# Motion Extraction 

Computer Vision

## Motion is a basic cue

Motion can be the only cue for segmentation
Biologically favoured because of camouflage


Computer Vision

## Motion is a basic cue

... which set in motion a constant, evolutionary race


Computer Vision

## Motion is a basic cue

## Motion can be the only cue for segmentation



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Motion is a basic cue

## Even impoverished motion data can elicit a strong percept



Computer Vision

Some applications of motion extraction
$\square$ Change / shot cut detection
$\square$ Surveillance / traffic monitoring
$\square$ Autonomous driving
$\square$ Analyzing game dynamics in sports
$\square$ Motion capture / gesture analysis (HCl)
$\square$ Image stabilisation
$\square$ Motion compensation (e.g. medical robotics)
$\square$ Feature tracking for 3D reconstruction
$\square$ Etc.!

Computer Vision

## Shot cut detection \& Keyframes



Computer Vision

## Human-Machine Interfacing



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## 3D: Structure-from-Motion

## Tracked Points gives correspondences



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## 3D: Structure-from-Motion

## Temple of the Masks, Edzna, Mexico



Computer
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## www.arc3d.b

K.U. Leuven


Computer Vision

```
in this lecture...
```

Several techniques, but... this lecture is restricted to the

## 1. detection of the "optical flow"

2. tracking with the "Condensation filter"

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## Definition of optical flow

> OPTICAL FLOW =  brightness patterns

Ideally, the optical flow is the projection of the threedimensional motion vectors on the image

Such 2D motion vector is sought at every pixel of the image (note: a motion vector here is a 2 D translation vector)


## Computer Vision <br> Caution required!

Two examples where following brightness patterns is misleading:

1. Untextured, rotating sphere

$$
\stackrel{\Downarrow}{\text { O.F. }}=0
$$

2. No motion, but changing lighting

$$
\stackrel{\Downarrow}{\text { O.F. } \neq 0}
$$

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## Caution required!



Computer Vision

## Qualitative formulation

Suppose a point of the scene projects to a certain pixel of the current video frame. Our task is to figure out to which pixel in the next frame it moves...

That question needs answering for all pixels of the current image.

In order to find these corresponding pixels, we need to come up with a reasonable assumption on how we can detect them among the many.

We assume these corresponding pixels have the same intensities as the pixels the scene points came from in the previous frame.

That will only hold approximately...

## Mathematical formulation

## Our mathematical representation of a video:

$I(x, y, t)=$ brightness at $(x, y)$ at time $t$

Optical flow constraint equation :

$$
\frac{d I}{d t}=\frac{\partial I}{\partial x} \frac{d x}{d t}+\frac{\partial I}{\partial y} \frac{d y}{d t}+\frac{\partial I}{\partial t}=0
$$

This equation states that if one were to track the image projections of a scene point through the video, it would not change its intensity. This tends to be true over short lapses of time.

## Mathematical formulation

Our mathematical representation of a video:
$I(x, y, t)=$ brightness at $(x, y)$ at time $t$

Optical flow constraint equation :

$$
\frac{d I}{d t}=\frac{\partial I}{\partial x} \frac{d x}{d t}+\frac{\partial I}{\partial y} \frac{d y}{d t}+\frac{\partial I}{\partial t}=0
$$

Note the different types of time derivatives !

## Mathematical formulation

Our mathematical representation of a video:
$I(x, y, t)=$ brightness at $(x, y)$ at time $t$

Optical flow constraint equation :

$$
\frac{d I}{d t} \left\lvert\,=\frac{\partial I}{\partial x} \frac{d x}{d t}+\frac{\partial I}{\partial y} \frac{d y}{d t}+\frac{\partial I}{\partial t}=0\right.
$$

Change of intensity when following a physical point through the images

Change of intensity when looking at the same pixel $(x, y)$ through the images

## Mathematical formulation

$\begin{gathered}\text { We will use as } \\ \begin{array}{c}\text { shorthand } \\ \text { notation for }\end{array}\end{gathered} \frac{d I}{d t}=\frac{\partial I}{\partial x} \frac{d x}{d t}+\frac{\partial I}{\partial y} \frac{d y}{d t}+\frac{\partial I}{\partial t}=0$

$$
u=\frac{d x}{d t}, \quad v=\frac{d y}{d t}
$$

$$
I_{x}=\frac{\partial I}{\partial x}, \quad I_{y}=\frac{\partial I}{\partial y}, \quad I_{t}=\frac{\partial I}{\partial t}
$$



1 equation per pixel

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## The aperture problem

$$
\begin{gathered}
u=\frac{d x}{d t}, \quad v=\frac{d y}{d t} \\
I_{x}=\frac{\partial I}{\partial x}, \quad I_{y}=\frac{\partial I}{\partial y}, \quad I_{t}=\frac{\partial I}{\partial t}
\end{gathered}
$$

$$
I_{x} u+I_{y} v+I_{t}=0
$$

Note that we can measure the 3 derivatives of $I$, but that $u$ and $v$ are unknown

1 equation in 2 unknowns... the `aperture problem’

## The aperture problem

$$
I_{x} u+I_{y} v+I_{t}=0 \Rightarrow\left(I_{x}, I_{y}\right) \cdot(u, v)=-I_{t}
$$

Aperture problem : only the component along the gradient can be retrieved

$$
\frac{I_{t}}{\sqrt{I_{x}^{2}+I_{y}^{2}}}
$$

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## The aperture problem

Computer Vision

Remarks


## Remarks

1. The underdetermined nature could be solved using higher derivatives of intensity
2. for some intensity patterns, e.g. patches with a planar intensity profile, the aperture problem cannot be resolved anyway.

For many images, large parts have planar intensity profiles... higher-order derivatives than $1^{\text {st }}$ order are typically not used (also because they are noisy)

## Computer

 Vision
## Horn \& Schunck algorithm

Breaking the spell via an ... additional smoothness constraint :

$$
e_{s}=\iint\left(\left(u_{x}^{2}+u_{y}^{2}\right)+\left(v_{x}^{2}+v_{y}^{2}\right)\right) d x d y
$$

to be minimized, besides the OF constraint equation term

$$
e_{c}=\iint\left(I_{x} u+I_{y} v+I_{t}\right)^{2} d x d y
$$

The integrals are over the image.

## Computer

 Vision
## Horn \& Schunck algorithm

Breaking the spell via an ... additional smoothness constraint :

$$
e_{s}=\iint\left(\left(u_{x}^{2}+u_{y}^{2}\right)+\left(v_{x}^{2}+v_{y}^{2}\right)\right) d x d y
$$

to be minimized,
besides the OF constraint equation term

$$
e_{c}=\iint\left(I_{x} u+I_{y} v+I_{t}\right)^{2} d x d y
$$

## The calculus of variations

look for functions that extremize functionals
(a functional is a function that takes a vector as its input argument, and returns a scalar)
like for our functional:
$\left.\iint\left(u_{x}^{2}+u_{y}^{2}\right)+\left(v_{x}^{2}+v_{y}^{2}\right)\right) d x d y$

$$
+\lambda \iint\left(I_{x} u+I_{y} v+I_{t}\right)^{2} d x d y
$$

what are the optimal $u(x, y)$ and $v(x, y)$ ?

## The calculus of variations

look for functions that extremize functionals

$$
\begin{aligned}
& I=\int_{x_{1}}^{x_{2}} F\left(x, f, f^{\prime}\right) d x \quad \text { with } f=f(x), f^{\prime}=\frac{d f}{d x} \\
& f\left(x_{1}\right)=f_{1} \quad \text { and } \quad f\left(x_{2}\right)=f_{2}
\end{aligned}
$$

## Calculus of variations

## Suppose

1. $f(x)$ is a solution
2. $\eta(x)$ is a test function with $\eta\left(x_{1}\right)=0$ and $\eta\left(x_{2}\right)=0$

We then consider

$$
I=\int_{x_{1}}^{x_{2}} F\left(x, f, f^{\prime}\right) d x \quad \text { with } f=f(x), f^{\prime}=\frac{d f}{d x}
$$

$$
I=\int_{x_{1}}^{x_{2}} F\left(x, f+\varepsilon \eta, f^{\prime}+\varepsilon \eta^{\prime}\right) d x \Longleftrightarrow \int f \rightarrow f+\varepsilon \eta
$$

Rationale: supposed $f$ is the solution, then any deviation should result in a worse $I$; when applying classical optimization over the values of $\varepsilon$ the optimum should be $\varepsilon={ }^{3} 0$

## Calculus of variations

## Suppose

1. $f(x)$ is a solution
2. $\eta(x)$ is a test function with $\eta\left(x_{1}\right)=0$

$$
\text { and } \eta\left(x_{2}\right)=0
$$

We then consider

$$
I=\int_{x_{1}}^{x_{2}} F\left(x, f+\varepsilon \eta, f^{\prime}+\varepsilon \eta^{\prime}\right) d x
$$

With this trick, we reformulate an optimization over a function into a classical optimization over a scalar... a problem we know how to solve ${\underset{y}{2}}$

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## Calculus of variations

## Suppose

1. $f(x)$ is a solution
2. $\eta(x)$ is a test function with $\eta\left(x_{1}\right)=0$ and $\eta\left(x_{2}\right)=0$

$$
I=\int_{x_{1}}^{x_{2}} F\left(x, f+\varepsilon \eta, f^{\prime}+\varepsilon \eta^{\prime}\right) d x
$$

for the optimum :

$$
\left.\frac{d I}{d \varepsilon}\right|_{\varepsilon=0}=0
$$

## Computer Vision

## Calculus of variations

## Suppose

1. $f(x)$ is a solution
2. $\eta(x)$ is a test function with $\eta\left(x_{1}\right)=0$ and $\eta\left(x_{2}\right)=0$

$$
I=\int_{x_{1}}^{x_{2}} F\left(x, f+\varepsilon \eta, f^{\prime}+\varepsilon \eta^{\prime}\right) d x
$$

for the optimum :

$$
\begin{aligned}
& \int_{x_{1}}^{x_{2}}\left(\eta(x) F_{\overparen{f}}+\eta^{\prime}(x) F_{\overparen{f}}\right) d x=0 \\
& f+\varepsilon \eta \text { with } \varepsilon=0 \quad f^{\prime}+\varepsilon \eta^{\prime} \text { with } \varepsilon=0
\end{aligned}
$$

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## Calculus of variations

$\int_{x_{1}}^{x_{2}}\left(\eta(x) F_{f}+\eta^{\prime}(x) F_{f^{\prime}}\right) d x=0$
Using integration by parts:
$\int_{x_{1}}^{x_{2}} \frac{d}{d x}(g h) d x=\int_{x_{1}}^{x_{2}}\left(\frac{d g}{d x} h+\frac{d h}{d x} g\right) d x=[g h]_{x_{1}}^{x_{2}}$
where
$[g h]_{x_{1}}^{x_{2}}=g\left(x_{2}\right) h\left(x_{2}\right)-g\left(x_{1}\right) h\left(x_{1}\right)$

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## Calculus of variations

$$
\int_{x_{1}}^{x_{2}}\left(\eta(x) F_{f}+\eta^{\prime}(x) F_{f^{\prime}}\right) d x=0
$$

Using integration by parts $\int_{x_{1}}^{x_{2}} \frac{d}{d x}\left(\eta(x) F_{f^{\prime}}\right) d x$ :
$\int_{x_{1}}^{x_{2}} \eta^{\prime}(x) F_{f^{\prime}}+\eta(x) \frac{d}{d x} F_{f^{\prime}} d x=\left[\eta(x) F_{f^{\prime}}\right]_{x_{1}}^{x_{2}}$

Computer Vision

## Calculus of variations

$$
\int_{x_{1}}^{x_{2}}\left(\eta(x) F_{f}+\eta^{\prime}(x) F_{f^{\prime}}\right) d x=0
$$

Using integration by parts $\int_{x_{1}}^{x_{2}} \frac{d}{d x}\left(\eta(x) F_{f^{\prime}}\right) d x$ :
$\int_{x_{1}}^{x_{2}} \eta^{\prime}(x) F_{f^{\prime}}+\eta(x) \frac{d}{d x} F_{f^{\prime}} d x=\left[\eta(x) F_{f^{\prime}}\right]_{x_{1}}^{x_{2}}$

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## Calculus of variations

$$
\int_{x_{1}}^{x_{2}}\left(\eta(x) F_{f}+\eta^{\prime}(x) F_{f^{\prime}}\right) d x=0
$$

Using integration by parts $\int_{x_{1}}^{x_{2}} \frac{d}{d x}\left(\eta(x) F_{f^{\prime}}\right) d x$ :

$$
\int_{x_{1}}^{x_{2}} \eta^{\prime}(x) F_{f^{\prime}} d x=\left[\eta(x) F_{f^{\prime}} x_{x_{1}}^{x_{2}}-\int_{x_{1}}^{x_{2}} \eta(x) \frac{d}{d x} F_{f^{\prime}} d x,\right.
$$

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## Calculus of variations

$$
\int_{x_{1}}^{x_{2}}\left(\eta(x) F_{f}+\eta^{\prime}(x) F_{f^{\prime}}\right) d x=0
$$

Using integration by parts $\int_{x_{1}}^{x_{2}} \frac{d}{d x}\left(\eta(x) F_{f^{\prime}}\right) d x$ :
$\int_{x_{1}}^{x_{2}} \eta^{\prime}(x) F_{f^{\prime}} d x=\quad-\int_{x_{1}}^{x_{2}} \eta(x) \frac{d}{d x} F_{f^{\prime}} d x$,
Therefore

$$
\int_{x_{1}}^{x_{2}} \eta(x)\left(F_{f}-\frac{d}{d x} F_{f^{\prime}}\right) d x=0
$$

regardless of $\eta(x)$, then $\quad F_{f}-\frac{d}{d x} F_{f^{\prime}}=0$

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## Calculus of variations

Generalizations
■ 1. $I=\int_{x_{1}}^{x_{2}} F\left(x, f_{1}, f_{2}, \ldots, f_{1}^{\prime}, f_{2}^{\prime}, \ldots\right) d x$
Simultaneous Euler-Lagrange equations, i.c. one for $u$ and one for $v$ :

$$
F_{f i}-\frac{d}{d x} F_{f_{i}^{\prime}}=0
$$

## Calculus of variations

Generalizations
■ 1. $I=\int_{x_{1}}^{x_{2}} F\left(x, f_{1}, f_{2}, \ldots, f_{1}^{\prime}, f_{2}^{\prime}, \ldots\right) d x$
Simultaneous Euler-Lagrange equations, i.c. one for $u$ and one for $v$ :

$$
F_{f i}-\frac{d}{d x} F_{f_{i}^{\prime}}=0
$$

As we add $\varepsilon_{1} \eta_{1}$ to $f_{1}$, and $\varepsilon_{2} \eta_{2}$ to $f_{2}$ then repeat, once deriving w.r.t. $\varepsilon_{1}$, once w.r.t. $\varepsilon_{2}$ thus obtaining a system of 2 PDEs

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## Calculus of variations

## Generalizations

■ 1. $I=\int_{x_{1}}^{x_{2}} F\left(x, f_{1}, f_{2}, \ldots, f_{1}^{\prime}, f_{2}^{\prime}, \ldots\right) d x$
Simultaneous Euler-Lagrange equations, i.c. one for $u$ and one for $v$ :

$$
F_{f i}-\frac{d}{d x} F_{f_{i}^{\prime}}=0
$$

■ 2. 2 independent variables $x$ and $y$

$$
I=\iint_{D} F\left(x, y, f+\varepsilon \eta, f_{x}+\varepsilon \eta_{x}, f_{y}+\varepsilon \eta_{y}\right) d x d y
$$

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## Calculus of variations

Hence

$$
0=\iint_{D}\left(\eta F_{f}+\eta_{x} F_{f_{x}}+\eta_{y} F_{f_{y}}\right) d x d y
$$

Now by Gauss' s integral theorem,

$$
\iint_{D}\left(\frac{\partial Q}{\partial x}+\frac{\partial P}{\partial y}\right) d x d y=\int_{\partial D}(Q d y-P d x)
$$

such that

$$
\begin{aligned}
\iint_{D} \frac{\partial\left(\eta F_{f_{x}}\right)}{\partial x}+\frac{\partial\left(\eta F_{f_{y}}\right)}{\partial y} d x d y & =\int_{\partial D}\left(\eta F_{f_{x}} d y-\eta F_{f_{y}} d x\right) \\
& =0
\end{aligned}
$$

## Calculus of variations

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$$
\begin{gathered}
\iint_{D} \frac{\partial\left(\eta F_{f_{x}}\right)}{\partial x}+\frac{\partial\left(\eta F_{f_{y}}\right)}{\partial y} d x d y=0 \\
\iint_{D}\left(\eta_{x} F_{f_{x}}+\eta_{y} F_{f_{y}}\right) d x d y+\iint_{D}\left(\eta \frac{\partial F_{f_{x}}}{\partial x}+\eta \frac{\partial F_{f_{y}}}{\partial y}\right) d x d y=0
\end{gathered}
$$

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 VisionCalculus of variations

$$
0=\iint_{D}\left(\eta F_{f}+\eta_{x} F_{f_{x}}+\eta_{y} F_{f_{y}}\right) d x d y
$$

$$
\iint_{D} \frac{\partial\left(\eta F_{f_{x}}\right)}{\partial x}+\frac{\partial\left(\eta F_{f_{y}}\right)}{\partial y} d x d y=0
$$

$\iint_{D}\left(\eta_{x} F_{f_{x}}+\eta_{y} F_{f_{y}}\right) d x d y=-\iint_{D} \eta\left(\frac{\partial F_{f_{x}}}{\partial x}+\frac{\partial F_{f_{y}}}{\partial y}\right) d x d y$
Consequently,

$$
0=\iint_{D} \eta\left(F_{f}-\frac{\partial}{\partial x} F_{f_{x}}-\frac{\partial}{\partial y} F_{f_{y}}\right) d x d y
$$

for all test functions $\eta$, thus

$$
F_{f}-\frac{\partial}{\partial x} F_{f_{x}}-\frac{\partial}{\partial y} F_{f_{y}}=0
$$

is the Euler-Lagrange equation

## Horn \& Schunck

The Euler-Lagrange equations :

$$
\begin{aligned}
& F_{u}-\frac{\partial}{\partial x} F_{u_{x}}-\frac{\partial}{\partial y} F_{u_{y}}=0 \\
& F_{v}-\frac{\partial}{\partial x} F_{v_{x}}-\frac{\partial}{\partial y} F_{v_{y}}=0
\end{aligned}
$$

In our case ,

$$
F=\left(u_{x}^{2}+u_{y}^{2}\right)+\left(v_{x}^{2}+v_{y}^{2}\right)+\lambda\left(I_{x} u+I_{y} v+I_{t}\right)^{2},
$$

so the Euler-Lagrange equations are

$$
\begin{gathered}
\Delta u=\lambda\left(I_{x} u+I_{y} v+I_{t}\right) I_{x}, \\
\Delta v=\lambda\left(I_{x} u+I_{y} v+I_{t}\right) I_{y}, \\
\Delta=\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}} \quad \text { is the Laplacian operator }
\end{gathered}
$$

## Horn \& Schunck

## Remarks :

1. Coupled PDEs solved using iterative methods and finite differences (iteration $i$ )

$$
\begin{aligned}
& \frac{\partial u}{\partial i}=\Delta u-\lambda\left(I_{x} u+I_{y} v+I_{t}\right) I_{x} \\
& \frac{\partial v}{\partial i}=\Delta v-\lambda\left(I_{x} u+I_{y} v+I_{t}\right) I_{y}
\end{aligned}
$$

2. More than two frames allow for a better estimation of $I_{\mathrm{t}}$
3. Information spreads from edge- and corner-type patterns

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Horn \& Schunck, example result


## Horn \& Schunck, remarks

1. Errors at object boundaries
(where the smoothness constraint is no longer valid)
2. Example of regularisation
(selection principle for the solution of
ill-posed problems by imposing an extra generic constraint, like here smoothness)

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## condensation filter

Computer Vision

## condensation filter

as an example of a `tracker', shifting the emphasis from pixels to objects...

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## Condensation tracker

$x_{t}$ state vector
$z_{t}$ observation vector $v_{t}$ noise in measurement model

## Condensation tracker

1. Prediction, based on the system model
$x_{t}=f_{t-1}\left(x_{t-1}, w_{t-1}\right)$
( $f=$ system transition function)
2. Update, based on the measurement model
$z_{t}=h_{t}\left(x_{t}, v_{t}\right)$
( $h=$ measurement function)
$Z_{t}=\left(z_{1}, \ldots, z_{t}\right)$ is the history of observations

## Condensation tracker

## Example

## dots indicate time derivatives

System model

$$
x_{t}=\left(p_{t}, \dot{p}_{t}\right)
$$

$p_{t}=p_{t-1}+\Delta t \dot{p}_{t-1}+w_{p, t-1} \quad$ position
$\dot{p}_{t}=\dot{p}_{t-1}+w_{\dot{p}, t-1}$
velocity

Measurement model

$$
z_{t}=p_{t}+v_{t}
$$

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## Condensation tracker

## Recursive Bayesian filter

Object not as a single state but a prob. distribution


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## Condensation tracker

## Recursive Bayesian filter

Object not as a single state but a prob. Distribution ( $p$ here means probability...)

1. Prediction

$$
p\left(x_{t} \mid Z_{t-1}\right)=\int p\left(x_{t} \mid x_{t-1}\right) p\left(x_{t-1} \mid Z_{t-1}\right) d x_{t-1}
$$

2. Update

$$
p\left(x_{t} \mid Z_{t}\right)=\frac{p\left(Z_{t} \mid x_{t}\right) p\left(x_{t} \mid Z_{t-1}\right)}{p\left(Z_{t} \mid Z_{t-1}\right)}
$$

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## Condensation tracker

## Recursive Bayesian filter

Object not as a single state but a prob. distribution

1. Prediction

$$
p\left(x_{t} \mid Z_{t-1}\right)=\int p\left(x_{t} \mid x_{t-1}\right) p\left(x_{t-1} \mid Z_{t-1}\right) d x_{t-1}
$$

2. Update

$$
p\left(x_{t} \mid Z_{t}\right)=\frac{p\left(z_{t} \mid x_{t}\right) p\left(x_{t} \mid Z_{t-1}\right)}{p\left(z_{t} \mid Z_{t-1}\right)}
$$

$p\left(z_{t} \mid Z_{t-1}\right)$ can be considered a normalization factor

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## Condensation tracker

## Recursive Bayesian filter

Object not as a single state but a prob. distribution

## Bayes' rule

$p(b \mid a) p(a)=p(a \mid b) p(b)=p(a, b)$ here $p\left(x_{t}, z_{t} \mid Z_{t-1}\right)$
2. Update

$$
p\left(x_{t} \mid Z_{t}\right)=\frac{p\left(z_{t} \mid x_{t}\right) p\left(x_{t} \mid Z_{t-1}\right)}{p\left(z_{t} \mid Z_{t-1}\right)}
$$

$p\left(z_{t} \mid Z_{t-1}\right)$ can be considered a normalization factor

Computer Vision

## Condensation tracker

## Recursive Bayesian filter

Object not as a single state but a prob. distribution

## Bayes' rule

$$
p\left(x_{t}, \mathrm{Z}_{t} \mid Z_{t-1}\right)=p\left(x_{t} \mid Z_{t}\right) p\left(\mathrm{Z}_{t} \mid Z_{t-1}\right)=p\left(\mathrm{Z}_{t} \mid x_{t}\right) p\left(x_{t} \mid Z_{t-1}\right)
$$

2. Update

$$
p\left(x_{t} \mid Z_{t}\right)=\frac{p\left(z_{t} \mid x_{t}\right) p\left(x_{t} \mid Z_{t-1}\right)}{p\left(z_{t} \mid Z_{t-1}\right)}
$$

$p\left(z_{t} \mid Z_{t-1}\right)$ can be considered a normalization factor

Computer Vision

## Condensation tracker

## Recursive Bayesian filter

Object not as a single state but a prob. distribution

1. Prediction
$p\left(x_{t} \mid Z_{t-1}\right)=\int p\left(x_{t} \mid x_{t-1}\right) p\left(x_{t-1} \mid Z_{t-1}\right) d x_{t-1}$
2. Update

$$
p\left(x_{t} \mid Z_{t}\right)=\frac{p\left(z_{t} \mid x_{t}\right) p\left(x_{t} \mid Z_{t-1}\right)}{p\left(z_{t} \mid Z_{t-1}\right)}
$$

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Computer Vision

## Condensation tracker

## Recursive Bayesian filter

Calculating $p\left(x_{t} \mid Z_{t-1}\right)=\int p\left(x_{t} \mid x_{t-1}\right) p\left(x_{t-1} \mid Z_{t-1}\right) d x_{t-1}$ numerically is very time consuming, and the prob. distributions have to be known...

Analytic solutions are only available for the simplest of cases, e.g. when distr. are Gaussian and the system and measurement models are linear...
(Kalman filter, 1960 - Kalman was prof. at ETH, D-ITET)


Computer Vision

## Condensation tracker

## Recursive Bayesian filter

Calculating $p\left(x_{t} \mid Z_{t-1}\right)=\int p\left(x_{t} \mid x_{t-1}\right) p\left(x_{t-1} \mid Z_{t-1}\right) d x_{t-1}$ numerically is very time consuming, and the prob. distributions have to be known...

Analytic solutions are only available for the simplest of cases, e.g. when distr. are Gaussian and the system and measurement models are linear...

That's where Condensation comes in, acronym for CONditional DENSity propagATION

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## In our example model is linear, distributions Gaussian



System model

$$
\begin{aligned}
& p_{t}=p_{t-1}+\Delta t \dot{p}_{t-1}+w_{p, t-1} \\
& \dot{p}_{t}=\dot{p}_{t-1}+w_{\dot{p}, t-1}
\end{aligned}
$$

Measurement model

$$
z_{t}=p_{t}+v_{t}
$$

## Computer Vision



## Condensation tracker

The probability distribution is represented by a sample set $S$ (set of selected states $s$ )

$$
S=\left\{\left(s^{(n)}, \pi^{(n)}\right) \mid n=1 \ldots N\right\}
$$

With $\pi$ a weight determining the sampling probability

## Condensation tracker

## 1. prediction

Start with $S_{t-1}$, the sample set of the previous step, and apply the system model to each sample, yielding predicted samples $s_{t}^{\prime(n)}$
2. update

Sample from the predicted set, where samples are drawn with replacement and with probability

$$
\pi^{(n)}=p\left(z_{t} \mid s_{t}^{\prime(n)}\right) \quad \text { (i.e. using meas. model) }
$$

In the limit (large N) equivalent to Bayesian tracker

Computer Vision

## Condensation tracker



## Condensation tracker

## NOTE

Sample may be drawn multiple times, but noise will yield different predictions for samples corresponding to the same state after drawing.

This diversification through noise is important, as otherwise fewer and fewer different samples would survive


## Condensation tracker

## Comparison with Kalman filter

## Condensation

Unrestricted system models Unrestricted noise models Multiple hypotheses

Discretisation error
Postprocessing for interpret.

## Kalman-Bucy

Linear system models Gaussian noise Unimodal

Exact solution Direct interpretation

Computer Vision

## Condensation tracker



Computer Vision

## Condensation tracker



Computer Vision

## Condensation tracker



Computer Vision

## Condensation tracker



Computer Vision

## Condensation tracker



Computer Vision


Computer Vision

## Condensation tracker



Computer Vision


Computer Vision

## Condensation tracker



Computer Vision

## ALERT

Recognition: No successful recognition Camera ${ }^{\boldsymbol{1}} \mathbf{1}$ Cam-Pos
Dale: Deterible, 01. 2000
Time: 11:36:30

previous next
play

loop IMAGE \#9

1


Computer Vision

## Condensation tracker

## Elliptical region with prescribed color histogram

System model

$$
\begin{array}{ll}
x_{t}=x_{t-1}+\Delta t \dot{x}_{t-1}+w_{x, t-1} & \\
y_{t}=y_{t-1}+\Delta t \dot{y}_{t-1}+w_{y, t-1} & \text { position } \\
\dot{x}_{t}=\dot{x}_{t-1}+w_{\dot{x}, t-1} & \\
\dot{y}_{t}=\dot{y}_{t-1}+w_{\dot{y}, t-1} & \text { velocity } \\
H_{x t}=H_{x t-1}+\Delta t \dot{H}_{x t-1}+w_{H_{x}, t-1} & \\
H_{y_{t}}=H_{y_{t-1}}+\Delta t \dot{H}_{y_{t-1}}+w_{H_{y}, t-1} & \\
\dot{H}_{x t}=\dot{H}_{x_{t-1}}+w_{\dot{H}_{x}, t-1} & \\
\dot{H}_{y_{t}}=\dot{H}_{y_{t-1}}+w_{\dot{H}_{y}, t-1} & \text { size chance }
\end{array}
$$

## Condensation tracker

Measurement model

$$
\pi=\frac{1}{\sqrt{2 \pi} \sigma} e^{-\frac{1-\rho}{2 \sigma^{2}}}
$$

with

$$
\rho=\sum_{u=1}^{m} \sqrt{p^{(u)} q^{(u)}}
$$

where $p$ and $q$ are the color histograms of a sample and the target, resp.

Computer
Vision

## Condensation tracker



Computer Vision

## Mean shift tracker



Computer Vision

## Mean shift tracker



Computer

## Condensation tracker

Computer Vision

## Condensation tracker



## Other approaches

1. Model-based tracking (application-specific)

- active contours (discussed with segmentation)
- analysis/synthesis schemes

2. Feature tracking (more generic)

- corner tracking (shown when we discuss 3D)
- blob/contour tracking
- intensity profile tracking
- region tracking

Computer Vision

## Model-based tracker


(EPFL)

Computer Vision

## Model-based tracker


(EPFL)

Computer Vision

## Motion capture for special effects



