





Stereo

The underlying principle is "triangulation" :





Simple configuration :



A simple stereo setup



identical cameras

coplanar image planes

□ aligned *x*-axes

A simple stereo setup



Reminder :

the camera projection can be formulated as

$$\rho p = KR^t (P - C)$$

for some non-zero $\ \rho \in \mathbb{R}$

Here R is the identity...

A simple stereo setup



$$\rho\begin{pmatrix}x\\y\\1\end{pmatrix} = K\begin{pmatrix}X\\Y\\Z\end{pmatrix} \quad \rho'\begin{pmatrix}x'\\y'\\1\end{pmatrix} = K\begin{pmatrix}X-b\\Y\\Z\end{pmatrix} \quad K = \begin{pmatrix}fk_x & 0 & 0\\0 & fk_y & 0\\0 & 0 & 1\end{pmatrix}$$

A simple stereo setup





Note that y = y'

A simple stereo setup

The 3D coordinates of the point are

$$X = b \frac{x}{(x - x')},$$
$$Y = b \frac{k_x}{k_y} \frac{y}{(x - x')},$$
$$Z = b k_x \frac{f}{(x - x')}.$$

(x - x') is the so-called *disparity* Stereo is imprecise for far away objects, but increasing *b* and/or *f* can increase depth resolution

A simple stereo setup

Notice: for this simple setup, same disparity means same depth







A simple stereo setup

Increasing b increases depth resolution



one has to strike a balance with visibility...

A simple stereo setup

Increasing f increases depth resolution



one has to strike a balance with visibility...

Remarks

1. increasing b and/or f increases depth resolution but reduces simultaneous visibility

2. iso-disparity loci are depth planes, not so for other configurations

I 3. human stereo vision only works up to \pm 10 m

☐ 4. the real problem is finding correspondences

A simple stereo setup



The HARD problem is finding the *correspondences*

Notice : no reconstruction for the untextured back wall...

A simple stereo setup



The HARD problem is finding the *correspondences*

Notice : no reconstruction for the untextured back wall...





Stereo, the general setup

we start by the relation between the two projections of a point

in the second image the point must be along the projection of the viewing ray for the first camera :



Stereo, the general setup

We cast this constraint in mathematical expressions :

 $p \ \mathrm{and} \ p$ ' are the two images of P

$$\mu p = K R^{t} (P - C)$$
$$\rho' p' = K' R'^{t} (P - C')$$

w.r.t. world frame P is on the ray with equation

$$P = C + \mu R K^{-1} p$$
 for some $\mu \in \mathbb{R}$

Stereo, the general setup

so, the ray is given by

$$P = C + \mu R K^{-1} p$$
 for some $\mu \in \mathbb{R}$

now we project it onto the second image in general, points project as follows :

$$\rho'p' = K'R'^t(P-C')$$

and thus, filling in the ray's equation

 $\rho' p' = \mu K' R'' R K^{-1} p + K' R'' (C - C')$

Stereo, the general setup

the projected ray was found to be

$$\rho' p' = \mu K' R'' R K^{-1} p + K' R'' (C - C')$$

the second term is the projection of the 1st camera's center, the so-called *epipole*

$$\rho'_e e' = K' R'^t (C - C')$$

the first term is the projection of the ray's point at infinity, the so-called *vanishing point*

finally, adopting the simplifying notation

$$A = \frac{1}{\rho'_e} K' R'' R K^{-1}$$
$$\rho' p' = \rho'_e (\mu A p + e')$$

A is the *infinity homography*

Stereo, the general setup

note that the epipole lies on all the epipolar lines



Stereo, the general setup

 $\rho'p' = \rho'_e(\mu Ap + e')$

Stereo, the general setup

$$\rho'p' = \rho'_e(\mu Ap + e')$$

expresses that p lies on the line l through the epipole e' and the vanishing point Ap of the ray of sight of p (in the 2nd image)

Stereo, the general setup

 $\rho'p' = \rho'_e(\mu Ap + e')$

the epipolar constraint (epipolar line)

we can rewrite this constraint as

$$\left|p'e'Ap\right| = p''(e' \times Ap) = 0$$

Stereo, the general setup

$$\left|p'e'Ap\right| = p''(e' \times Ap) = 0$$

can be written, given



$$|p'e'Ap| = p''[e']_{\times}Ap$$

 $F = [e']_{\times} A$ is the *fundamental matrix*

F is a 3x3 matrix, but has rank 2

Stereo, the general setup

$$p'^{t}[e']_{\times}Ap = 0 \quad \rightarrow \quad p'^{t}Fp = 0$$

The 3-vector p''F contains the line coordinates of the epipolar line of p' (i.e. a line in the 1st image)

The 3-vector F p contains the line coordinates of the epipolar line of p (i.e. a line in the 2nd image)

Hence, the epipolar matrix works in both directions

Stereo, the general setup





Andrea Fusiello, CVonline

Epipolar geometry cont'd



Epipolar geometry cont'd

• Epipolar lines are in mutual correspondence



• allows to separate matching problem: matching pts on an epipolar line to pts on the corresponding epipolar line

Exploiting epipolar geometry

Separate 2D correspondence search problem to 1D search problem by using two view geometry





Epipolar geometry cont'd



Stereo, the general setup

• one point yields one equation $p'^t F p = 0$ that is linear in the entries of the fundamental matrix F

so, we can actually obtain F without any prior knowledge about camera settings if we have sufficient pairs of corresponding points !!

- F can be computed *linearly* from 8 pairs of corresponding points,
 i.e. already from 8 `correspondences' (not 9, as this is a homogeneous system and one coefficient can be fixed to value 1 to fix the scale !)
- F being rank 2 yields an additional, but non-linear constraint. Thus, 7 correspondences suffice to *non-linearly* solve for F

Stereo, the general setup

Remarks :

- Of course, in practice one wants to use as many Correspondences as available, e.g. for obtaining a least-squares solution, based on the linear system, followed by a step to impose rank 2.
- Often, F is found through a procedure called RANSAC (RANdom Sample Consensus). It starts from a randomly drawn subset of correspondences of minimal size (e.g. 8), and then keeps on drawing until a subset is found that yields an F so that many correspondences are seen to obey the epipolar constraint. RANSAC is good to fend off against correspondences that are wrong (`outliers')
Relations between 3 views

one could use more than 2 images, e.g. 3 suppose P projects to p,p', and p'' p'' is found at the intersection of epipolar lines :



fails when the epipolar lines coincide





Correspondence problem : constraints

Reducing the search space :

- 1. Points on the epipolar line
 - 2. Min. and max. depth \Rightarrow line segment
 - 3. Preservation of order
- 4. Smoothness of the disparity field

Correspondence problem : methods

1. correlation

deformations...

 \Box small window \Rightarrow noise!

 \Box large window \Rightarrow bad localisation

2. feature-based

mainly edges and corners

□ sparse depth image

3. regularisation methods

Stereo, the general setup

3D reconstruction

 $P = C + \mu R K^{-1} p$ $P = C' + \mu' R' K'^{-1} p'$

Yields 6 equations in 5 unknowns X,Y, Z and $\mu,\,\mu'$

However, due to noise and errors, the rays may not intersect!

⇒ e.g. use the middle where the rays come closest









3D city models – ground level

Mobile mapping example – for measuring



3D city models – ground level

Can also be turned into 3D for visualisation, but one needs to stay close to the camera viewpoints.

The example shown is of Quebec

3D city models – ground level



Uncalibrated reconstruction

From 2 views...







If the camera translates...

An affine reconstruction can be made

A projective reconstruction is always possible (if no pure rot.)

Uncalibrated reconstruction

From 3 general views taken with the same camera parameters...





A metric reconstruction is possible



Uncalibrated reconstruction







Points and cameras



Depth map



3D models

Uncalibrated reconstruction



Uncalibrated reconstruction - example



Univ. of Leuven

Shape-from-stills

Input Images shots taken with Canon EOS D60

(Resolution: 6,3 Megapixel)

copyright Eyetronics

Shape-from-stills



Webservice, free for non-commercial use









CAMERA'S CENTER OF PROJECTION



Active triangulation

INTERSECTION LASER PLANE & OBJECT SURFACE

LASER WITH CYLINDRICAL LENSE IN FRONT

INTERSECTION LASER PLANE & VIEWING RAY

A plane and a line do normally intersect... Noise has little Influence on the triangulation

POINT ON THE LASER LINE SEEN BY THE CAMERA

CAMERA'S CENTER OF PROJECTION

Active triangulation



Active triangulation



Active triangulation

Camera image



Active triangulation





Active triangulation

Example 1 Cyberware laser scanners



Desktop model for small objects

Medium-sized objects

Body scanner

Head scanner







Active triangulation

Example 2 Minolta



Portable desktop model



Structured light

patterns of a special shape are projected onto the scene

deformations of the patterns yield information on the shape

Focus is on combining a good resolution with a minimum number of pattern projections

Serial binary patterns

A sequence of patterns with increasingly fine subdivisions

Yields 2ⁿ identifiable lines for only n patterns



Reducing the nmb of projections: colour

Binary patterns Yields 2ⁿ identifiable lines for only n patterns Using colours, e.g. 3, Yields 3ⁿ identifiable lines for only n patterns



Interference from object colours...

One-shot implementation

3D from a single frame – KULeuven '96:



One-shot implementation

KULeuven '81: checkerboard pattern with column code



example :

3D reconstruction for the example



An application in agriculture


One-shot 3D acquisition Leuven ShapeCam



Shape + texture often needed

Higher resolution

Texture is also extracted







James Bond

Die another day

Lara Croft

Thomb Raider

Active triangulation

Recent, commercial example

Kinect 3D camera, affordable and compact solution by Microsoft.

Projects a 2D point pattern in the NIR, to make it invisible to the human eye

Kinect: 9x9 patches with locally unique code

Kinect as one-shot, low-cost scanner

Excerpt from the dense NIR dot pattern:

http://research.microsoft.com/apps/video/default.aspx?ideo/defau

Face animation - input

Face animation – replay + effects

Facial motion capture

motion capture for League of Extraordinary Gentlemen

Facial motion capture

LC015 Eyetronics Face Capture Test V01 1/291

03 / 11 / 2003

Phase shift

Phase shift

$$I_{r} = A + R\cos (\phi - \theta)$$
$$I_{g} = A + R\cos (\phi)$$
$$I_{b} = A + R\cos (\phi + \theta)$$

 detect phase from 3 subsequently projected cosine patterns, shifted over 120 degrees
unwrap the phases / additional stereo
texture is obtained by summing the 3 images / color camera w. slower integration

Phase shift

 $=\frac{I_r + I_g + I_b}{3}$ $\phi = \arctan\left(\tan\left(\frac{\theta}{2}\right)\frac{I_r - I_b}{2I_g - I_r - I_b}\right)$

4D acquisition

Motion retargetting, from 3D phase shift scans

Face/Off: Live Facial Puppetry

PaperID 102

Time-of-flight

measurement of the time a modulated light signal needs to travel before returning to the sensor

this time is proportional to the distance

waves :

- 1. radar
- 2. sonar
- 3. optical radar

low freq. electromagnetic acoustic waves optical waves

working principles :

- 1. pulsed
- 2. phase shifts

Time-of-flight

Example 1: Cyrax

Example 2: Riegl

Time-of-flight: example

Cyrax ™ 3D Laser Mapping System

Accurate, detailed, fast measuring

Integrated modeling

Pulsed laser (time-of-flight)

No reflectors needed

Cyrax

2mm - 6mm accuracy

Distance = $C \times \Delta T \div 2$

Cyrax is *also* a visualization tool

Cyrax detects the <u>intensity</u> of each reflected laser pulse and <u>colors</u> it

Step 1: Target the structure

Step 2: Scan the structure

Step 3: Color the points

Step 4: Model fitting in-the-field

Result

Project: As-built of Chevron hydrocarbon plant

- 400'x500' area
- 10 vessels; 5 pumps
- 6,000 objects
- 81 scans from 30 tripod locations
- *Cyrax* field time = 50 hrs

Added Value

- Greater detail & no errors
- Higher accuracy
- Fewer construction errors
- 6 week schedule savings

Application Modeling movie sets

Lidar data with Riegl LMS-Z390i

courtesy of RWTH Aachen, L. Kobbelt et al.

Comparison Lidar - passive

Image courtesy of Tippett Studio

Shape-from-texture

assumes a slanted and tilted surface to have a homogeneous texture

inhomogeneity is regarded as the result of projection

e.g. anisotropy in the statistics of edge orientations

₩

orientations deprojecting to maximally isotropic texture



50 Tim f f



Shape-from-contour

makes assumptions about contour shape

E.g. the maximization of area over perimeter squared (compactness)

ellipse $\xrightarrow{\Downarrow}$ circle

E.g. assumption of symmetry Symmetric contours \rightarrow surface of revolution



Shape-from-contour









Shape-from-silhouettes



Shape from silhouettes - uncalibrated

tracking of turntable rotation

- volumetric modeling from silhouettes
- triangular textured surface mesh



Turntable sequence

Camera tracking

VRML model



Outdoor visual hulls







Outdoor visual hulls





REAL-TIME FOCUS RANGE SENSOR

SHREE K. NAYAR MASAHIRO WATANABE MINORI NOGUCHI COLUMBIA UNIVERSITY







Shape-from-shading

Uses directional lighting, often with known direction

local intensity is brought into correspondence with orientation via *reflectance maps*

orientation of an isolated patch cannot be derived uniquely

extra assumptions on surface smoothness and known normals at the rim



Photometric stereo

constraint propagation eliminated by using light from different directions

simultaneously when the light sources are given different colours

Mini-dome for photometric stereo

Instead of working with multi-directional light applied simultaneously with the colour trick, one can also project from many directions in sequence...

Mini-dome for photometric stereo

KATHOLIEKE UNIVERSITEIT

Mini-dome



Mini-dome

Computer



Mini-dome for photometric stereo

Example for tablet with first world map known, an exhibit at the British Museum:

http://homes.esat.kuleuven.be/~mproesma/mptmp/cuneiform

Mini-dome for photometric stereo



3D and recognition integrated

3D City Modeling using Cognitive Loops





ETH Eidgenössische Technische Hochschule Zürich Swiss Federal Institute of Technology Zurich

Multi-walker tracker



Computer Vision Strongest 3D cues for us are 2D...

