Stochastic Misalignment Model for magneto-inductive SISO and MIMO Links

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(extended slide set)
Outline

1. Introduction to magneto-inductive Links
2. Dipole Model applied to SISO and MIMO Links
3. Stochastic Coupling Model
4. Model Error, Rates, Localization
Magneto-inductive Links: Applications

- NFC
- Mid-range RFID
- Wireless Powering
- Communication in harsh media
  - Underground
  - Underwater

Magneto-inductive SISO (1 × 1) Link

- $M = f(\text{coil shapes, arrangement})$
- e.g. Neumann Formula
- $\rightarrow$ Unsuited for link analysis

\[ v_r = j\omega M \, i_t \]

**Mutual Inductance $M$ multiplies channel gain**

\[ M = \frac{\mu}{4\pi} \int_{(1)} \int_{(2)} \frac{ds_1 \cdot ds_2}{\|s_1 - s_2\|} \]
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Mutual Inductance under the Dipole Assumption

- **Assumption 1:** $d > \text{coil diameters} \implies \mathbf{b} \approx \text{dipole field}$
- **Assumption 2:** $\mathbf{b}$ constant across Rx surface

\[
M = \frac{K}{d^3} J
\]

Alignment Factor (unitless)
\[
J = r^T \mathbf{e} t, \quad -1 \leq J \leq 1
\]
\[
\mathbf{e} = \frac{3}{2} \mathbf{e} \mathbf{e}^T - \frac{1}{2} \mathbf{I}
\]

\[
K = \frac{\mu}{2\pi} N_t N_r S_t S_r
\]

cf. Kisseleff et al., «Interference Polarization in magnetic induction based WUSNs”, *PIMRC 2013*
Alignment Factor: Example 1

\[ J = r^T \mathcal{E} t = +1 \quad \ldots \quad \text{Coaxial (optimal)} \]
Alignment Factor: Example 2

\[ J = r^T \mathcal{E} t = +1/\sqrt{2} \]
Alignment Factor: Example 3

\[ J = r^T \mathcal{E} t = 0 \quad \ldots \quad \text{outage} \]

\[ M = \frac{K}{d^3} J \]

\[ \mathcal{E} = \frac{3}{2} ee^T - \frac{1}{2} I \]
Alignment Factor: Example 4

\[ J = r^T \mathcal{E} t = -1 \quad \ldots \quad \text{Coaxial (optimal)} \]
Alignment Factor: Example 5

\[ J = r^T \mathcal{E} t = -\frac{1}{2} \quad \text{... Coplanar} \]
**Alignment Factor: Example 6**

\[ J = r^T \mathcal{E} t = 0 \quad \ldots \text{Magic Angle (outage)} \]

\[ \| \mathcal{E} t \| = 1/2 \]

\[ \| \mathcal{E} t \| = 1 \]

**Repetition**

\[ M = \frac{K}{d^3} J \]

\[ \mathcal{E} = \frac{3}{2} e e^T - \frac{1}{2} I \]
Extension to collocated, orthogonal Coil Arrays

- \( \mathbf{v}_r = j \omega \mathbf{M} \mathbf{i}_t \) with 3 \times 3 mutual inductance matrix \( \mathbf{M} \)
- Orientations \( \mathbf{T} := [\mathbf{t}_1, \mathbf{t}_2, \mathbf{t}_3] \), \( \mathbf{R} := [\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3] \)

\[
\mathbf{M} = \frac{K}{d^3} \mathbf{R}^T \mathbf{E} \mathbf{T} \quad \overset{\text{EVD}}{=} \quad \frac{K}{d^3} \mathbf{R}^T \mathbf{U} \begin{bmatrix}
1 & -\frac{1}{2} \\
-\frac{1}{2} & -\frac{1}{2}
\end{bmatrix} \mathbf{U}^T \mathbf{T}
\]
Beamforming & Spatial Multiplexing

- Multiplexing Gain = 3
- Strongest stream is 6 dB above others

$$M = \frac{K}{d^3} R^T U \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} U^T T$$

$$\implies y \propto \frac{K}{d^3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} x$$

$$J = 1$$

$$J = -\frac{1}{2}$$

$$J = -\frac{1}{2}$$
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1 × 1 Link: 
Tx fixed, Rx has random Orientation

\[ M = \frac{K}{d^3} J \rightarrow \text{Random Variable} \]

- \( J \) is a function of orientations.
- **Idea:** assume random orientations
- For the shown link, \( J \) has uniform distribution between \( \pm \| \mathbf{E}_t \| \).
1 × 1 Link: Random Node Orientations

Marginalization yields

\[ f_J(J) = \begin{cases} \frac{\text{arsinh} \sqrt{3}}{\sqrt{3}} & \frac{\text{arsinh} \sqrt{3}}{\sqrt{3}} - \frac{\text{arsinh} \sqrt{4J^2-1}}{\sqrt{3}} \bigg| J \bigg| \leq \frac{1}{2} \\
0 & \frac{1}{2} < \bigg| J \bigg| \leq 1 \\
1 & 1 < \bigg| J \bigg| 
\end{cases} \]

Statistical Moments

\begin{align*}
E(J^2) &= \frac{1}{6} \\
E(J^4) &= \frac{3}{50}
\end{align*}
1 × 3 Beamforming: Tx fixed, Rx has random Orientation

Optimal beamforming achieves performance of a 1 × 1 link with field-aligned Rx coil, i.e.

\[ |J| = \|\mathcal{E}t\| \]

\[ \|\mathcal{E}t\| = 1/2 \]

\[ \|\mathcal{E}t\| = 1/\sqrt{2} \]

\[ \|\mathcal{E}t\| = 1 \]
1 × 3 Beamforming: Random Node Orientations

Uniformly Random Orientations

\[ f_J(J) = \begin{cases} 
\frac{2}{\sqrt{3}} \frac{|J|}{\sqrt{4J^2 - 1}} & \frac{1}{2} \leq |J| < 1 \\
0 & \text{otherwise}
\end{cases} \]

\[ \mathbf{v}_r = \mathbf{b}_r^H \mathbf{v}_r \]
3 × 3 Beamforming: Random Node Orientations

Full misalignment mitigation:

\[ |J| = 1 \]
regardless of arrangement!
$1 \times 3$ Coil Selection:
Tx fixed, Rx has random Orientation
1 × 3 Coil Selection: Random Node Orientations

Misalignment limited to

\[
\frac{1}{2\sqrt{3}} \leq |J|
\]
$3 \times 3$ Coil Selection: Random Node Orientations

Misalignment limited to
$0.4797883 \leq |J|$

Worst-case value is the greatest real root of

$$(24J^3 - 8J^2 - J + 1)^2 - 16J^3$$
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Model Error in terms of Achievable Rate (SISO)

- Circuit Model of Communications Link

- Rate Statistics for several $d$

Flat circular coils, $\Omega = 5$ cm, 5 turns. $f_c = 25.4$ MHz, $B = 100$ kHz, $P_T = 1$ $\mu$W

Great match between proposed stochastic model and Neumann model!
RSSI-based Ranging Application

- Observed $M_{\text{meas}}$ yields likelihood function $L(d)$ of range $d$

$$L(d) = f_{M|d}(M_{\text{meas}}|d) = \frac{d^3}{K} \cdot f_J \left( \frac{d^3}{K} M_{\text{meas}} \right)$$

$$\hat{d}_{\text{ML}} = \sqrt[3]{\frac{K}{2M_{\text{meas}}}}$$
Conclusions

- Dipole model enables analytic treatment of misalignment.
- A $3 \times 3$ link corresponds to one coaxial and two coplanar $1 \times 1$ links.
- Mutual inductance between nodes of unknown orientations can be described with closed-form PDFs.
- Stochastic results can be used for localization.
Thank you for your attention!