Real-Time Control of Distribution Grids

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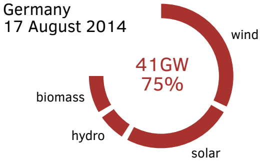
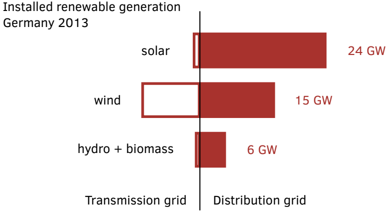
Future power distribution grids





Micro-generation

Most of the renewable generators are

- interfaced to the grid via **power converters**
- **small** in size (rated power)
- connected to **medium- and low- voltage** levels
- connected to resistive **distribution grids** (limited transfer capacity)

Power distribution grid congestion

Power distribution feeders have limited **power transfer capacity**:

- overvoltage
- line current limits
- transformer power rating

Set-point scheduling

(S6) Set-points $P_{\text{ref}}, Q_{\text{ref}}, v_{\text{ref}}$ need to

- be consistent with the physics of the grid
- satisfy operational constraints
- minimize an economic dispatch criterion
- react to changing demands
- react to time-varying primary sources

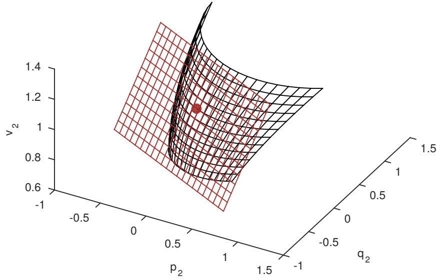
THE SET-POINT SCHEDULING PROBLEM

Nonlinear power flow equations

- $x = \begin{bmatrix} v & \theta & p & q \end{bmatrix}$
- Set of all grid states that satisfy the **AC power flow equations**

$$\rightarrow \textbf{power flow manifold} \quad \mathcal{M} := \{x \mid f(x) = \mathbf{0}\}$$

- Regular submanifold of dimension $2n$



Consistency constraint

Set-points on the manifold at all times

$$x(t) \in \mathcal{M}$$

$\rightarrow$ set-point updates need to satisfy

$$\dot{x}(t) \in T_{x(t)}\mathcal{M}$$

$\rightarrow$ Bolognani & Dörfler (2015)
"Fast power system analysis via implicit linearization of the power flow manifold"

Set-point specifications as an OPF

Real-time Optimal Power Flow (OPF)

- Minimize cost of generation
- Satisfy AC power flow laws
- Respect generation capacity
- No over-/under-voltage
- No congestion

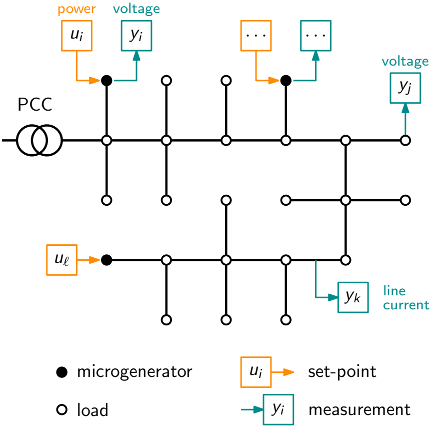
$$
\begin{aligned}
\text{minimize} \quad & \text{cost}(p_G) \\
\text{subject to} \quad & \begin{bmatrix} p_0 \\ p_G \\ p_L \end{bmatrix} + j \begin{bmatrix} q_0 \\ q_G \\ q_L \end{bmatrix} = \text{diag}(\nu)\overline{Y\nu} \\
& p_G^{\min} \leq p_G \leq p_G^{\max}, \quad q_G^{\min} \leq q_G \leq q_G^{\max} \\
& v^{\min} \leq v \leq v^{\max} \\
& |p_{kl} + jq_{kl}| \leq s_{kl}^{\max}
\end{aligned}
$$

Prototype of real-time OPF

$$
\begin{aligned}
\text{minimize} \quad & J(x) \\
\text{subject to} \quad & x \in \mathcal{K} = \mathcal{M} \cap \mathcal{X}
\end{aligned}
$$

$\phi : \mathbb{R}^n \to \mathbb{R}$	objective function
$\mathcal{M} \subset \mathbb{R}^n$	AC power flow equations
$\mathcal{X} \subset \mathbb{R}^n$	operational constraints

Transmission vs distribution grids



Transmission grids

- grid model
- power demand predictions

→ **offline programming (OPF)**

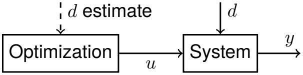
Distribution grids

- poor models
- unknown demands
- time-varying parameters
- faster than tertiary control
- sparse actuation
- sparse measurements

→ **real-time (feedback) decision**

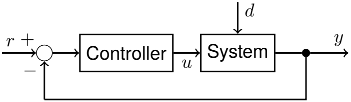
A feedback approach to set-point scheduling

Feedforward optimization



- **complex optimal decision**
- **operational constraints**
- **MIMO (multi-input/output)**
- highly model-based

Feedback control



- **robust to model uncertainty**
- **fast response**
- requires exogenous set-points
- suboptimal resource use

Autonomous optimization

An **autonomous feedback** approach to **optimal** real-time operation to inherit the best of the two worlds

Related works

- **power system operations** (& other infrastructures)
  - Frequency Control
  - Voltage Control
  - general AC OPF

Work from 2013-now by:

Low, Li, Dörfler, Bolognani, Simpson-Porco, Zhao, Dall'Anese, Simonetto, De Persis, Gan, Topcu, Bernstein, Jokic, ... → **survey [Molzahn et al., 2018]**

- **other related approaches**:
  - **process control**: reducing the effect of model uncertainty in succ. optimization
    Optimizing Control [Garcia & Morari, 1981/84], *Self-Optimizing Control* [Skogestad, 2000], *Modifier Adaptation* [Marchetti et. al, 2009], *Real-Time Optimization* [Bonvin, ed., 2017], ...
  - **extremum-seeking**: derivative-free, but hard for higher dimensions & constraints [Ariyur & Krstic, 2003], [Grushkovskaya et al., 2017], [Feiling et al., 2018], ...
  - **congestion control** in communication networks [Kelly et al. 1998], [Low et al. 2002]
    real-time iteration [Diel et al. 2005], real-time MPC [Zeilinger et al. 2009]

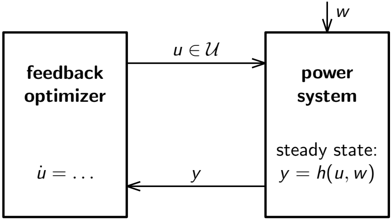
FEEDBACK OPTIMIZATION DESIGN

Design of optimizing feedback

- Algebraic constraints on the state x
  - **Implicit:** power flow equations $f(x) = 0$
  - **Explicit:** steady state of local controllers + physics $y = h(u, w)$
    - u **controllable inputs** (generator P/V set-points, controllable loads, etc.)
    - w **uncontrollable inputs** (loads, uncontrollable generators, etc.)
    - y **measured state** on the grid (possibly filtered through state estimator)

Equivalent optimization problem

$$
\begin{aligned}
\text{minimize}_{u,y} \quad & J(u, y) \\
\text{subject to} \quad & y \in \mathcal{Y} \\
& u \in \mathcal{U} \\
& y = h(u, w)
\end{aligned}
$$



- **Input saturation:** $u \in \mathcal{U}$ at all times (hard constraint)
- **Closed-loop trajectory:** $y = h(u, w) \in \mathcal{Y}$ at steady state
- **Optimality:** The closed-loop system converges to the solution of the OPF

Online optimization in closed loop

Optimization perspective

Algorithms as dynamical systems

[Lessard et al., 2014], [Wilson et al., 2018]

→ **implemented via the physics**

Control perspective

Existing feedback systems interpreted as solving opt. problem

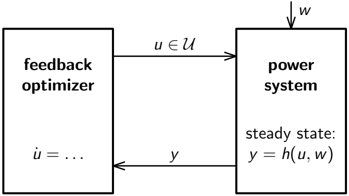
→ **general objective + constraints**

"Certainty equivalence" design

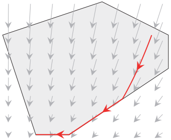
- $\mathcal{Y} \to$ penalty function in $J(u, y)$
- **assume steady-state** $y = h(u, w)$

$$\text{minimize}_u \quad \underbrace{J(u, h(u, w))}_{:=\phi(u)}$$

$$\text{subject to} \quad u \in \mathcal{U}$$



Projected gradient descent $\dot{u} = \Pi_{T_u \mathcal{U}} \left[-Q \nabla \phi(u) \right]$



Feedback optimizer

$$\dot{u} = \Pi_{T_u\mathcal{U}}\left[-Q\nabla\phi(u)\right] \qquad \phi(u) = J(u,h(u,w)) = J(u,y)|_{y=h(u,w)}$$

$$= \Pi_{T_u\mathcal{U}}\left[-Q\underbrace{\left(\nabla_u J(u,y) + \nabla_u h(u,w)^\top \nabla_y J(u,y)\right)}_{:=\widehat{\nabla\phi}(u,y) \quad \text{feedback evaluation of the gradient}}\right]$$

- **Input saturation:** $u \in \mathcal{U}$ at all times
- **Metric:** $Q \geq 0$, possibly state-dependent
- **Robust / Model-free:** $\nabla_u h(u,w) \approx$ sensitivities, w unknown
- **Feasibility:** guaranteed by stability of local controllers + physics

Existence of $\nabla_u h(u,w)$ depends on the existence of the explicit map h

$$f(y,u,w) = 0 \quad \rightarrow \quad y = h(u,w) \quad \rightarrow \quad \text{invertibility of Jacobian } \nabla_y f$$

- We recover standard **voltage "stability"** results for power systems [Tamura 1983] [Sauer 1990] [Dobston 2011]

Optimization algorithms as dynamical systems

Design of **feedback optimizer** $\rightarrow$ continuous-time limit of iterative algorithms

- Gradient Flows on Matrix Manifolds

  [Brockett, 1991], [Bloch et al., 1992], [Helmke & Moore, 1994], ...

- Interior-point methods

  [Karmarkar, 1984], [Khachian, 1979], [Faybusovich, 1992], ...

- Acceleration & Momentum methods

  [Su et al., 2014], [Wibisono et al, 2016], [Krichene et al., 2015], [Wilson et al., 2016], [Lessard et al., 2016], ...

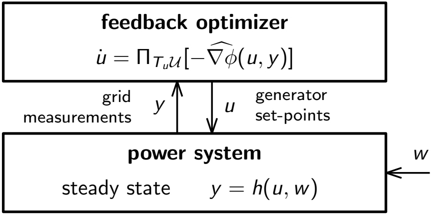
- Saddle-Point Flows

  [Arrow et al., 1958], [Kose, 1956], [Feijer & Paganini, 2010], [Cherukuri et al., 2017], [Holding & Lestas, 2014], [Cortés & Niederländer, 2018], [Qu & Li, 2018], ...

In continuous-time, most algorithms reduce to either (projected) **gradient flows** (w/ w/o momentum), (projected) **Newton flows**, or (projected) **saddle-point** flows.

CLOSED-LOOP STABILITY

Stability



Two reasons why it may not converge to the OPF solution

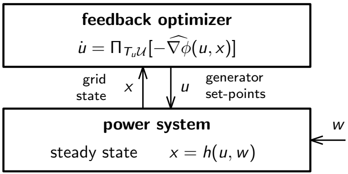
- **Irregularity of the domain**

$\rightarrow$ Hauswirth, Bolognani, & Dörfler (2018)
"Projected Dynamical Systems on Irregular, Non-Euclidean Domains for Nonlinear Optimization"

- **Interplay with the dynamics of the grid** ($y \approx h(u, w)$)

$\rightarrow$ Hauswirth, Bolognani, Hug, & Dörfler (2019)
"Timescale Separation in Autonomous Optimization"

Gradient-based feedback optimization



Optimization Dynamics

Variable-metric gradient descent

$$\dot{u} = -Q(u)\nabla\phi(u)$$

where

- $Q(u) \succ 0$ for all $u \in \mathbb{R}^p$
- $\phi(u) := J(u, h(u,w))$
- $\nabla\phi(u) = \nabla_u J + \nabla h^T \nabla_y J$

Plant Dynamics

Exponentially stable system

$$\dot{x} = f(x,u)$$

with steady-state map $x = h(u,w)$

Interconnection

$$\dot{x} = f(x, u)$$

$$\dot{u} = -Q(u)\left(\nabla_u J(u, x) + \nabla h^T \nabla_y J(u, x)\right)$$

Gradient-based Feedback Optimization

Theorem

Assume

- Physical system **exponentially stable** with Lyapunov function $W(x,u)$ s.t.

$$\dot{W}(x,u) \leq -\gamma \|x - h(u)\|^2 \qquad \|\nabla_u W(x,u)\| \leq \zeta \|x - h(u)\| .$$

- $J(u,x)$ has compact level sets and **L-Lipschitz** gradient.

Then, all trajectories converge to the set of KKT points whenever

$$\sup_{u \in \mathbb{R}^p} \|Q(u)\| < \frac{\gamma}{\zeta L} .$$

Furthermore,

- Asymptotically stable equilibrium $\Rightarrow$ strict local minimizer
- Strict local minimizer $\Rightarrow$ stable equilibrium

$\rightarrow$ If J convex and $h(u,w)$ linear, then convergence to set of global minimizers.

Gradient-based Feedback Optimization

Vanilla GD

Choose $Q = \varepsilon I_n$.
Stability is guaranteed if

$$\varepsilon \leq \frac{\gamma}{\zeta L}$$

$\Rightarrow$ prescription on global control gain

Projected GD

Control signal u constrained to set $\mathcal{U}$ (in case of actuator saturation).

$$\dot{u} = \Pi_{T_u \mathcal{U}}[-\varepsilon \nabla \phi(u)]$$

$\Rightarrow$ stable if $\varepsilon \leq \frac{\gamma}{\zeta L}$ (same bound)

Newton GD

Choose $Q(u) = (\nabla^2 J(u, h(u)))^{-1}$
(if J μ-strongly cvx and twice diff'ble)
Stability is guaranteed if

$$\frac{L}{\mu} \leq \frac{\gamma}{\zeta}$$

$\Rightarrow$ invariant under scaling of J

Not

- Subgradient methods
- Accelerated gradient method

Saddle flows

$$\begin{aligned} \text{minimize}_{u,y} \quad & J(u,y) \\ \text{subject to} \quad & g(y) \leq 0 \\ & u \in \mathcal{U} \\ & y = h(u,w) \end{aligned}$$

- "certainty-equivalence" $y = h(u,w)$
- Lagrangian

$$\mathcal{L}(u,\lambda) := J(u,y) + \lambda^\top g(y)\Big|_{y=h(u,w)}$$

Primal gradient descent / Dual gradient ascent

$$\begin{aligned} \dot{u} &= \Pi_{T_u\mathcal{U}}\left[-\nabla_u \mathcal{L}(u,\lambda)\right] \\ &= \Pi_{T_u\mathcal{U}}\left[-\nabla_u J(u,y) - \underbrace{\nabla_u h(u,w)^\top}_{\text{model}} \nabla_y J(u,y) - \underbrace{\nabla_u h(u,w)^\top}_{\text{model}} \nabla_y g(y)^\top \lambda\right] \\ \dot{\lambda} &= \Pi_{\geq 0}\left[\nabla_\lambda \mathcal{L}(u,\lambda)\right] \\ &= \Pi_{\geq 0}\left[g(y)\right] \end{aligned}$$

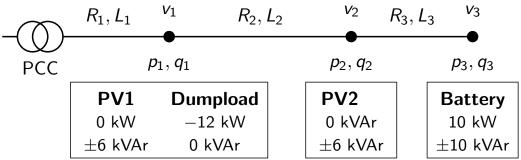
Stability analysis requires special care (exponential stability of saddle flows).

ENGINEERING

Experimental result

TEAMVAR

Experimental distribution feeder SYSLAB at DTU, Denmark.



u **control inputs:** reactive power injection PV1, PV2, Battery

y **measurement:** voltage magnitude PV1, PV2, Battery

optimization problem

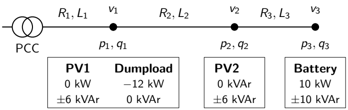
$$\begin{aligned} \text{minimize} \quad & \sum_i q_i / q_i^{\max} \\ \text{subject to} \quad & q_i \in [q_i^{\min} \; q_i^{\max}] && \mathcal{U} \\ & v_i \in [v^{\min} \; v^{\max}] && \mathcal{Y} \end{aligned}$$

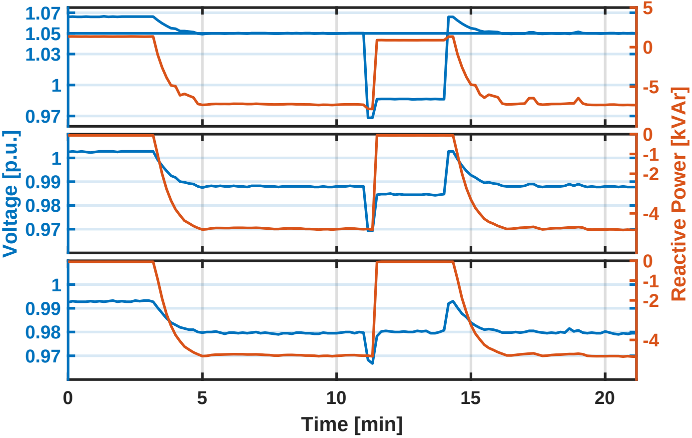
projected saddle flow

Feedback optimization (experiment)

Model-free

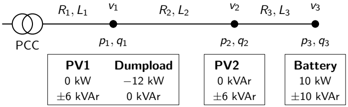
$$\nabla_u h = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

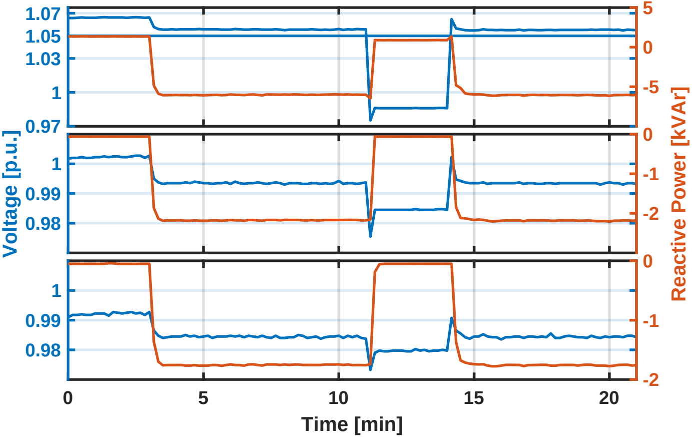




Feedforward OPF (experiment)

- Fast
- Fragile
- Requires full state





Distributed feedback

Example: projected gradient

$$\dot{u} = \Pi_{T_u\mathcal{U}}\left[-Q\left(\nabla_u J(u,y) + \nabla_u h(u,w)^\top \nabla_y J(u,y)\right)\right]$$

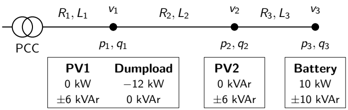
- Cost function $J(u,y)$ is usually **separable** in y
- Choose Q in order to make **both** $Q\nabla_u J(u,y)$ and $Q\nabla_u h(u,w)^\top$ **sparse**
- **Warning:** $\Pi_{T_u\mathcal{U}}$ needs to be computed according to the same metric

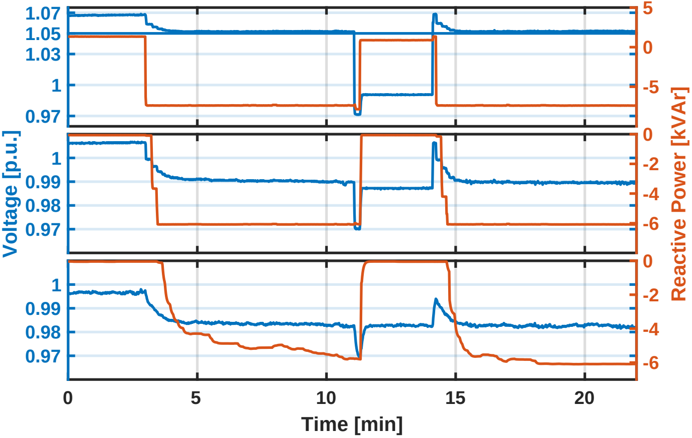
$$\Pi_{T_u\mathcal{U}}[z] = \arg\min_{\xi\in T_u\mathcal{U}} \|\xi - v\|_Q$$

- If Q is sparse, then $\Pi_{T_u\mathcal{U}}$ is a **QP** with sparse linear constraints
  $\rightarrow$ distributed solver (no sensing/actuation)

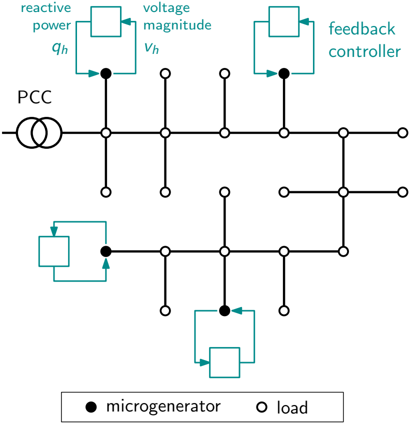
$\rightarrow$ Bolognani, Carli, Cavraro, & Zampieri (2019)
"On the Need for Communication for Voltage Regulation of Power Distribution Grids"

Distributed feedback optimization (experiment)



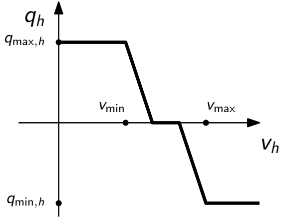


From distributed to decentralized?



Tempting idea: choose Q to have completely decentralized feedback

→ VDE-AR-N 4105-2018 (2018)
→ EU Commission Regulation 2016/631 (2016)
→ IEEE Standard 1547-2018 (2018)



$$q_h(t+1) = f_h(v_h(t))$$

Impossible.

→ Bolognani, Carli, Cavraro, & Zampieri (2019)
"On the Need for Communication for Voltage Regulation of Power Distribution Grids"

CONCLUSIONS

Conclusions

- Generator set-points need to be dynamically generated in order to
  - relief **congestion in the distribution grid**
  - respond dynamically to **time-varying** parameters

- **Feedback optimization** / **autonomous optimization** schemes
  - are essentially **model-free**
  - do not require **full-state monitoring**
  - have **certified performance / stability**

- The design of feedback optimization schemes taps directly into **iterative optimization methods**
  - performance / tuning is well-understood
  - potential for distributed (but not decentralized) implementation

$\rightarrow$ Dörfler, Bolognani, Simpson-Porco, & Grammatico (2019)
"Distributed Control and Optimization for Autonomous Power Grids"

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