A Fast Method for Real-Time Chance-Constrained Decision with Application to Power Systems

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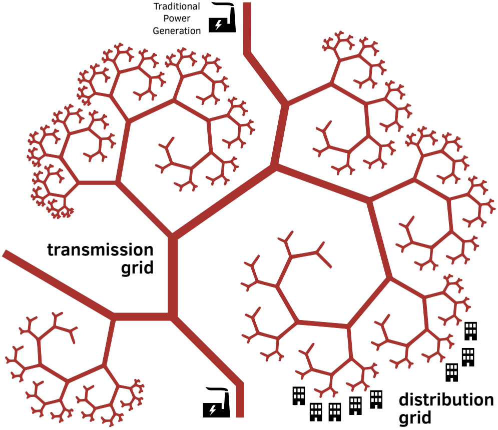
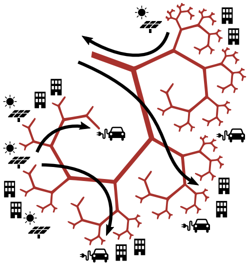
Power distribution systems

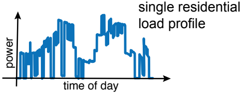


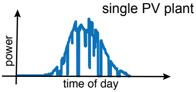
- **Traditional role**
  deliver power from the utility grid to the loads
- **Physical constraints**
  Kirchhoff laws
- **Operational constraints**
  limits on voltages, line currents, transformer loading, etc.
- **Fit-and-forget**
  historically designed according to worst-case possible demand

New challenges

- **Distributed microgeneration**
  - conventional and renewable sources
  - congestion and under-/over-voltage
- **Variable renewable energy sources**
  - poor short-range prediction
  - correlated uncertainty
- **Electric mobility**
  - large peak (power) & total (energy) demand
  - flexible but spatio-temporal patterns

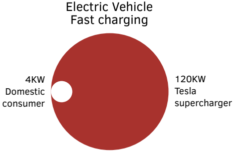






Decision under uncertainty

Based on the available **measurements**, determine the **control signals** such that the **state of the grid** satisfies given **operational constraints**.



Problem statement

- **Grid state** $x = \begin{bmatrix} v & \theta & p & q \end{bmatrix}$

  $v_i e^{\theta_i}$ bus complex voltage
  $p_i + jq_i$ bus complex power injection

- Linearized **power flow equations**

  $\left.\frac{\partial f}{\partial x}\right|_{x^*} (x - x^*) = 0$

- Algebraic **control inputs** u

  e.g. controllable load p_i
  e.g. microgenerator reactive power q_i

- Stochastic **disturbances** w

  e.g. uncontrollable load p_i

- Polytopic **operational constraints**

  e.g. voltage limits $v_{\min} \leq v_i \leq v_{\max}$

- Linear ***measurements*** y

  e.g. voltage magnitude v_i

Chance-constrained decision

$$\text{minimize} \quad f(u)$$

subject to

$$\mathbb{P}\left[Au + Bw \leq z \mid \mathbf{Hw} = \mathbf{y} \right] \geq 1 - \epsilon$$

where

- u is the control input
- $f(u)$ is a convex objective
- w is the disturbance
- $\mathbb{P}$ is the probability w.r.t. w
- ϵ is the violation probability
- $\mathbf{Hw} = \mathbf{y}$ is the measurement

Scenario approach

Let $w^{(i)}$ be samples of the stochastic disturbance w.

M. C. Campi, S. Garatti (2008)

The solution of the deterministic problem

$$\begin{aligned} \text{minimize} \quad & f(u) \\ \text{subject to} \quad & Au + Bw^{(i)} \leq z \quad i = 1, \ldots, N \end{aligned}$$

satisfies $\mathbb{P}\left[Au + Bw \leq z\right] \geq 1 - \epsilon$ with confidence larger than $1 - \beta$ if

$$\sum_{i=0}^{N-1} \binom{N}{i} \epsilon^i (1-\epsilon)^{N-i} \leq \beta,$$

How to use the real-time measurement $Hw = y$ ***?***

Scenario approach with measurements

- **scenario approach:** stochastic constraint

  → (very) large set of deterministic constraints

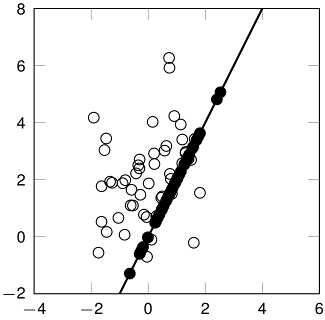
- two **sources of information** on the unknown w

  1. **historical samples** $w^{(i)}$ of prior distribution

  → classic scenario-based approach

  2. **online measurements** y from the system

  → use measurements to reduce uncertainty?



- **re-sampling solution:** scenario approach based on conditional distribution

→ historical samples discarded, high computational and memory demand

- today: **online computation** of posterior distribution after measurement

Approximate conditioning

Affine transformation

$$\hat{w}_y = w + K(y - Hw)$$

where

$$K = \Sigma H^\top \left(H \Sigma H^\top\right)^{-1}$$

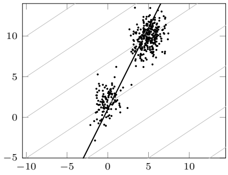
and $\Sigma = \mathrm{cov}(w)$

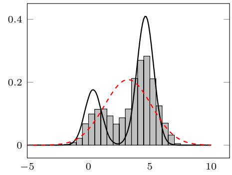
→ **projection** of uncertainty in the subspace $\{y = Hw\}$

→ **Gaussian case:** recovers the conditional distribution

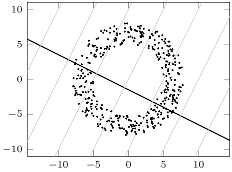
→ outperforms **conventional approximation** of the prior by a Gaussian distribution

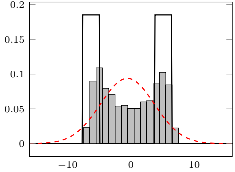
Bimodal distribution	Mean	Variance	Skewness	Kurtosis
True posterior	3.35	4.23	-0.74	2.00
Gaussian approximation	3.20	3.57	0	3
Affine transformation	3.20	3.57	-0.54	2.35





Annular distribution	Mean	Variance	Skewness	Kurtosis
True posterior	-0.6	32.9	0	1.08
Gaussian approximation	-0.6	17.8	0	3
Affine transformation	-0.6	17.8	0	1.60





Affine transformation of the feasible region

The feasible polytope $Au + B\hat{w}_y \leq z$ can be approximated as

$$Au + B\left(w + K\left(y - Hw\right)\right) \leq z$$

which is a function of the **a-priori disturbance** w and of the ***measurement*** y.

Scenario approach: replace w with finitely many historical samples $w^{(i)}$

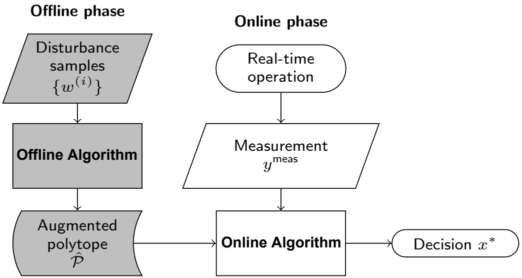
$$\bigcap_{i=1}^{N} Au + B\left(w^{(i)} + K\left(y - Hw^{(i)}\right)\right) \leq z \quad \rightarrow \quad \text{polytope } \hat{\mathcal{P}} \text{ in } u \text{ and } y$$

Offline algorithm = scenario approach & affine transformation

Given historical samples $\{w^i\}$ and the measurement matrix H:

- compute $K = \Sigma H^\top \left(H \Sigma H^\top\right)^{-1}$ from the sample covariance Σ
- construct augmented polytope $\hat{\mathcal{P}}$ and compute minimal representation

Real-time decision



Online algorithm = slicing of constraints with measurements

Given the augmented polytope $\hat{\mathcal{P}}$ and the measurement y^{meas}, solve

$$\text{minimize}_{u,y}\ f(u) \qquad \text{subject to} \qquad (u,y) \in \hat{\mathcal{P}} \cap \{\, y = y^{meas} \,\}$$

Example: IEEE 123-bus test system

- **Actuation**
  distributed microgeneration

$$p_i \,,\quad i \in \mathcal{G}$$

- **Control cost**
  cost of curtailment

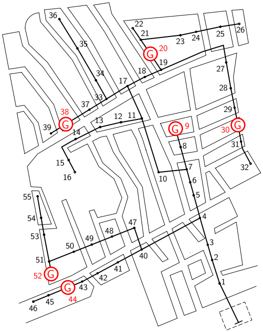
$$\text{maximize} \sum_{i \in \mathcal{G}} p_i$$

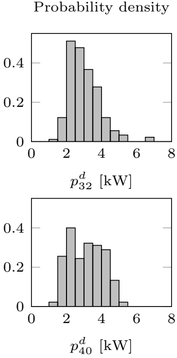
- **Operational constraints**
  overvoltage limits

$$v_i \leq v_{\max} \;\; \forall i$$

- **Scalar measurement**
  total demand

$$y = \sum_i p_i$$

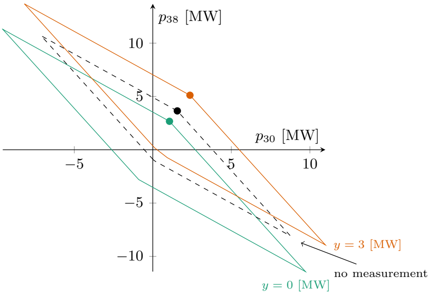




Historical samples
power demands of 1200 households from the Danish city of Horsens (provided by DSO NRGi)

(note: demands are not monitored in real time)

Example: IEEE 123-bus test system



Computation time

Offline	Compute Σ and K	
	Construct augmented polytope $\hat{\mathcal{P}}$	
	Compute minimal representation of $\hat{\mathcal{P}}$	
	Total offline computation time	*55 min*
Online	Slice $\hat{\mathcal{P}}$ at $y = y^{\text{meas}}$ to obtain $\mathcal{P}$	
	Total online computation time	*1.8 ms*

Memory footprint

Offline	Augmented polytope $\hat{\mathcal{P}}$	48620 constraints
Online	Minimal representation of $\hat{\mathcal{P}}$	12 constraints

- Ignoring measurements yield the smaller feasible region
  uncertainty ⇒ conservativeness
- Incorporating measurement in real time yields
  - increased **security** (y = 0 MW)
  - increased **efficiency** (y = 3 MW)
- The **computation time** and **memory footprint** of the online algorithm is minimal
  - real-time ready
  - can be embedded in sensors/actuators