



Comparative Analysis of SWE models

- Case studies of floods in the North of Madagascar -



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 - Context
 - How to model a flood ?
 - Problems
 - Objective

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 - The Shallow Water Equation
 - How to solve SWE ?
 - Simulation software
 - Study areas

- 3 Results and discussion
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 - On a flooding case

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Context

Flooding as the main risk in Madagascar

- Madagascar is the 3^{graverd} most exposed country in the world to tropical cyclones and first among African countries.
- These cyclones bring heavy rains causing flooding.

Identification of flood zones.

- Several methods are possible depending on the need for detail.
- Hydrodynamic method often discarded
- Need to make complex calculations
- Requiring specific tools
- Lack of data (hydrograph, DTM, etc.)

How to model a flood ?

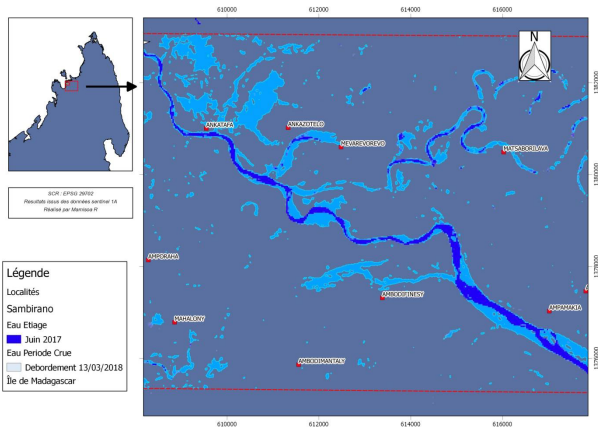
Empirical Methods

Simplified Conceptual Methods

Hydrodynamic Methods

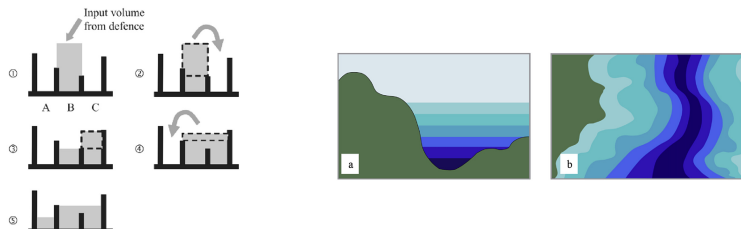
Empirical Methods

- Based on field measurements, surveys, interviews, satellite images, etc.
- Reflect the past and often considered robust and accurate
- Rapid but not detailed results



Simplified Conceptual Methods

- Widely referred to as 0D models
- Do not involve all the physical processes of flooding



- "*RFSM*" Rapid Flood Spread Method : consists in dividing the area into surfaces according to the topography and pouring the quantity of water for the flood then loading and/or emptying each elementary surface
- "*Planner method*" : considers the spreading of the flood by flat surfaces above the DTM

Problems

Identification of flood zones by remote sensing

- Satellite image processing
- Fast and can cover a large area
- Lack of frequency values of floods (useful for planning)
- Just maps of the past
- Optical image processing combined with radar images
- Results after the passage of the flood and depending on the passage of satellites

Flooding dynamics

Knowing the velocity field and the depths of the water

⇒ It is necessary to solve the Shallow Water Equations (hydrodynamic methods)

Hydrodynamic methods

- Can be in 1D, **2D** or 3D according to the spatial representation of the flow
- Based on the resolution of the system of the SWE
- Can represent all the physical processes of the flood
- Calculations relative to the available computing capacities
- Several software and solver developed

Developer	1D	2D	1D-2D	3D
EDF	Mascaret	TELEMAC 2D		TELEMAC 3D
US Army Corps of Engineers	HEC-RAS	HEC-RAS 2D		
Iric		Nays2D Flood, Mflow 02		
ETH Zurich	BASEChain	BASEPlane	BASEMENT	

Objective and methodological approaches

Global objective

Compare the three methods through the results of practical flooding cases. The comparison is made on the quality and the practical aspect of the method : precision, practicability, accessibility of the means, necessary resources (free software, calculation time etc.)

Identification of the means to be used

Identify and use software tools to model floods, taking into account the different criteria

Application for comparison purposes

- Section of the Sambirano River for the three methods FEM, FDM, and FVM
- Inundable area of the Mananjeba River around the village of Marivorahona for the FDM and FVM

The Shallow Water Equation

SWE : choice of laws

Formulation of the equations of motion

2 equations based on 2 physical laws (3 available) :

- 1 Conservation of Mass (CM)
- 2 Conservation of the Quantities of Movements (CQM)
- 3 Conservation of Energy (CE)

⇒ Choice of the couple (1 + 2) because it allows to study the flow when the variables are both continuous and discontinuous.

⇒ The couple (1 + 3) allows to make the study only for discontinuous variables.

SWE : principle and approach

Basic principle

- Start from the local equations of CM and CQM
- Integrate along the vertical to average them
- Simplify by eliminating weak terms

Identification of terms to be overlooked

- To carry out the complete integrations
- Try to interpret the results obtained

Usual approach

- 1 Show the quantities that control the flow
- 2 Eliminate the terms of weak influence
- 3 Do the integration

SWE (suite)

Different types of formulation depending on the variables chosen and also on the type of conservative or non-conservative formulation

1D Formulation

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q$$

$$\frac{1}{A} \frac{\partial Q}{\partial t} + \frac{1}{A} \frac{\partial \left(\frac{Q^2}{A} \right)}{\partial x} + g \frac{\partial h}{\partial x} - g (S_0 - S_f) = 0$$

With $Q = uA$ the flow rate; u the mean velocity; A the area of the normal section normal section, h the water depth, and S_0 and S_f the bottom and friction slopes

2D Formulation

$$\frac{\partial h}{\partial t} + \frac{\partial(hu)}{\partial x} + \frac{\partial(hv)}{\partial y} = S$$
$$\frac{\partial(hu)}{\partial t} + \frac{\partial}{\partial x} \left(hu^2 + \frac{1}{2}gh^2 \right) + \frac{\partial(huv)}{\partial y} = 0$$
$$\frac{\partial(hv)}{\partial t} + \frac{\partial(huv)}{\partial x} + \frac{\partial}{\partial y} \left(hv^2 + \frac{1}{2}gh^2 \right) = 0$$

With u and v values of the velocity vector, h the depth and S the source terms

- The 1D and 2D hypotheses can be used separately or combined according to the cases in the physical reality
- \Rightarrow The resolution of these equations allows to characterize the dynamics of the flow

How to solve SWE

Finite difference method

- Purely mathematical approach based on Taylor's theorem.

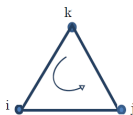
$$\frac{U_{i-1} - U_{i+1}}{2\Delta x} = \frac{\partial U}{\partial x}$$

- Grid (mesh) aligned
- Simple coding
- Values on the nodes

Does not allow to adapt to a complex practical reality and to solve "naturally" the discontinuities (Riemann problem) except special schemes (Example : mesh in curvilinear coordinates + use of a CIP scheme (Constrained Interpolation Pseudo-Particles), (*Ratsaramody et al., 2022*))

Finite elements method

- Based on the weak formulation
- Advanced theoretical basis
- Flexible grid
- Values on nodes
- Non conservative



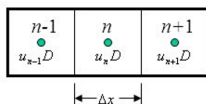
$$h = N_i h_i, \quad u = N_i u_i, \quad v = N_i v_i$$

$$N_i = a_i + b_i x + c_i y$$

$$a_i = (x_j y_k - x_k y_j) / 2s, \quad b_i = (y_j - y_k) / 2s, \quad c_i = (x_k - x_j) / 2s$$

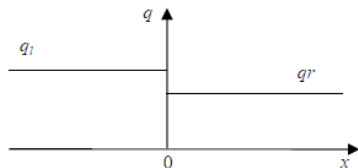
Finite volumes method

- Based on the integral formulation of the equation
- Flow principle : more physical because based on the conservation principle
- Well adapted for the conservation laws
- Values on the centroid of the mesh
- Flexible for curved limits



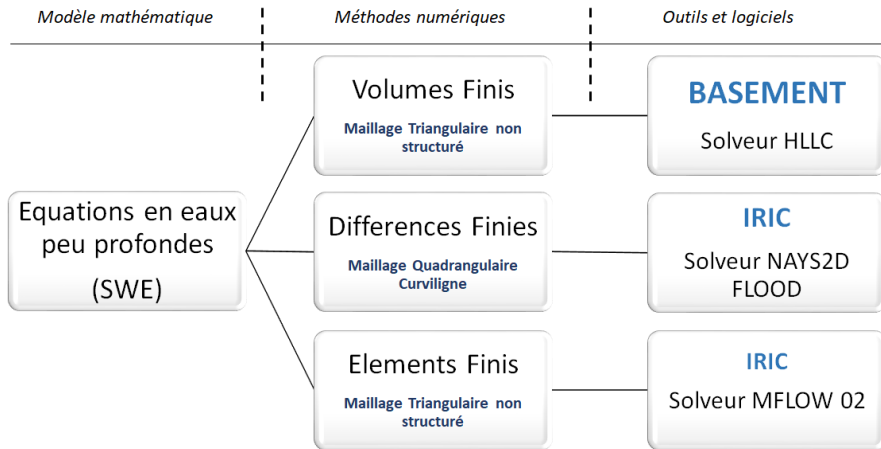
$$\frac{\partial u D}{\partial x} = \frac{u_{n+1} D_{n+1} - u_{n-1} D_{n-1}}{2\Delta x}$$

Riemann problem



- Represents the discontinuities
- Several adapted solvers (Roe solver, HLL, HLLC, etc.) with the finite volume method
- Difficulty (or impossibility) to integrate with other methods

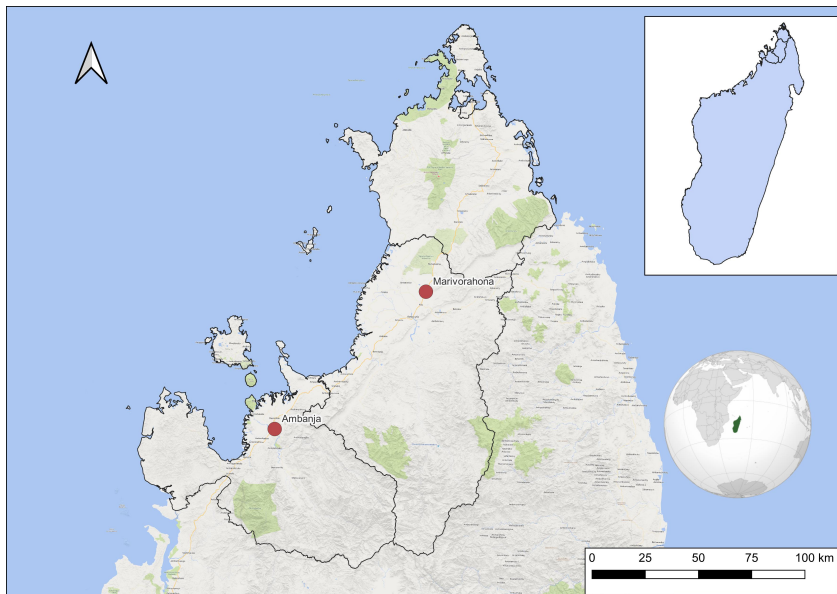
Methods and software used



All calculations were done using a laptop computer with an 8th generation i7 core, 24 GB of RAM and 12 Logical Processors.

APPLICATIONS

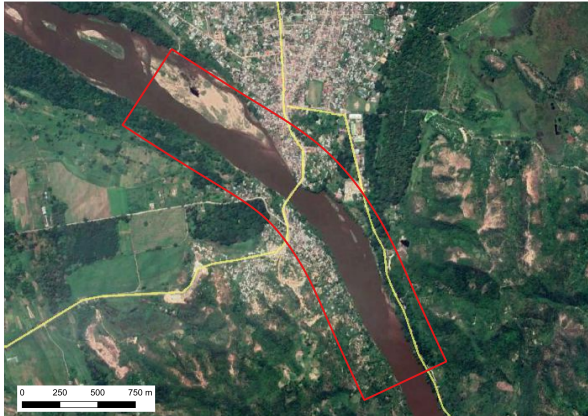
Global location of sites



Comparison of numerical methods

0

Characteristic of the section : location

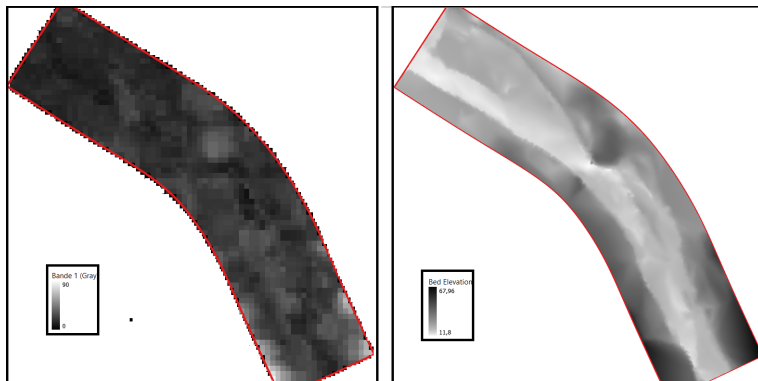


- Represents a constriction at the bridge
- Presents an overflow during floods
- High resolution topography and bathymetry

Section characteristics : topographic surveys

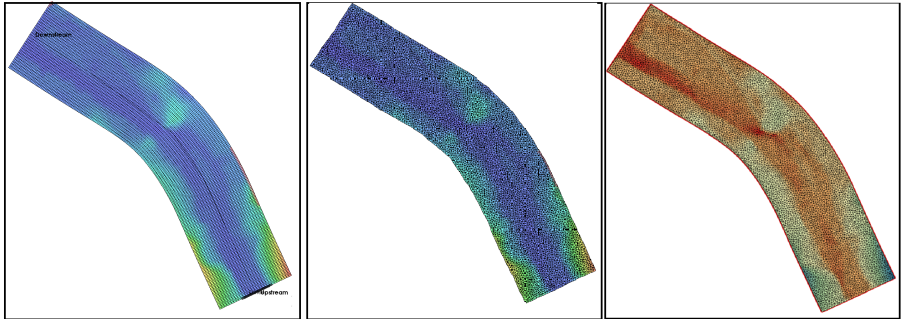


Section characteristics : available data



- Visible differences in spatial resolution
- Represents better the reality
- Difficulty of obtaining

Characteristic of the section : Mesh

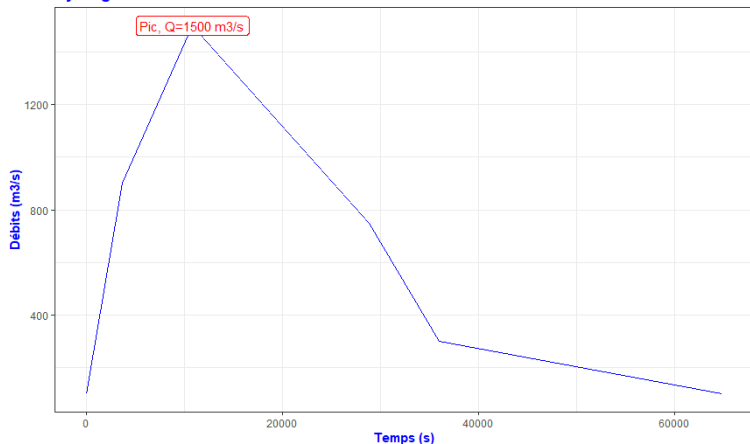


From left to right

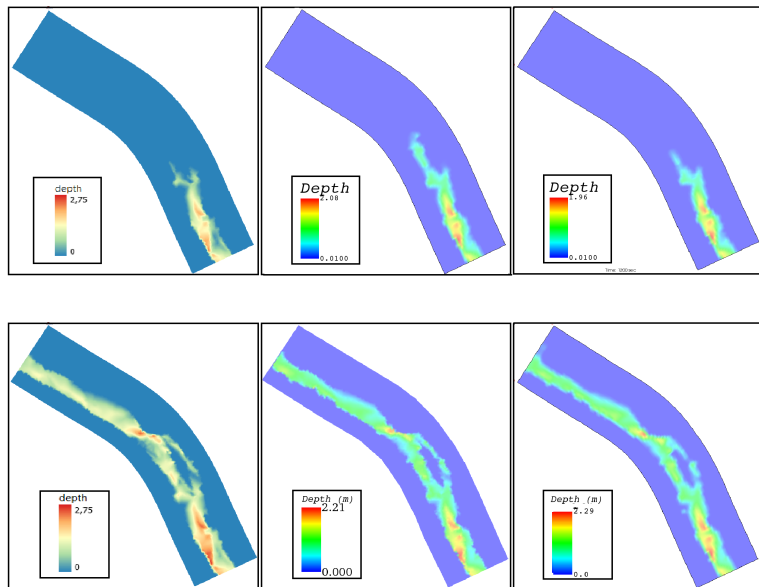
- Finite differences : Curvilinear quadrangle
- Finite elements : Unstructured triangular
- Finite volumes : Unstructured triangular

Flood hydrograph used

Hydrogramme à l'entrée

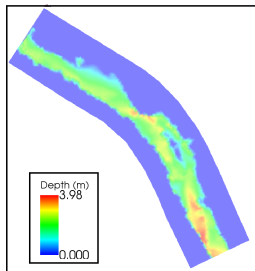


Initialization of the calculations

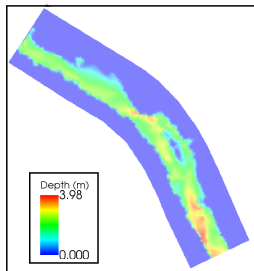


At $t = 2600s$

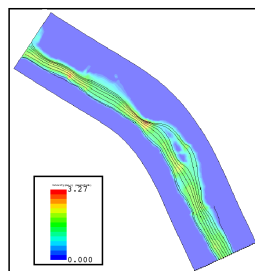
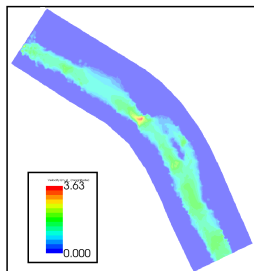
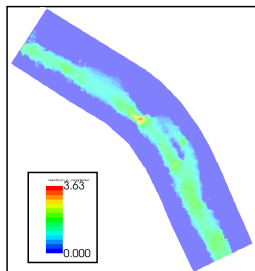
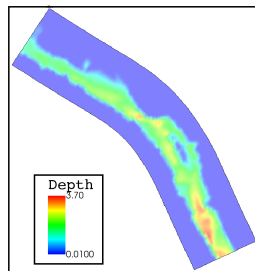
Volumes finis



Differences finis

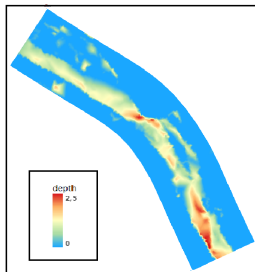


Elements finis

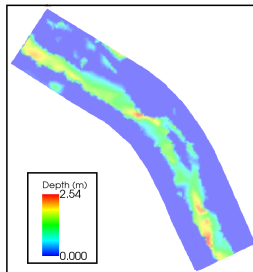


At $t = 64800s$

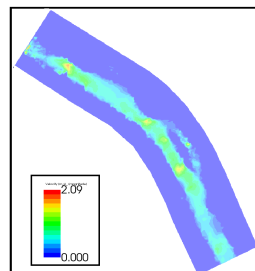
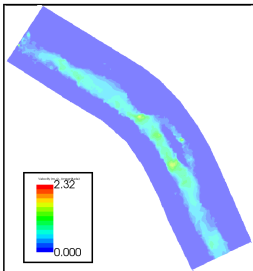
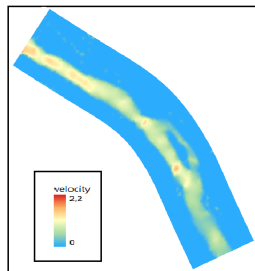
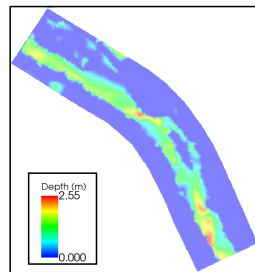
Volumes finis



Différences finies



Elements finis



Conclusions

Quality of the numerical results

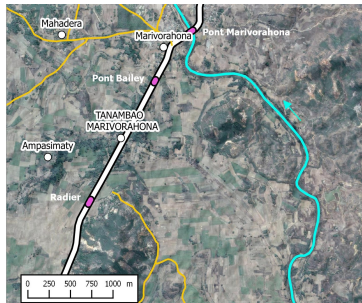
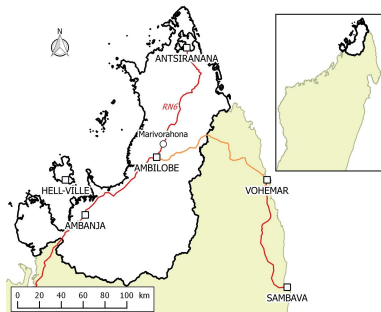
- All three methods give similar results
- The finite volume method has more advantages for the dry/wet transition
- The finite element method presents a difficulty in implementation (non acceptance of the free exit condition)

Speed of calculation

- Finite element methods (using the Mflow 02 solver) are not parallelizable, so the calculation time is very high
- Finite volumes (with BASEMENT) and finite differences (Nays2D Flood) can be parallelized so the calculation time is reduced depending on the number of available CPUs
- Coupling of BASEMENT with GPU calculation (using graphic processors) which greatly reduces the calculation time

Realistic modeling of the Mananjeba floodplain, at the village of Marivorahona

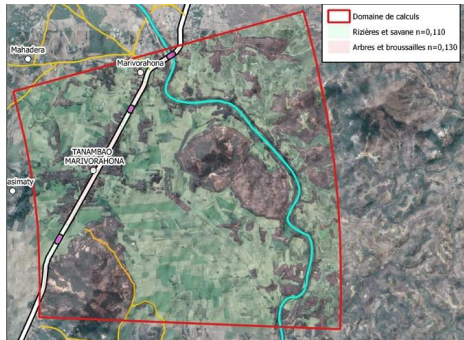
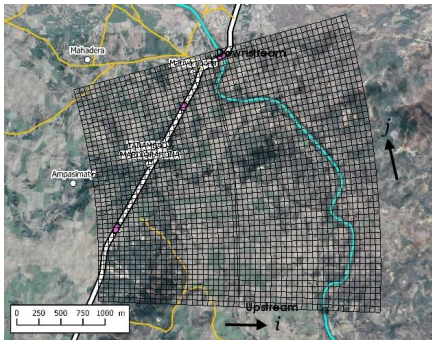
Mananjeba : presentation and reasons for choice



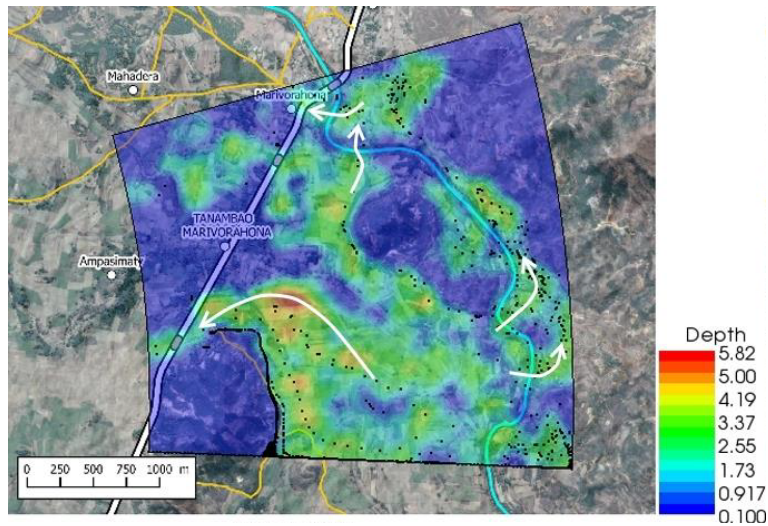
- Existing model with DF method and possibility of improvement
- Existence of a realistic hydrograph
- DTM improved at the level of the village of Marivorahona (surveyed with a drone)

Existing model : description

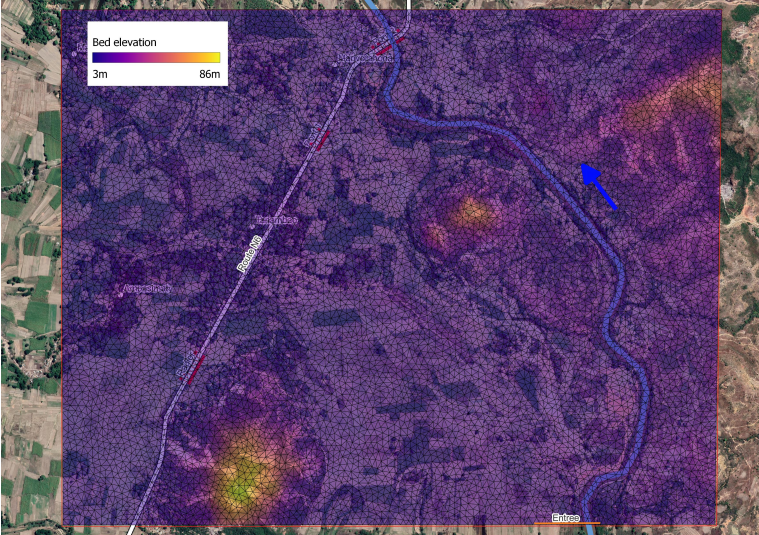
- Calculation with the Nays2D Flood solver (FD method)
- Meshing in curvilinear coordinates
- Use of the CIP scheme
- Simplified flood hydrograph with a peak of $1244 \text{ m}^3/\text{s}$



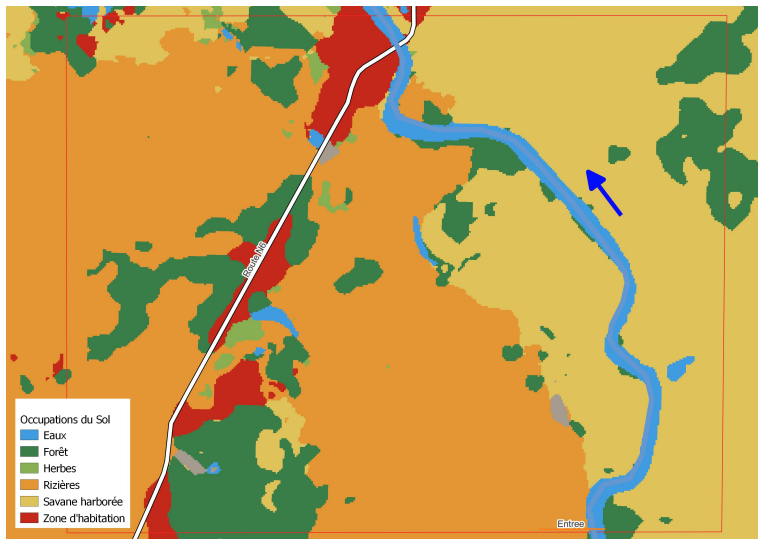
Mananjeba : result with the FD method



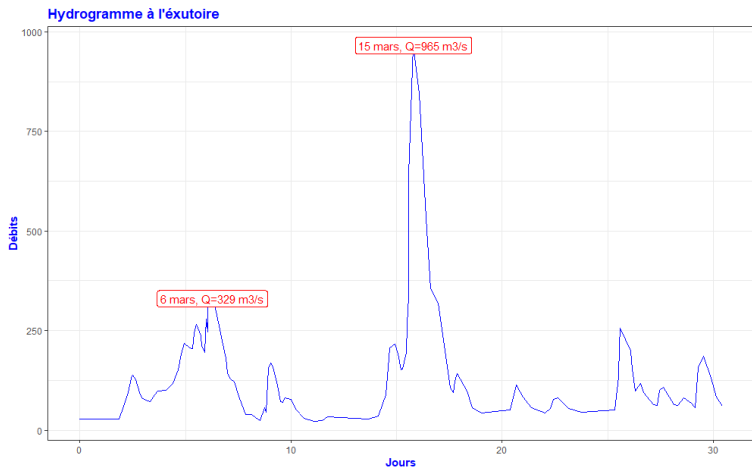
New modeling : mesh



Land use



Mananjeba : flow rate chronicle for the month of March 2018

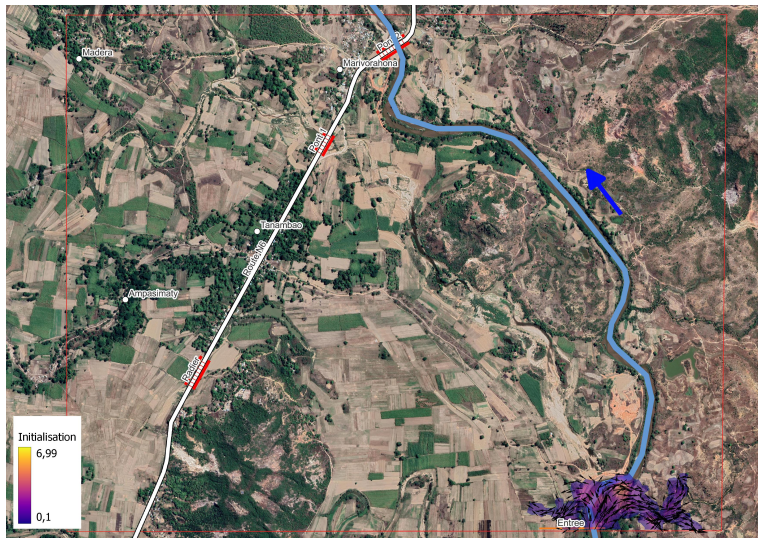


- Hydrograph from a hydrological model (*Ratsaramody et Randriamparany, 2021*)

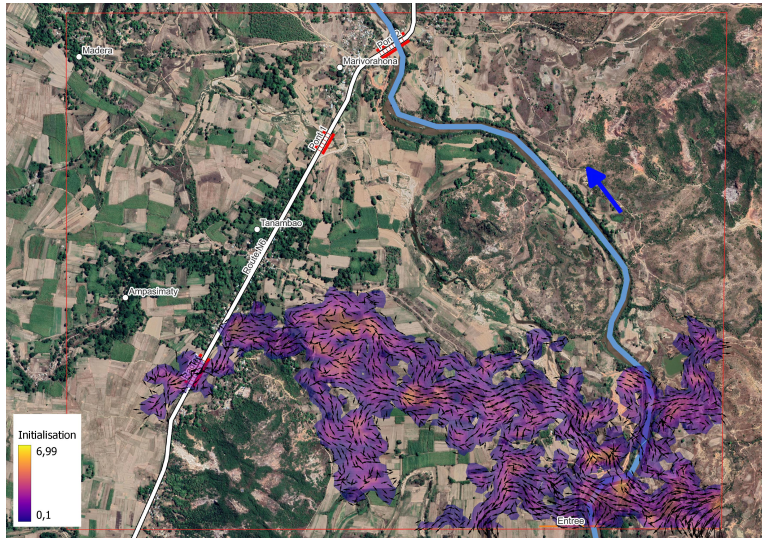
Mananjeba : elements of the area to be modeled



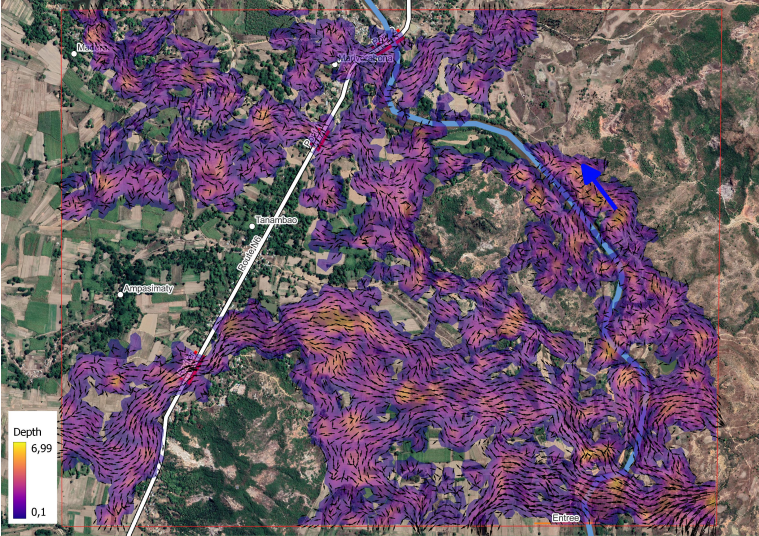
Mananjeba : at the beginning



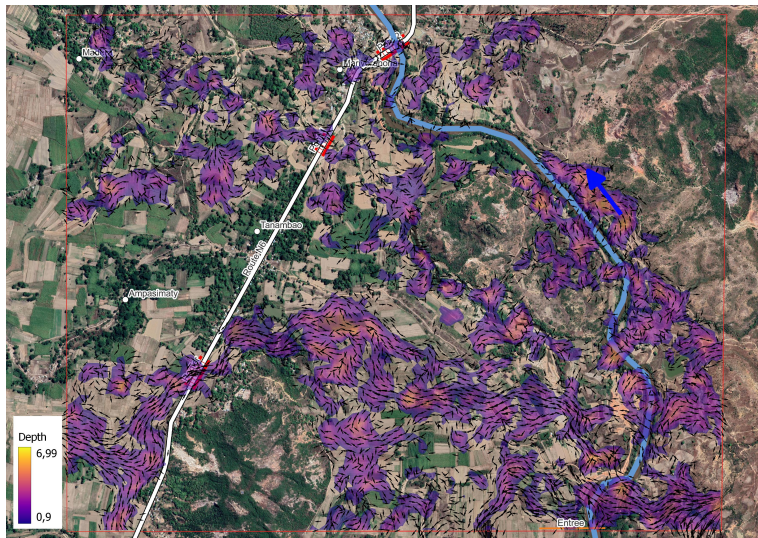
Mananjeba : $at = 10h$ (March 01, 2018)



Mananjeba : status on March 16, 2018 (peak hydrograph)



Mananjeba : status on March 25, 2018



Observation

The objective was to improve the existing model :

- Change the DTM
- Recalculate with the finite volumes method

The two methods produce similar results but :

- the numerical results are improved around the village of Marivorahona,
- the calculation is faster with the finite volume method,

General conclusions

On the global functioning

- The finite difference method tends to deviate from the physical reality of the flow
- The classical finite element method is not adapted either to the discontinuous character of the solutions or to the conservative character of the system
- The finite volume method is the most adapted to the modeling of flows in the natural environment

Sur les résultats numériques

- DTM quality is the most important factor for a flood model
- Reducing the time step has a minimal impact on the accuracy of the results

Recommendation and perspectives

Choice of the hydrodynamic modeling tool

It must :

- be open source (or at least free)
- meet the theoretical requirements
- simple to use
- be coupled with GIS tools

⇒ BASEMENT

Equipment used

- Use of equipment and new technologies for data acquisition
⇒ Drone for DTMs
- Need for significant computing capacity for large areas
⇒ Installation of a powerful computing server in the hydraulics laboratory

Thanks to all