Regelsysteme 2

Scaling:

$D_e = \hat{e}_{max}$, $D_u = \hat{u}_{max}$, $D_d = \hat{d}_{max}$, $D_r = \hat{r}_{max}$

$d = D_d^{-1}\hat{d}$, $u = D_u^{-1}\hat{u}$, $y = D_e^{-1}\hat{y}$,

$e = D_e^{-1}\hat{e}$, $r = D_e^{-1}\hat{r}$ or $r = D_r^{-1}\hat{r}$

$G = D_e^{-1}\hat{G}D_u$, $G_d = D_e^{-1}\hat{G}_d D_d$

CL-Transfer functions

$L = GK$

$S = (I + GK)^{-1} = (I+L)^{-1}$

$T = (I+GK)^{-1}GK = (I+L)^{-1}L$

$S + T = I$

$y = Tr + SG_d d - Tn$

$e = y - r = -Sr + SG_d d - Tn$

$u = KSr - KSG_d d - KSn$

SISO-Limitations

Dist. Rej.: $\omega_c > \omega_d$ or $|S(j\omega)| \leq |1/G_d(j\omega)|\ \forall\omega$

Ref. Track.: $|S(j\omega)| \leq 1/R$ up to some ω_r

Delay: $\omega_c < \frac{1}{\theta}$

RHP-Zero: $\omega_c < \frac{z}{2}$

RHP-Pole (OL): $\omega_c > 2p$ to stabilize

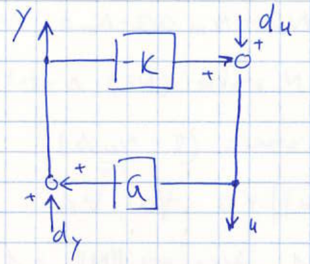
MIMO-poles

The pole polynomial $\phi(s)$ corresponding to a minimal realization is the least common denominator of all non-zero minors of $G(s)$.

MIMO-zeros

The zero polynomial $z(s)$ corresponding to a minimal realization is the greatest common divisor of all order-rank(G) minors of $G(s)$, adjusted such that $\phi(s)$ is their denominator.

Internal stability



$u = (I+KG)^{-1}d_u - K(I+GK)^{-1}d_y$

$y = (I+GK)^{-1}d_y + G(I+KG)^{-1}d_u$

For IS, all 4 TF stable.

For stable G: Only check $Q = K(I+GK)^{-1}$

Controllability

$C = [B \ AB \ A^2B \dots A^{n-1}B]$

must have full rank.

Observability

$O = [C \ CA \ CA^2 \dots CA^{n-1}]^T$

must have full rank.

Closed-loop stability

MIMO Nyquist

$\det(I + L(s))$ Nyquist plot must

i) make P_{ol} anti-clockwise encirclements of the origin

ii) not pass trough the origin

Spectral radius

$\rho(L(j\omega)) = \max_i |\lambda_i(L(j\omega))| < 1\ \forall\omega$

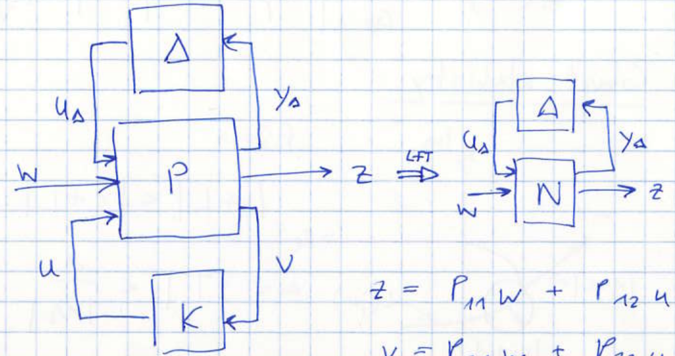
Is conservative (no phase information)

Small-gain theorem

$\|L(j\omega)\| < 1\ \forall\omega$, for any norm

Is conservative (no phase and $\rho(L) \leq \|L\|$)

General control problem



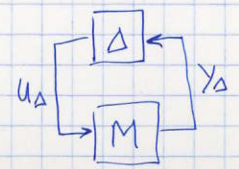
$z = P_{11}w + P_{12}u$

$v = P_{21}w + P_{22}u$

Lower LFT: $N = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}$

Upper LFT: $F_u = N_{22} + N_{21}\Delta(I - N_{11}\Delta)^{-1}N_{12}$

For robust stability analysis:

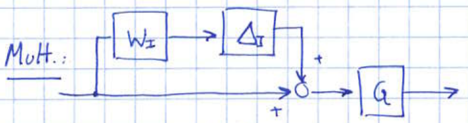


With $M = N_{11}$

SISO Robustness

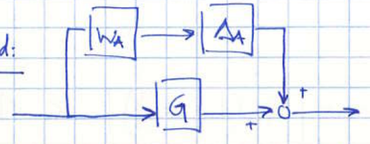
Uncertainty

Mult.:



$$G_p = G\left(1 + w_I(s)\,\Delta_I\right)$$

Add:



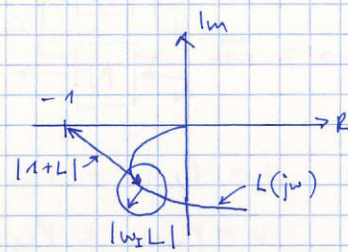
$$G_p = G + w_A \Delta_A$$

Smallest radius:

$$l_I(\omega) = \max_{G_p} \left| \frac{G_p - G}{G} \right| \leq |w_I(j\omega)|$$

$$l_A(\omega) = \max_{G_p} \left| G_p - G \right| \leq |w_A(j\omega)|$$

Robust stability

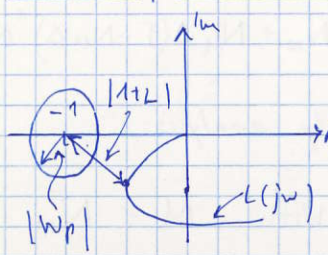


$RS \Leftrightarrow$

$$|w_I L| < |1 + L| \quad \forall\omega$$

$$\Leftrightarrow |T| < \frac{1}{|w_I|}$$

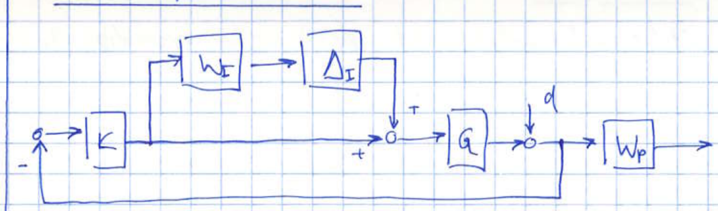
Nominal performance

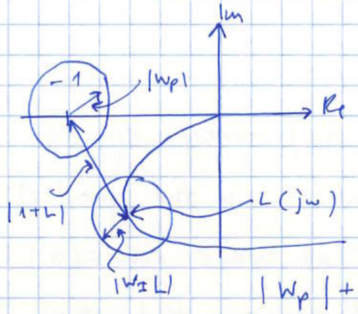


$NP \Leftrightarrow$

$$|S| < \frac{1}{|w_p|} \quad \forall\omega$$

Robust performance





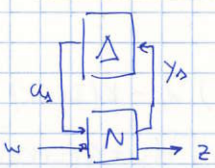
$RP \Leftrightarrow$

$$\max_{\omega}\left(|w_p S| + |w_I T|\right) < 1$$

$$|w_p| + |w_I L| < |1 + L|$$

MIMO Robustness

Robust stability



$$F_u(N,\Delta) = N_{22} + N_{21}\Delta(I - N_{11}\Delta)^{-1}N_{12}$$

Assume N stable (NS), Δ stable

$\Rightarrow$ only check $(I - \underbrace{N_{11}}_{=M}\Delta)^{-1}$

$$RS \Leftrightarrow \det(I - M\Delta(j\omega)) \neq 0 \quad \forall\omega, \forall\Delta$$

$$\Leftrightarrow \bar{\sigma}(M(j\omega)) < 1 \quad \forall\omega \Leftrightarrow \|M\|_\infty < 1$$

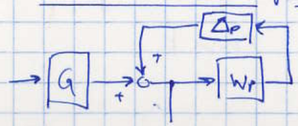
Structured singular value

Find the smalles structured Δ (measured in terms of $\bar{\sigma}(\Delta)$) which makes the matrix $I - M\Delta$ singular, then $\mu(M) = 1/\bar{\sigma}(\Delta)$.

$$\mu(M)^{-1} = \min_{\Delta}\{\bar{\sigma}(\Delta) \mid \det(I - M\Delta) = 0,\ \Delta \text{ structured}\}$$

$$RS \Leftrightarrow \mu(M) < 1$$

Performance as perturbation



Here: $RS \Leftrightarrow RP$

$$\hat{\Delta} = \begin{bmatrix} \Delta & 0 \\ 0 & \Delta_p \end{bmatrix}$$

Robust performance

$NS \Leftrightarrow N$ (internally) stable

$NP \Leftrightarrow \bar{\sigma}(N_{22}) = \mu_{\Delta p} < 1 \quad \forall\omega$, and NS

$RS \Leftrightarrow \mu_\Delta(N_{11} = M) < 1 \quad \forall\omega$, and NS

$RP \Leftrightarrow \mu_{\hat{\Delta}}(N) < 1 \quad \forall\omega$, and NS

H_2/H_∞ optimal control

H_2:
- Minimizes energy (rms) in error signal from unit intensity white noise
- Minimizes all singular values
- Is a norm

H_∞:
- Minimizes worst case deviation for any input
- Minimizes peak of largest singular value
- Is a norm

S/T Loop shaping

Dist. rej.: $\bar{\sigma}(S)$ small at low freq.
Noise att.: $\bar{\sigma}(T)$ small at high freq.
Ref track.: $\bar{\sigma}(T) \approx \underline{\sigma}(T) \approx 1$ at $\omega < \omega_B$
Energy red.: $\bar{\sigma}(KS)$ small at $\omega < \omega_B$
Robust. stab.: $\underbrace{\bar{\sigma}(KS)}_{\text{add. }\Delta} / \underbrace{\bar{\sigma}(T)}_{\text{mult. }\Delta}$ small at $\omega < \omega_B$

Weights:

$$w(s) = \frac{s/M + \omega_B^*}{s + \omega_B^* A}$$