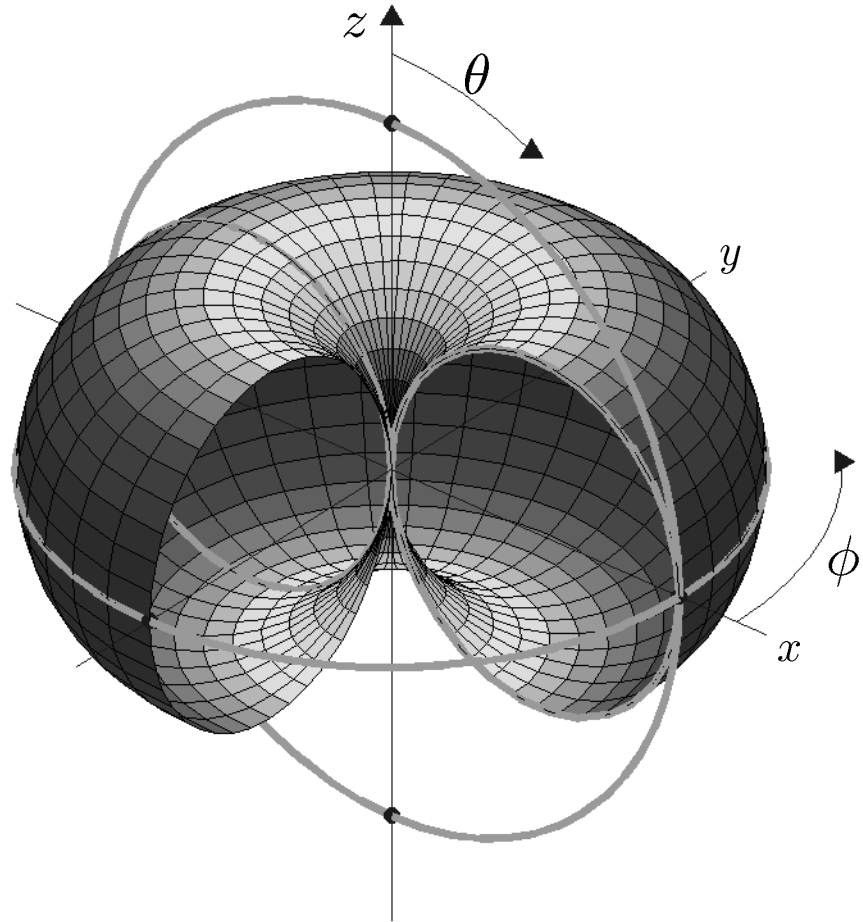
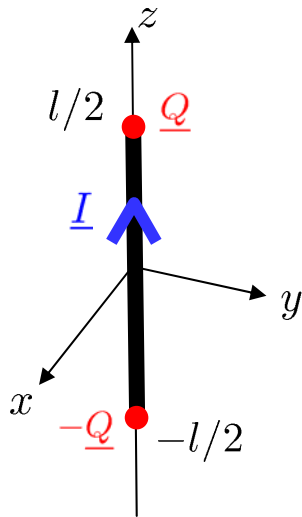
$$\underline{H}_\phi = \underline{p} \frac{k^2}{Z_w} \frac{e^{-jkr}}{r} \sin \theta = \frac{1}{Z_w} \underline{E}_\theta$$

$$S = \frac{|\underline{I}|^2 Z_w}{8} \left(\frac{l}{\lambda}\right)^2 \sin^2 \theta$$

Nur für $\frac{l}{\lambda} \ll 1$

$$S = r^2 \frac{1}{2} \Re \left\{ \underline{\vec{E}} \times \underline{\vec{H}}^* \right\} \cdot \underline{\vec{e}}_r = \frac{r^2}{2} \Re \left\{ \underline{E}_\theta \underline{H}_\phi^* \right\} = \frac{|\underline{p}|^2 k^4}{2Z_w} \sin^2 \theta$$

Hertz'scher Dipol: Strahlungsdiagramm



$$S = \frac{|\underline{I}|^2 Z_w}{8} \left(\frac{l}{\lambda} \right)^2 \sin^2 \theta$$

Hertz'scher Dipol: Abgestrahlte Leistung

$$\underline{E}_\theta = \underline{p} k^2 \frac{e^{-jkr}}{r} \sin \theta$$

$$\underline{H}_\phi = \frac{1}{Z_w} \underline{E}_\theta$$

$$S = \frac{|\underline{I}|^2 Z_w}{8} \left(\frac{l}{\lambda} \right)^2 \sin^2 \theta$$

$$S_{av} = \frac{P_{rad}}{4\pi} = \frac{|\underline{I}|^2 Z_w}{12} \left(\frac{l}{\lambda} \right)^2$$

Abgestrahlte Leistung

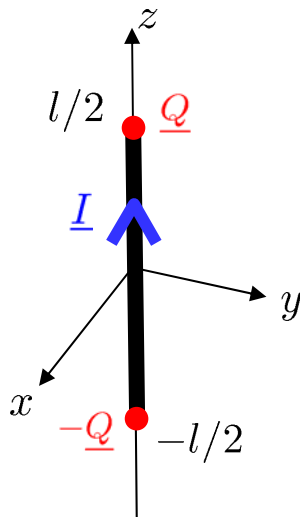
$$P_{rad} = \int_0^\pi \int_0^{2\pi} S(\theta) d\phi \sin \theta d\theta = \frac{|\underline{I}|^2 Z_w \pi}{3} \left(\frac{l}{\lambda} \right)^2$$

$$\int_0^\pi \sin^3 \theta d\theta = \frac{4}{3}$$

$$D = \frac{S_{max}}{S_{av}} = \frac{3}{2}$$

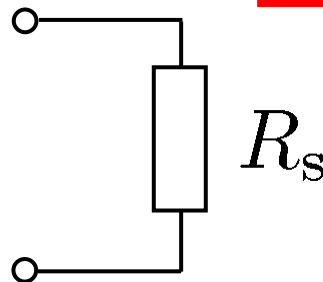
Richtfaktor ("*directivity*")

Hertz'scher Dipol: Strahlungswiderstand



$$P_{\text{rad}} = \frac{|\underline{I}|^2 Z_w \pi}{3} \left(\frac{l}{\lambda} \right)^2 = \frac{|\underline{I}|^2}{2} R_s$$

$$R_s = Z_w \frac{2\pi}{3} \left(\frac{l}{\lambda} \right)^2$$



Sehr viel kleiner als
die Wellenimpedanz
des Raumes!!

Hertz'scher Dipol

E-Feld: Pfeile

H-Feld: Dreiecke

