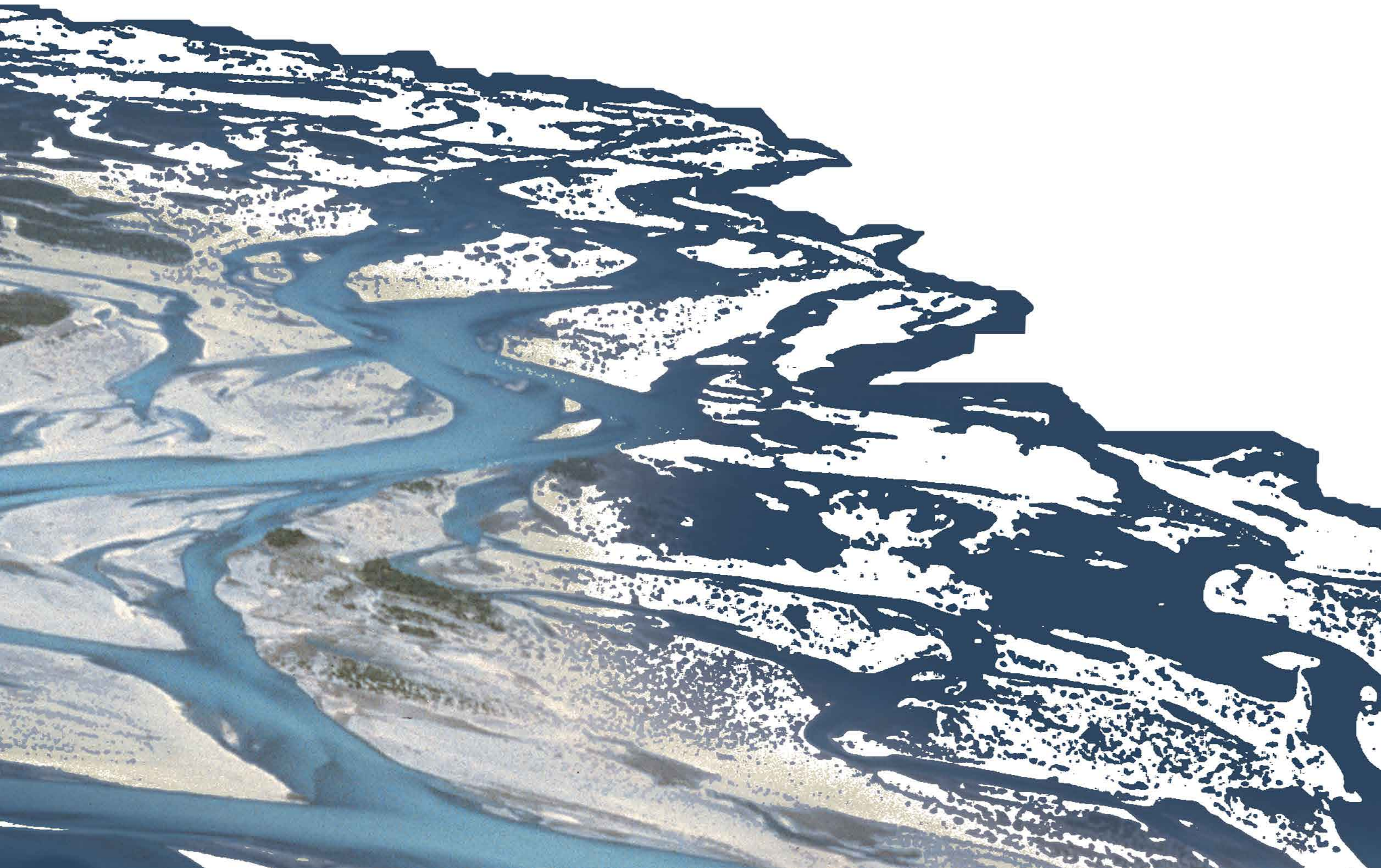
BASEMENT

BASIC SIMULATION ENVIRONMENT
FOR SIMULATION OF ENVIRONMENTAL FLOW
AND NATURAL HAZARD SIMULATION

SYSTEM MANUALS

VERSION 3.1
November 2020



Preamble

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For Website:

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BASIC SIMULATION ENVIRONMENT
FOR SIMULATION OF ENVIRONMENTAL FLOW
AND NATURAL HAZARD SIMULATION

REFERENCE MANUAL

VERSION 3.1
November 2020

Contents

1

Mathematical Models

1.1 Hydrodynamics

1.1.1 Introduction

Mathematical models of the so-called *shallow water* type govern a wide variety of physical phenomena. Especially the one-dimensional (1D) de Saint-Venant equations (SVE) or two-dimensional (2D) shallow water equations (SWE) are of practical interest with regard to water flows with a free surface under the influence of gravity. Applications of the models include e.g.:

- River hydrodynamics
- Propagation of flood waves
- Dam break waves
- Flooding and inundation
- Ecological assessment based on flow quantities

The 2D SWE are based on the following set of hypotheses:

- the water is assumed to be incompressible; i.e. the water density ρ is constant
- the vertical acceleration of the water particles are assumed to be small compared to the longitudinal component of the acceleration. As a consequence the pressure distribution is hydrostatic;
- the bottom slope is small enough for the longitudinal coordinate to coincide with the horizontal axis;
- the flow regime is turbulent. As a consequence the head loss, mainly due to friction against the bottom, is proportional to the square of the flow velocity.

1.1.2 Governing Equations

The governing equations are obtained under shallow water conditions imposing mass conservation for the fluid and solid phases and the momentum principle to a flow in an open channel with a fixed bottom.

Introducing a Cartesian reference system (x, y, z) in which the z axis is vertical and the $x - y$ plane is horizontal with respect to gravity g , the system of governing equations can be written as

$$\begin{cases} \frac{\partial h}{\partial t} + \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = S_h \\ \frac{\partial q_x}{\partial t} + \frac{\partial}{\partial x}\left(\frac{q_x^2}{h} + \frac{1}{2}gh^2\right) + \frac{\partial}{\partial y}\left(\frac{q_x q_y}{h}\right) + gh\left(S_{bx} + S_{fx}\right) = 0 \\ \frac{\partial q_y}{\partial t} + \frac{\partial}{\partial x}\left(\frac{q_y q_x}{h}\right) + \frac{\partial}{\partial y}\left(\frac{q_y^2}{h} + \frac{1}{2}gh^2\right) + gh(S_{by} + S_{fy}) = 0 \ , \end{cases} \tag{1.1}$$

where:

h	$[m]$	water depth
g	$[m/s^2]$	gravity acceleration
u (v)	$[m/s]$	depth averaged velocity in x (y) direction
q_x (q_y)	$[m^2/s]$	discharge per unit width in x (y) direction
S_h	$[m/s]$	lateral inflow/outflow discharge per unit width
S_{fx} (S_{fy})	$[-]$	friction terms in x (y) direction .

The bed slope source terms

$$S_{bx}, S_{by}$$

are evaluated as follows:

$$S_{bx} = -\frac{\partial z_B}{\partial x} \quad ; \quad S_{by} = -\frac{\partial z_B}{\partial y} \tag{1.2}$$

1.1.3 Closure Relations

In order to solve system (eq. 1.1) we need to specify the closure relations for the friction terms S_{fx}, S_{fy} and the value of lateral inflow/outflow discharge per unit width S_h.

1.1.3.1 Friction Terms

The governing equations (eq. 1.1) have been derived under the hypothesis of turbulent flow, hence the friction terms can be assumed proportional to the square of the depth-averaged velocity and can be written as:

Several formulae are available for S_f. All these formulae use hypothesis (H3) of a turbulent flow regime, hence the assumption that the slope of the energy line is proportional to the square of the flow velocity u. The most frequently used laws are

Adopting a quadratic friction law, the friction term is proportional to the square of the depth-averaged velocity and can be written as:

$$S_{fx} = \frac{u|\vec{u}|}{gc_f^2 h} \quad ; \quad S_{fy} = \frac{v|\vec{u}|}{gc_f^2 h} \tag{1.3}$$

where g is the gravity acceleration, u and v are the depth averaged velocities in x and y direction, $|\vec{u}| = \sqrt{u^2 + v^2}$ is the magnitude of the velocity vector and c_f is the dimensionless friction coefficient.

Several formulae are available for the dimensionless friction coefficient c_f. Here it is quantified using both a power or a logarithmic for which are described in the next sections.

1.1.3.1.1 Power Law

The Manning-Strickler power law is widely used in practice and it requires that either the Strickler's k_{str} $[m^{1/3}/s]$ or the Manning's n coefficients ($k_{str} = n^{-1}$) is specified.

In this case the dimensionless friction coefficient c_f is calculated as

$$c_f = \frac{k_{str} h^{1/6}}{\sqrt{g}} \tag{1.4}$$

1.1.3.1.2 Logarithmic Law

The following approaches are implemented to determine the friction coefficient c_f:

Chézy:

$$\begin{aligned} c_f &= 5.75 \log \left(12 \frac{R}{K_s}\right) && \text{for} \quad R > K_s \\ c_f &= 5.75 \log (12) && \text{for} \quad R < K_s \ , \end{aligned} \tag{1.5}$$

where K_s $[m]$ is the bed roughness height which is commonly taken to be proportional to a representative sediment size d_x. For rivers, K_s can be assumed $K_s = n_k d_{90}$ where $n_k = 2 \div 3$.

Bezzola:

In this closure relation, proposed by Bezzola (2002), c_f is given as a function of the roughness sublayer height y_R $[m]$ (usually for rivers $y_R \approx 1.0 d_{90}$ is a good approximation). This approach is also valid for small values of the relative submergence h/y_r Bezzola (2002).

$$\begin{cases} c_f = 2.5\sqrt{1 - \frac{y_R}{h}} \ \ln\left(10.9\frac{R}{y_R}\right), & \text{for} \quad \frac{h}{y_R} > 2 \\ c_f = 1.25\sqrt{\frac{h}{y_R}} \ \ln\left(10.9\frac{R}{y_R}\right), & \text{for} \quad 0.5 \le \frac{h}{y_R} \le 2 \\ c_f = 1.5, & \text{for} \quad \frac{h}{y_R} < 0.5 \end{cases} \tag{1.6}$$

1.1.3.2 Lateral Inflow/Outflow

S_h is used to represent additional sources of water like rainfall and springs or water abstraction (sink) and are allocated on a set of elements defined by regions. The external

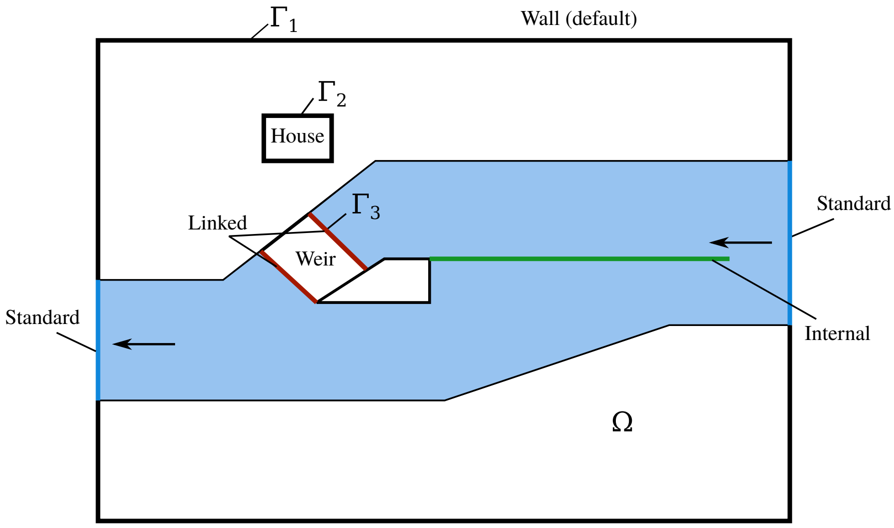

Figure 1.1 *Modeling domain and types of boundary conditions available. The flow is from right to left and a side weir (green line) divides the channel into a lower and an upper channel through the weir.* ***Standard*** *or* ***linked*** *boundary conditions must be provided at* Γ_1, Γ_2 *and* Γ_3 *while* ***internal*** *boundary conditions can be specified in any place within* Ω

source can be specified as total discharge $[m^3/s]$ or distributed over time $[mm/h]$. Different approaches are used to manage the behaviour of the external sources:

- Exact: The specified water volume is added or extracted (non conservative)
- Available: The specified water volume to extract is limited by the available water volume in the elements (conservative)
- Infinity: All available water will be abstracted (conservative)

Addition of water always follows the "Exact" behaviour as there is no upper limit. The abstraction of water could also follow the "Exact" behaviour but the simulation might end abruptly if the available water volume is smaller than the volume prescribed. Therefore, the "Available" behaviour aims to avoid this situation. The "Infinity" behaviour abstracts all available water volume.

1.1.4 Boundary Conditions

After the specification of the *closure relations* there are now three equations and three unknowns, namely h, q_x and q_y. In principle, given initial and boundary conditions, one should be able to solve system (eq. 1.1) for h, q_x and q_y as functions of space x, y, and time t. Given the modeling domain described in Figure 1.1, boundary conditions are required at the domain boundary Γ and optionally can be specified within the interior domain Ω.

Therefore, three different types of boundary conditions can be defined:

- Standard boundary conditions: located at the domain boundary Γ_i
- Linked boundary conditions: located at the domain boundary Γ_i or inside the domain Ω

- Internal boundary conditions: located inside the domain Ω

Standard boundaries (at Γ) represent the limits of the computational domain possibly including also buildings, weirs or structures for water intake (see Figure 1.1).

1.1.4.1 Standard Boundary Conditions

At the standard boundaries, two different types of boundary conditions can be specified: wall or flow boundaries. Flow boundary conditions allow the flow to enter or leave the domain while wall boundary conditions express no mass flux over the boundary. By default, the extneral boundaries of the domain are set as wall boundaries.

1.1.4.1.1 Wall Boundaries

The *Wall* or *reflective* boundary consider the boundary at Γ_i and suppose it physically consists of a fixed, reflective impermeable wall. Then the physical situation is modelled imposing that:

$$\rho\vec{u}\cdot\vec{n} = 0 \quad ; \quad \frac{\partial\vec{u}}{\partial\vec{n}} = 0 \tag{1.7}$$

Where $\vec{n}$ is the outward directed unit vector perpendicular to the wall and $\vec{u} = (u, v)^T$ is the velocity vector. The static pressure is assumed to be zero.

1.1.4.1.2 Flow Boundaries

The *Flow* boundary conditions are defined as *inflow* if they let water entering or as *outflow* if they let water leaving the domain. Flow boundaries are further distinguished into *Standard* and *Linked*. The former are applied on the boundary domain Γ, while the latter establish a *link* between two portions of the domain.

Standard

Inflow boundaries:

This boundary requires the specification of a value for the total volume discharge Q, $[m^3/s]$, which is then divided by the length of the boundary Γ and projected orthogonally to the boundary to obtain the values of q_x and q_y. In case of supercritical flow the following possibilities to specify the value of the water depth h are possible:

- *uniform_in*: h is calculated assuming that local uniform flow conditions. The calculation proceeds as follows:

  $$h = \sqrt[3]{\frac{(Q/b)^2}{gc_f^2 s}} \tag{1.8}$$

  where c_f is the Chézy coefficient, b is the entire length of the boundary Γ and s is the value of the local bed slope that must be specified.

- *froude_in*: In this case the flow depth h is calculated as follows:

$$h = \sqrt[3]{\frac{(Q/b)^2}{gFr^2}} \tag{1.9}$$

where b is the entire length of the boundary and Fr is the value of the local Froude number that must be specified

- *zhydrograph*: The water surface elevation (wse) at the boundary must be specified by the user. The depth is calculated as:

$$h = wse - z_B \tag{1.10}$$

where z_B is the bottom elevation at the boundary. The flow velocity at the boundary is set to zero.

Outflow boundaries:

At the outflow boundaries a value for the water depth h must be specified. These are the possible options:

- *uniform_out*: the water depth h is calculated using equation (eq. 1.8) specifying a value for the total discharge Q and a local bed slope s. Uniform flow is calculated based on given slope and cell state at boundary (eq. 1.8).

- *weir_out_constant* and *weir_out_dynamic*: These boundary conditions establishe a relation between the approaching discharge q constant and the water depth using the Poleni weir formula:

$$q = \frac{2}{3}\mu\sqrt{2g(h_{up} - w)^3} \tag{1.11}$$

where h_{up} is the water depth of the approaching flow and w is the weir elevation. The Poleni factor μ can be either set as constant ($\mu = 0.75$ by default) or dynamically evaluated as:

$$\mu = \frac{0.611}{a}\frac{0.75}{b}\frac{h_{up} - z_w}{w} \tag{1.12}$$

where a and b must be specified by the user in the case of *weir_out_dynamic* (default values are $a = 0.611$ and $b = 0.075$).

- *hqrelation_out*: The discharge is determined as a function of the water surface elevation, thus a stage-discharge-relation has to be specified.

- *zhydrograph*: Sets a fixed water surface elevation (wse) at the boundary. The wse [m] at the boundary must be specified by the user. The depth is calculated as:

$$h = wse - z_B \tag{1.13}$$

where z_B is the bottom elevation at the boundary. The flow velocity is calculated with the Riemann solver (HLLC).

- *zero_gradient_out* (scientific use only): Transmissive, or transparent boundaries allow the passage of waves without any effect on them. This is mathematically obtained imposing over the entire length of the boundary that:

$$\rho\vec{u}\cdot\vec{n} = \text{const} \quad ; \quad \frac{\partial \vec{u}}{\partial \vec{n}} = 0 \tag{1.14}$$

  In this case there is no need to specify further parameters.

  Note: This is boundary condition should ***not*** *be used for practical problems and is intended for scientific use only.*

1.1.4.2 Linked Boundary Conditions

This type of boundaries establish a *link* between within a certain region of the domain where equations are not solved. Once this domain portion is identified the two boundaries, between which the link is established, must be specified. Let us call them Γ_{in} and Γ_{out}. Then, one inflow boundary condition must be specified at Γ_{in} and one outflow boundary condition at Γ_{out} while in the remaining boundaries wall conditions are automatically assigned. Not necessarily, Γ_{in} and Γ_{out} must have the same number of elements.

Linked boundaries can describe a $h - Q$ relation or a weir, i.e.:

- *weir_linked_constant* and *weir_linked_dynamic*: Similar to the standard weir boundary, the weir height w has to be specified. No kinetic energy is considered.
- *hqrelation_linked*: The flux is calculated given a h-Q relation (see description of the h-Q relation for standard boundaries).
- *2way_hqrelation_linked*: The internal boundary works as dynamic wall that is controlled by water surface elevation thresholds. If the upper water surface elevation threshold is reached, the internal boundary is removed until the water level reaches the lower water surface elevation, where the wall is re-established.
- *zhydrograph_linked*: Sets a fixed water surface elevation (WSE) at the upstream boundary. The flux is caculated with the Riemann solver and used as inflow in the downstream boundary. Input of kinetic energy is neglected at the downstream boundary.
- *zhydrograph_linked_kinE*: Sets a fixed water surface elevation (WSE) at the upstream boundary. The flux is caculated with the Riemann solver and used as inflow in the downstream boundary. Input of kinetic energy is taken into account at the downstream boundary, using the flow depths of the cells adjacent to the boundary.

1.1.4.3 Internal Boundary Conditions

The internal boundary condition allows a direct cell-cell relation due to the exact same number of elements on the left and on right side of the boundary. Internal boundary conditions can be used to specify internal walls, dynamic walls or an h-Q relation.

- *wall_internal*: The wall conditions (eq. 1.7) are applied on both sides of the internal boundary.

- *dynamic_wall_internal*: The wall conditions are applied on the internal boundary until reaching a threshold value (time or water depth) after which the wall is removed.
- *hqrelation_internal*: A stage-discharge relation is applied on one side of the internal boundary, while on the other side, wall conditions apply (unidirectional flow).

1.1.5 Flood Tracking

The flood tracking aims at extracting the flood arrival time, the maximum water depth, flow velocity, specific discharge and bed shear stress along the numerical simulation and over a selected domain area. The flood tracking provides outputs within a tracking time step defined by the user.

1.2 Morphodynamics

1.2.1 Introduction

Morphodynamic models provide scientific frameworks for advancing our understanding of river systems. The research on involved topics is an important and socially relevant undertaking regarding our environment. Nowadays numerical models are used for different purposes, from answering questions about basic morphodynamic research to managing complex river engineering problems. Due to increasing computer power and the development of advanced numerical techniques, morphodynamic models are now more and more used to predict the bed patterns evolution to a broad spectrum of spatial and temporal scales. The development and the success of application of such models are based upon a wide range of disciplines from applied mathematics for the numerical solution of the equations to geomorphology for the physical interpretation of the results.

Applications of morphodynamic models include:

- Damming of river basins
- Morphological changes due to width changes (e.g. River widenings)
- Effects of sediment mining
- River straightening

1.2.2 Bedload Sediment Transport

1.2.2.1 Governing Equations for Uniform Sediment Transport

The governing equations are obtained under shallow water conditions imposing mass conservation for the fluid and solid phases and the momentum principle to a flow in an open channel with a cohesionless bottom.

Introducing a Cartesian reference system $(x; y; z)$ in which the z axis is vertical and the $x - y$ plane is horizontal, the system of governing equations is described by the system of equations (eq. 1.1) for hydrodynamics coupled with one equation for the conservation of the total sediment mass. The conservation of sediment mass is ensured by the Exner equation (eq. 1.15), named after the Austrian sedimentologist Felix M. Exner (Exner, 1925). The Exner equation allows to describe the bed evolution due to erosion or deposition, which results in the elevation change of the actual bed level z_B:

$$(1-p)\frac{\partial z_B}{\partial t} + \frac{\partial q_{B_x}}{\partial x} + \frac{\partial q_{B_y}}{\partial y} - Sl_b = 0 \tag{1.15}$$

where p is the porosity, Sl_b is the source term per unit width specifying local input or output of sediment material (e.g. slope collapse or excavation) and $\vec{q}_B = \begin{pmatrix} q_{B_x} \\ q_{B_y} \end{pmatrix}$ is the specific bedload flux. The Exner equation describes the bed evolution due to erosion or deposition processes, which results in changes of the bed level z_B.

The Exner equation is solved in a decoupled way, meaning that the shallow water equations and the Exner equation are solved in sequence. This approach makes the assumption that the bedload flux is much slower than the water flow velocity (Soares-Frazão and Zech, 2011).

1.2.2.2 Threshold Conditions for Sediment Movement

The key dimensionless parameter quantifying sediment mobility is the Shields parameter defined as:

$$\theta = \frac{|\vec{\tau_b}|}{(\rho_s - \rho)gd} \tag{1.16}$$

where $vec\tau_b$ is the bed shear stress (drag force acting on the particle), d is the sediment diameter, ρ and ρ_s are the water and sediment density, respectively. The Shields parameter can be interpreted as the ratio scaling the impelling force of flow drag acting on a particle to the Coulomb force resisting motion acting on the same particle.

The bed shear stress $\vec{\tau_b} = \begin{pmatrix} \tau_{bx} \\ \tau_{by} \end{pmatrix}$ is usually estimated by a closure condition using an empirical or semi-empirical formula. Here we use the quadratic friction law which relates the depth-averaged velocities to the bed shear stress as follows:

$$\tau_{bx} = \rho\frac{|\vec{u}|u}{c_f^2} \quad ; \quad \tau_{by} = \rho\frac{|\vec{u}|v}{c_f^2} \tag{1.17}$$

where $\vec{u} = \begin{pmatrix} u \\ v \end{pmatrix}$ is the flow velocity vector, ρ is the density of water and c_f is the dimensionless Chézy friction coefficient as defined in Section 1.1.3.1.

When a granular bed is subjected to a turbulent flow, it is found that virtually no motion of the grains is observed below a critical value (θ_{cr}) of the Shields parameter. According to the Shields' theory Shields (1936), θ_{cr} can be expressed as a function of the Reynolds number $Re^* = \frac{du_*}{\nu}$. Alternatively, the diagram of incipient motion (see Figure 1.2) can be plotted as a function of the dimensionless grain diameter D^* ($\theta_{cr} = f(D^*)$), where

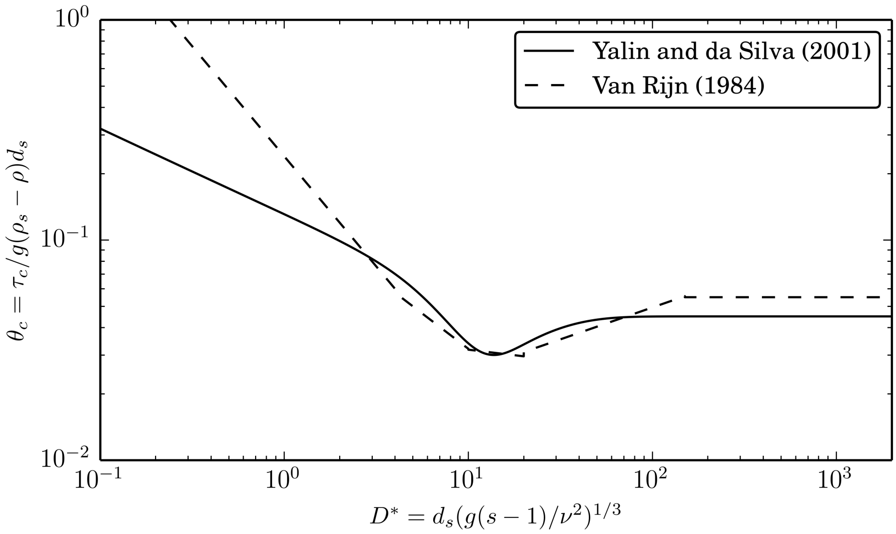

Figure 1.2 *Modified Shields diagram for initiation of sediment motion*

$$D^* = d\left[\frac{g(s-1)}{\nu^2}\right]^{1/3}$$

.

The curve representing the particle incipient motion ($\theta = \theta_{cr}$) can be divided into three parts in the log-log graph:

- for $D^* \leq 3$, can be approximated by a linear segment;
- for $3 \leq D^* \leq 100$ this is represented by a curve with a relative minimum;
- for \$D^* > 100 \$ by a constant trend.

An approximation of the original Shields diagram was proposed by van Rijn (1984):

$$\begin{array}{lll} \theta_{cr} = 0.24(D^*)^{-1} & \text{for} & 1 \leq D^* \leq 4 \\ \theta_{cr} = 0.14(D^*)^{-0.64} & \text{for} & 4 < D^* \leq 10 \\ \theta_{cr} = 0.04(D^*)^{-0.1} & \text{for} & 10 < D^* \leq 20 \\ \theta_{cr} = 0.013(D^*)^{0.29} & \text{for} & 20 < D^* \leq 150 \\ \theta_{cr} = 0.055 & \text{for} & D^* > 150 \end{array} \tag{1.18}$$

Another explicit formulation of the Shields curve was proposed by Yalin and Silva (2001). It reads

$$\theta_{cr} = 0.13 D^{*-0.392} \exp(-0.015 D^*) + 0.045\left(1 - \exp(-0.068 D^*)\right) \tag{1.19}$$

1.2.2.2.1 Influence of Local Slope on Incipient Motion

The threshold condition for incipient motion of grains developed by Shields is valid for almost horizontal bed. In case of sloped bed in flow direction or transverse to it, the stability of grains is either increased or reduced due to the gravity. The critical shear stress value can be adapted consequently to account for the influence of local slopes. One approach is to multiply the critical shear stress for almost horizontal bed θ_{cr} with the correction factors k_l and k_t for the local bed slope in the longitudinal and transversal flow direction. In the following, the critical Shields stress corrected for arbitrary bed slope δ is referred to as $\theta_{c,\delta}$, defined as:

$$\frac{\theta_{c,\delta}}{\theta_{cr}} = k_l k_t = k \tag{1.20}$$

The correction factor k_l and k_t are calculated as suggested by van Rijn (1989):

$$k_l = \cos\delta_l \left(1 - \frac{\tan\delta_l}{\tan\gamma}\right) \tag{1.21}$$

$$k_t = \cos\delta_t \sqrt{1 - \frac{\tan^2\delta_t}{\tan^2\gamma}} \tag{1.22}$$

where δ_l is the angle between the horizontal and the bed along flow direction, δ_t is the slope angle transversal to the flow direction and γ is the angle of repose of the sediment material.

Other formulations are also available, as for example the one proposed by Chen et al. (2010):

$$k = \frac{1}{\tan\gamma}\left(\cos^2\left(\frac{\pi}{2} - \delta_l\right) - 1 + \frac{1 + \tan^2\gamma}{(1 + \tan^2\delta_l + \tan^2\delta_t)}\right)^{0.5} + \cos\left(\frac{\pi}{2} - \delta_l\right) \tag{1.23}$$

1.2.2.3 Closure Relations for Bedload Transport

In order to solve system (eq. 1.1) and equation (eq. 1.15) we need to specify the closure relations. For the friction terms S_{fx}, S_{fy} and the value of lateral inflow/outflow discharge per unit width S_h we can use the relations already introduced in the Hydrodynamic part (Section 1.1.3). For the Exner equation we need relations quantifying the bedload discharge. Let us now introduce the dimensionless bedload transport rate Φ also known as the Einstein bedload number, first introduced by Hans Albert Einstein in 1950, and given by

$$\Phi = \frac{q_B}{\sqrt{(s-1)gd^3}} \tag{1.24}$$

where $s = \rho_s/\rho$.

It is common practice to quantify bedload transport empirically relating Φ with either the Shields stress θ or the excess of the Shields stress θ above some appropriately defined "critical" Shields stress $(\theta - \theta_{cr})$. The critical Shields stress θ_{cr} is defined so as to fit experimental or field data and provide a threshold for which the bedload transport rate is too low to be of interest. The Shields parameter, takes the following form

$$\theta = \frac{h\sqrt{S_{fx}^2 + S_{fy}^2}}{(s-1)d} \tag{1.25}$$

where h is the water depth, S_{fx} and S_{fy} the friction slope in x- and y-direction respectively, $s = \rho_s/\rho_w$, and d is the grain size diameter. Note that Eq. 1.16 and Eq. 1.25 are equivalent.

In what follows, we describe the bedload transport formulas that are implemented to calculate the transport capacity $q_B = |\vec{q_B}|$, where the specific bedload flux vector $\vec{q_B} = (q_{B_x}, q_{B_y})$ generally has the same direction as the water flow.

For practical purposes, the bedload transport formula can be calibrated by an additional pre-factor (*factor*). The bedload transport capacity is obtained from the closure relation scaled by this pre-factor.

1.2.2.3.1 Meyer-Peter and Müller (1948)

The bedload transport formula of Meyer-Peter and Müller (Meyer-Peter and Müller, 1948) defines the specific bedload transport rate q_B as:

$$q_B = \alpha(\theta - \theta_{cr})^m \sqrt{(s-1)gd^3} \tag{1.26}$$

Herein, α denotes the bedload coefficient (originally $\alpha = 8$), m the bedload exponent (originally $m = 1.5$), θ is the dimensionless bed shear stress (Shields parameter), θ_{cr} is the critical dimensionless bed shear stress, d is the grain diameter, $s = \rho_s/\rho$ and g stands for the gravitational acceleration. Meyer-Peter and Müller observed in their experiments that the first grains moved already for $\theta_{cr} = 0.03$. But as their experiments took place with steady conditions they used a value for which already 50% of the grains where moving. They proposed the value of $\theta_{cr} = 0.047$. The formula of Meyer-Peter and Müller is applicable in particular for coarse sand and gravel with grain diameters larger than 1 mm (Malcherek, 2001).

The bedload coefficient α, the exponent m and the critical Shields parameter θ_{cr} can be adapted by the user in the MPM-like formula.

1.2.2.3.2 Grass Formula

The Grass formula (Grass, 1981) proposes a simple bedload transport formula, where q_b is a function of the flow velocity u and a dimensional constant α and does not require the evaluation of the Shields stress:

$$q_B = \alpha(u - u_c)^m \cdot \sqrt{(s-1)gd^3} \tag{1.27}$$

where $\alpha \in [0, 1]$ is a dimensional constant that encompasses the effects of grain size and kinematic viscosity and is usually determined from experimental data, u_c is the critical velocity and the exponent m is usually set to $m = 3$. The threshold condition for incipient motion of grains is typically set to zero, meaning that the bedload transport and the fluid motion start simultaneously. The coefficient α characterizes the interaction between the bed and the fluid. If $\alpha = 0$, no sediment transport occurs. If $\alpha = 1$ the interaction between the bed and fluid is the largest.

1.2.2.3.3 Engelund and Hansen (1972)

Engelund and Hansen (1972) proposed a transport formula for uniform bed material taking into account at the same time the presence of both bed- and suspended-load. This formula is commonly used as a bulk load formula and reads

$$q_B = 0.05\sqrt{(s-1)gd^3} \cdot c_f^2 \theta^{2.5} \tag{1.28}$$

where d denotes the median sediment size of the bed material, c_f the dimensionless Chézy friction coefficient and θ is the dimensionless bed shear stress (see eq. 1.25). The Engelund and Hansen formula for bedload transport does not consider the critical shear stress as threshold condition for incipient motion.

1.2.2.3.4 Smart & Jäggi (1983)

Smart and Jaeggi (1983) developed a bedload transport formula for steep channels using their own experimental results and the results of Meyer-Peter and Müller (Meyer-Peter and Müller, 1948). The specific bedload transport rate q_B is defined as:

$$q_B = \frac{\alpha}{(s-1)} \left(\frac{d_{90}}{d_{30}}\right)^{0.2} J^{0.6} |\vec{q}| (J - J_{cr}) \tag{1.29}$$

where s is the sediment density coefficient ($s = \rho_s/\rho$), $|\vec{q}|$ is the magnitude of the specific discharge and d_{30} and d_{90} are the characteristic grain size diameters, i.e. 30 % resp. 90 % (by weight) of the bed material are smaller. The energy slope J and the critical slope for the initiation of the bedload transport J_{cr} calculated as

$$J = \frac{\theta(s-1)d_m}{h} \tag{1.30}$$

$$J_{cr} = \frac{\theta_{cr}(s-1)d_m}{h} \tag{1.31}$$

where θ is the dimensionless bed shear stress (see eq. 1.25), θ_{cr} the critical dimensionless bed shear stress, d_m the mean grain size diameter and h the water depth. Smart and Jaeggi (1983) recommend values of $\alpha = 4$ and $\theta_{cr} = 0.05$. The scope of applictation is for bed slopes $0.005 \leq$ J ≤ 0.2 (Smart and Jaeggi, 1983).

1.2.2.4 Correction of Bedload Direction

The 2D projection of the solid discharge along x and y is obtained through standard procedures, that are mostly based on empirical basis and which account for the downward effect of gravity on sediment particles due to local bed slope and the presence of spiral flow motion in curved reaches.

1.2.2.4.1 Lateral Bed Slope Effect

Empirical bedload formulas were originally derived for situations where bed slope equals flow direction. However, in case of lateral bed slope with respect to flow direction, the bedload direction differs from the flow direction due to gravity acting on the bed material.

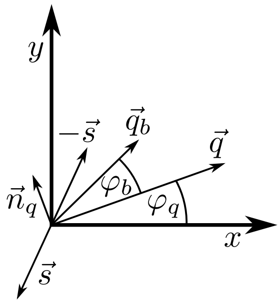

Figure 1.3 *Bed load transport deviation angle φ_b from the flow direction $\vec{q}$ due to the lateral bed slope $\vec{s}$ (Vonwiller, 2017)*

Figure 1.3 illustrates the deviation of the bedload transport direction due to lateral bed slope in a Cartesian coordinate system.

The bedload direction is corrected for lateral bed slope based on the following approach (e.g. see Ikeda (1982) and Talmon et al. (1995)):

$$\tan\varphi_b = -f(\theta)\cdot\ \vec{s}\cdot\ \vec{n}_q \quad \text{for} \quad \vec{s}\cdot\vec{n}_q < 0 \tag{1.32}$$

$$f(\theta) = N_l\left(\frac{\theta_{cr}}{\theta}\right)^{M_l} \tag{1.33}$$

where φ_b = bedload direction with respect to the flow vector $\vec{q}$, N_l = lateral transport factor ($0.75 \leq N_l \leq 2.63$), M_l = lateral transport exponent (typically $M_l = 0.5$), $\vec{s} = \left(\frac{\partial z_B}{\partial x}, \frac{\partial z_B}{\partial y}\right)$ bed slope (positive uphill, negative downhill), $\vec{n}_q$ = unit vector perpendicular to $\vec{q}$ pointing in downhill direction ($\vec{s}\cdot\vec{n}_q < 0$) , θ = effective dimensionless shear stress and θ_{cr} = critical dimensionless shear stress of sediment.

The direction of the bedload transport under the influence of lateral bed slope is written as:

$$\frac{q_{B_y}}{q_{B_x}} = \tan(\varphi_b + \varphi_q) \tag{1.34}$$

1.2.2.4.2 Curvature Effect

Curvature in rivers may cause deviation of the bedload direction from the depth averaged flow direction. Due to three dimensional spiral flow motion, the bedload direction tends to point towards the inner side of the curve, while the flow direction points towards the outer side (Figure 1.4). This curvature effect is taken into account according to an approach proposed by Engelund (1974) , where the deviation angle φ_c of the bottom shear stress $\vec{\tau_b}$ (positive counterclockwise and vice versa) from the main flow direction is determined as

$$\tan\varphi_c = \frac{|\vec{\tau}_{bn}|}{|\vec{\tau}_{bs}|} = -N_*\frac{h}{R} \tag{1.35}$$

where $\vec{\tau}_{bn}$ and $\vec{\tau}_{bs}$ are the bed shear stress normal to and in the flow direction respectively, h denotes the water depth, N_* is a curvature factor, and R denotes the radius of the river

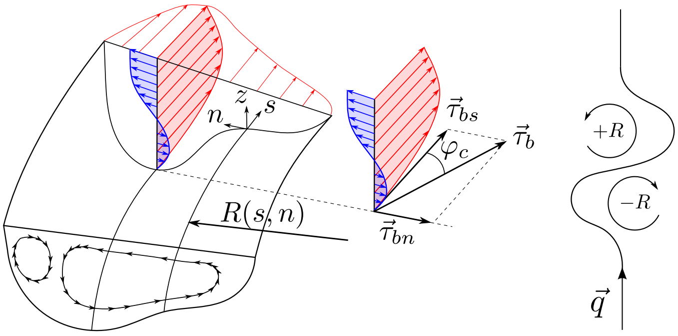

Figure 1.4 *Effect of spiral motion in river bend on bed shear stress $\vec{\tau}_b$ with deviation angle from main flow direction φ_c (Vonwiller, 2017)*

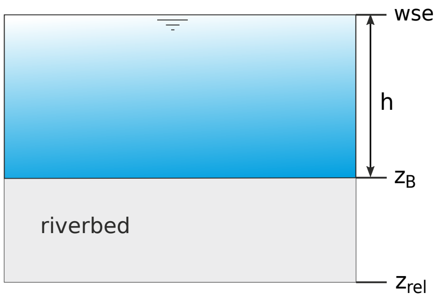

Figure 1.5 *Fixed bed concept and definition*

bend (positive for curvature in counterclockwise direction and vice versa).

Note that the curvature factor N_* mainly depends on bed roughness. Therefore, $N_* \approx 7$ for natural streams (Engelund, 1974), and values up $N_* \approx 11$ for laboratory channels (Rozovskii, 1961).

1.2.2.5 Fixed Bed Concept

Morphodynamic simulations generate deposition and erosion patterns of the riverbed. Erosion processes, if not limited, can proceed indefinitely in the vertical direction. This limit can be imposed by defining a non-erodible fixed bed elevation z_{rel}, below which the river bed is considered as *fixed*. This threshold also determines the amount of sediment available for transport (see Figure 1.5). The fixed bed elevation can either be assigned via regions or via a separate .2dm mesh file. When the fixed bed elevation is specified via regions, the fixed bed elevation must be provided relative to the initial bottom elevation z_B with $z_{rel} \leq 0$. When the fixed bed elevation is specified via a mesh, the elevation in the separate .2dm mesh file must correspond to the absolute fixed bed elevation z_{fix} [m]. Moreover, the fixed bed mesh file must have the exact same topology as the original computational mesh. In case the specified mesh only has elevation information on the nodes, the elevation is interpolated with the same method as specified in the INTERPOLATION block (default: mean). If the fixed bed elevation of the fixed bed mesh exceeds the bottom elevation of the computational mesh, the fixed bed elevation is defined at the elevation of the computational mesh.

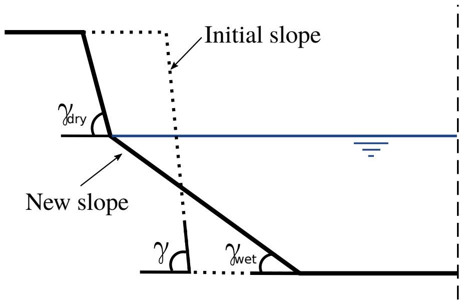

Figure 1.6 *Critical failure angles for slope collapse*

The accuracy of the fixed bed correction is guaranteed by defining the maximal overshoot below the fix bed elevation and the maximal number of iterations required for the correction.

1.2.2.6 Gravitational Transport

Gravitational induced riverbank or sidewall failures are significant aspects concerning erosion and transport modelling. Such processes may play an important role in many situations, such as meandering streams, river widenings or failures of erodible embankment structures due to overtopping waters. Such slope failure processes take place mostly discontinuous and can deliver significant contributions to the total sum of transported material. The modes of slope failures can differ largely (falls, topples, slides, etc.) and depend on the soil material, the degree of soil compaction and the pore pressures within the soil matrix. Here, a simplified, geometric approach is applied to be able to consider some aspects of this purely gravitational induced transport. The main idea of the implemented geometrical approach is to assume that a slope failure takes place if the local bed slope γ becomes steeper than a critical slope γ_{cr} (Figure 1.6).

$$q_{B,grav} = \begin{cases} 0 & \text{if}\,(\gamma \leq \gamma_{cr}) \\ f(\gamma, \gamma_{cr}) & \text{if}\,(\gamma > \gamma_{cr}) \end{cases} \tag{1.36}$$

The sliding material is moved from the sediment element with higher elevation to the lower situated element until the stable condition: $\gamma \leq \gamma_{cr}$ is reached. Two characteristic critical slope angles are defined in this approach to have some flexibility in modelling the complex geotechnical aspects. The critical angles can be characterized as:

- critical angle for dry or partially saturated bank material γ_{dry}, which may greatly exceed the material's angle of repose (up to nearly vertical walls) due to negative pore pressures,
- critical angle for fully saturated and over flown material γ_{wet} which is in the range of the material's angle of repose

1.2.2.6.1 Calculation Proceedure

The flux due to gravitational transport $q_{B,grav}$ is calculated with the following proceedure, by looping over each element of the computational grid:

1. In a first steep, the local bed slope γ_i is calculated with respect to each neighbouring element i, where z is the bed elevation of the main element, z_i is the bed elevation of the neighbour element i and d_i is the distance between the element centers.

$$\gamma_i = arctan\left(\frac{z - z_i}{d_i}\right) \tag{1.37}$$

2. For local bed slopes γ_i exceeding the critical slope γ_{cr}, the new bed elevations z_{new} and $z_{i,new}$ are determined such that the stable condition $\gamma_i = \gamma_{cr}$ is reached for all neighbouring cells i. The critical angle γ_{cr} is selected according to eq. 1.38, where h is the water depth in the main element and h_{min} is the user-specified minimum water depth.

$$\gamma_{cr} = \begin{cases} \gamma_{wet} & \text{if}\, h \geq h_{min} \\ \gamma_{dry} & \text{if}\, h < h_{min} \end{cases} \tag{1.38}$$

3. If the calculated bed elevation change of the main element $\delta_z = z - z_{new}$ is smaller than the user-specified parameter *min_bed_change* (default: 0.001 m) $\delta_{z,min}$, no gravitational transport occurs to avoid oscillatory behaviour and to reduce the computational effort. If the minimum bed elevation change $\delta_{z,min}$ is exceeded, the specific gravitational flux $q_{B,grav,i}$ [m^2/s] to each neighbour element i is calculated from the bed elevation change in element i according to eq. 1.39, where A_i is the area of element i, and l_i is the length of the edge connecting the main element to its i^{th} neighbour element. If the calculated bed elevation change of the main element $\delta_z = z - z_{new}$ is larger than the user-defined maximum bed elevation change $\delta_{z,max}$, the gravitational flux is limited, such that $\delta_z = \delta_{z,max}$. The maximum bed elevation change $\delta_{z,max}$ is calculated by eq. 1.40, where $r_{b,max}$ is the user-specified parameter *max_bed_change_rate* and Δt is the current update time step.

$$q_{B,grav,i} = \begin{cases} 0 & \text{if}\, \delta_z < \delta_{z,min} \\ \frac{(z_{i,new} - z_i) \cdot A_i}{l_i} & \text{if}\, \delta_z \geq \delta_{z,min} \\ \frac{(z_{i,new} - z_i) \cdot A_i}{l_i} \cdot \frac{\delta_{z,max}}{\delta_z} & \text{if}\, \delta_z \geq \delta_{z,max} \end{cases} \tag{1.39}$$

$$\delta_{z,max} = r_{b,max} \cdot \Delta t \tag{1.40}$$

4. If a non-erodbile fixed bed elevation z_{fix} is specified, the gravitational transport flux is corrected the same way as the bedload transport flux (see Section 1.2.2.5).

5. Finally, the balancing of the gravitational fluxes and the determination of the new bed elevations z is achieved by solving the Exner equation using the same numerical approaches as outlined for the bed load transport. This procedure ensures that fixed bed elevations are taken into account and the mass continuity is fulfilled.

1.2.2.6.2 Time Scale of the Gravitational Transport Process

Since the sediment movement due to gravitational transport during one update time step is limited to adjacent elements, it may take many update time steps to reach a stable condition on a larger scale, e.g. a bank slope spanning over multiple elements. The time scale until a globally stable condition is reached, is influenced by the update time step (update_time), the maximum bed change rate (max_bed_change_rate) and the grid resolution. The parameter *update_time* (default: 0.0 s) determines at which frequency the gravitational transport proceedure (steps 1-5 above) is executed. Generally, a smaller update time step reduces the time scale until a globally stable condition is reached. The default behaviour is to set the update time step value to 0.0 seconds, which results in the gravitational transport proceedure being executed at the same time step as determined from the hydraulic CFL-criterium (see Section 2.3.4).

Furthermore, the speed of the gravitational transport process can be limited with the parameter *max_bed_change_rate* (default: 1.0 m/s). This parameter represents a maximum rate at which the bed elevation of a cell can be lowered due to gravitational transport and determines the maximum bed elevation change during one update time step. Generally, a smaller value may increase the time scale until a globally stable condition is reached.

Since sediment movement due to gravitational transport is limited to adjacent elements, also the grid resolution effects the time scale until a globally stable condition is reached. A finer grid resolution (smaller elements) increases the necessary number of update cycles the reach a stable slope over a specific length. However, a finer grid resolution may also decrease the hydraulic time step and therefore may the increase frequency at which the gravitational transport procedure is executed.

1.2.2.6.3 Bed slope at Stable Condition

The implemented approach for modelling gravitational transport processes ensures that the local bed slope does not exceed the user-specified critical angle. Due to the spatial discretization with constant bed elevations within an element, the local bed slope is calculated between two element centers. On a larger scale (e.g. a bank slope spanning over multiple elements), the bed slope may deviate from the user-specified critical angle after reaching stable conditions. The stable bed slope over multiple elements, generally exceeds the critical bed slope.

To illustrate this effect, a simple test case was simulated. The rectangular computational domain is initially split into two parts: the left side with a bed elevation of 2 meters and the right side with a bed elevation of 0 meters. The domain is completely dry and the critical angle for dry material is set to $\gamma_{dry} = 35°$. The simulation only considers only gravitational tranport and is stopped after a stable condition is reached. The initial and final bed elevation profile along the centerline are illustrated in Figure 1.7. The initial profile exhibits the nearly vertical bed step. The final profile at stable conditions exhibits a slope of approximately 41°, which exceeds the critical angle of $\gamma_{dry} = 35°$. The reason for this deviation on a larger scale is that the distance which is considered for the calculation of the slope is not a straight line, but a line connecting the element centers. This is illustrated in Figure Figure 1.8. Calculating the large scale slope considering the length of the green line results in a slope of approximately 41°, while considering the length of the red line results in a slope of approx. 35°.

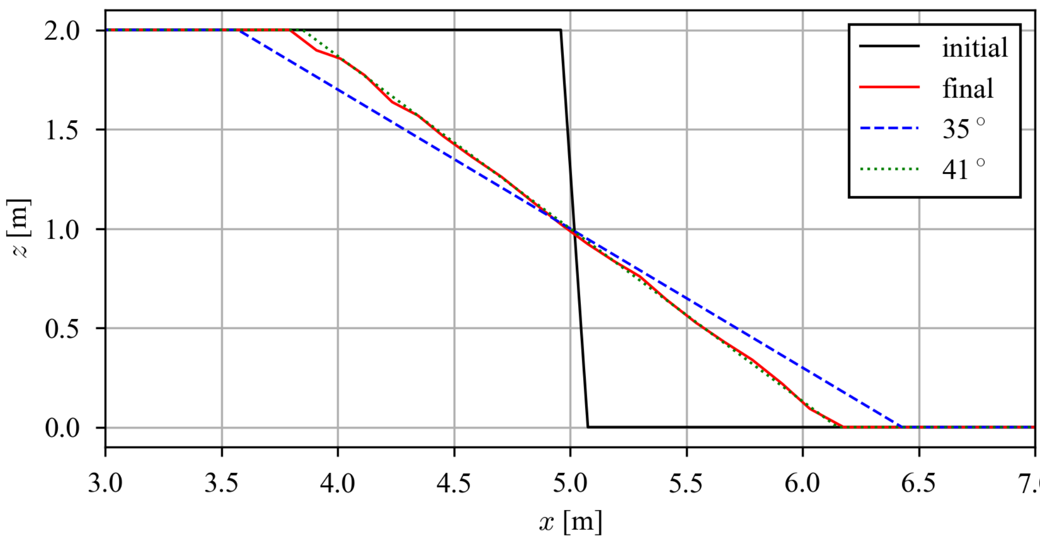

Figure 1.7 *Initial and final bed elevation profiles along the centerline are compared to profiles with angles of 35° and 41°. The slope of the bed profile at stable conditions (final) exceeds the critical angle of* γ_{dry} *= 35°.*

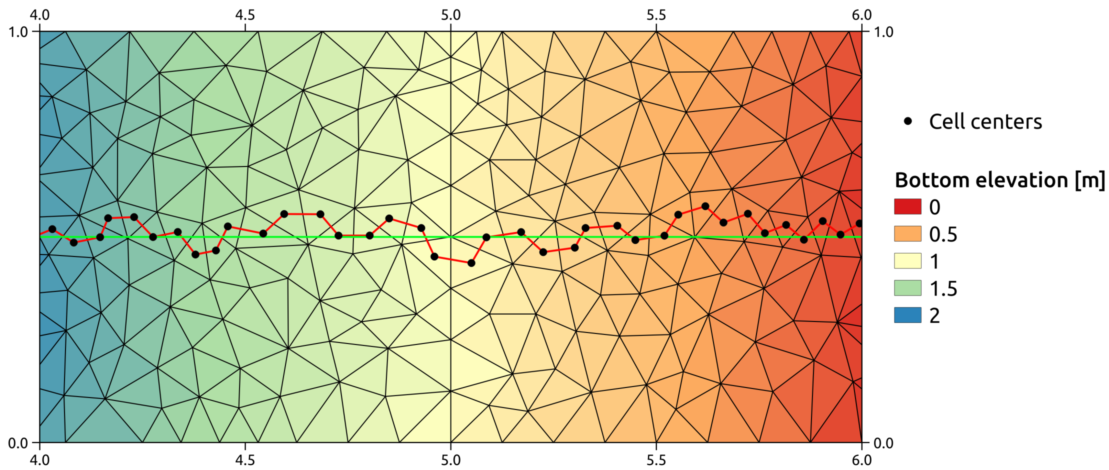

Figure 1.8 *The red line connects the element centers and represents the distance, which is taken into account for the* ***local slope*** *calculation and thus for the gravitational transport. The green line represents the direct distance, which is taken into account for the* ***large scale slope****. The slope along the red line corresponds to 35°, while the slope along the green line corresponds to 41°.*

1.2.2.7 External Sources Terms

The source term Sl_b represents additional sediment mass input or output (sink) that occurs locally on the computational domain on a set of elements defined by regions. The source can be specified as total volume flux including porosity $[m^3/s]$. Different approaches are used to manage the behaviour of the external sources in case of a negative flux (sink):

- Exact: The specified sediment volume is added or extracted (non conservative)
- Available: The specified sediment volume to extract is limited by the defined fixed bed elevation of the elements (conservative)
- Infinity: All available sediment will be abstracted (conservative)

Addition of sediment always follows the "Exact" behaviour as there is no upper limit. The abstraction of sediment could also follow the "Exact" behaviour but the simulation might end abruptly if the available sediment volume is smaller than the volume abstracted. Therefore, the "Available" behaviour aims to avoid this situation. The "Infinity" behaviour abstracts all available sediment volume.

1.2.2.8 Boundary Conditions

After the specification of the *closure relations* for the sediment transport, the system of governing equations (eq. 1.1) and (eq. 1.15) can be solved within the modeling domain described in Figure 1.1, provided boundary conditions (morphologic boundary conditions) are specified at the domain boundary Γ. For the sediment transport only *external boundaries* that allow sediment flowing into or out of the domain can be specified. A morphologic boundary condition can be co-located with a hydraulic boundary condition. In case no hydraulic boundary condition is specified, the boundary will behave as a wall and sediment transport will not occur.

1.2.2.9 Upstream Boundary Condition

- *equilibrium_in*: After erosion or deposition up to a user specified reference bed elevation (*reference_bed_elevation*) this upstream boundary condition grants a equilibrium condition, i.e. the same amount of sediment leaving the first computational cell in flow direction enters the cell from the upstream boundary. This leads to a constant bed elevation at the boundary condition.
- *sedimentograph*: based on a sediment hydrograph describing the bedload inflow as function of time (constant or variable). The bedload is defined as a volumetric flow rate $Q_b = \dfrac{\mu_s}{\rho_s}$ $[m^3/s]$, where μ_s is the sediment mass flow rate $[kg/s]$ and ρ_s the sediment density $[kg/m^3]$. Notice that the porosity is not considered in the bedload input and is specified separately as own parameter value. The volumetric flow rate is either distributed using a geometrical weighting (*sedimentograph*), using wetted area weighting (*sedimentograph_warea*) or using wetted conveyance weighting (*sedimentograph_conveyance*). **Note:** When using the pre-factor (*factor*) described in Section 1.2.2.3, it is automatically applied to the volumetric flow rate at the boundary for these types of boundary conditions, i.e. the sediment hydrograph is scaled by the pre-factor.

- *transport_capacity*: the sediment inflow is defined by calculating the equilibrium transport capacity according to the hydraulic state at the boundary. The bedload is defined as a compact volumetric flow rate (without porosity) Q_b $[m^3/s]$. The volumetric flow rate is either distributed using a geometrical weighting (*transport_capacity*), using wetted wetted area weighting (*transport_capacity_warea*) or using wetted conveyance weighting (*transport_capacity_conveyance*). **Note:** When using the pre-factor (*factor*) described in Section 1.2.2.3, the pre-factor is implicitly included in the volumetric flow rate at the boundary for this type of boundary condition, i.e. the transport capacity at the boundary is scaled by the pre-factor. Additionally, an independent scaling factor can be specified (*boundary_factor*), only applying to these types of the boundaries.

For the sediment discharge and transport capacity boundary condition types, the specific sediment discharge q_b is distinguished by three weighting schemes:

1. Geometrical weighting with respect to the total nodestring length L_n.

$$q_b = \frac{Q_b}{L_n} \qquad \left[\frac{m^3}{s \cdot m}\right] \tag{1.41}$$

2. Wetted area weighting

$$q_b = \frac{Q_b}{A_{w,tot}} \cdot h \qquad \left[\frac{m^3}{s \cdot m^2}\right] \tag{1.42}$$

3. Conveyance weighting

$$q_b = \frac{Q_b}{K_{tot}} h\sqrt{c_f h} \qquad \left[\frac{m^3}{s \cdot m}\right] \tag{1.43}$$

with $K_{tot} = A_{w,tot}\sqrt{c_f h}$ the total conveyance and c_f the friction coefficient.

1.2.2.10 Downstream Boundary Condition

One downstream boundary condition is available:

- *equilibrium_out*: After erosion or deposition up to a user specified reference bed elevation (*reference_bed_elevation*) this downstream boundary condition grants a equilibrium condition, i.e. all sediment entering the last computational cell will leave the cell over the downstream boundary. This leads to a constant bed elevation at the boundary condition.

1.2.2.11 Linked Boundary Condition

One linked boundary condition is available:

- *equilibrium_linked*: At the upsteam boundary, erosion or deposition is possible up to a user specified reference bed elevation (*reference_bed_elevation*). After reaching the reference elevation, this boundary condition grants a equilibrium condition, i.e. all sediment leaving the computational cells on the upstream side is entering at the downstream boundary with a lag of one timestep. This leads to a constant bed elevation at the upstream boundary.

1.3 Passive tracers

1.3.1 Introduction

A multitude of dissolved species are present in environmental flows. In the context of hydraulic and environmental engineering, numerical modelling of scalar transport becomes a relevant tool mostly because of its versatility. In terms of advective phenomena, the most common applications include

- Pollutant fate and transport
- Accumulation or depletion of nutrients
- Calculation of water residence times
- Flow visualization

1.3.2 Transport of passive species

1.3.2.1 Governing Equations for passive specie transport

The governing equations are obtained under the shallow water framework and impose mass conservation for both fluid and dissolved phases.

In a Cartesian frame of reference $(x; y; z)$ in which the z axis is vertical and the horizontal lies in the $x - y$ plane, the system of governing equations is formed by (eq. 1.1) for hydrodynamics and is coupled with multiple equations for the conservation of the total tracer masses. The conservation of each tracer mass is ensured by the scalar continuity equation (eq. 1.44), which is tightly coupled to the shallow water equations. This equation allows to describe the evolution of the specie concentration as:

$$\frac{\partial q_S}{\partial t} + \frac{\partial}{\partial x}\left(\frac{q_x q_S}{h}\right) + \frac{\partial}{\partial y}\left(\frac{q_y q_S}{h}\right) - S_s = 0 \tag{1.44}$$

where $q_S = h\phi_s$ is the volume of specie s per unit width, with ϕ_s being it's volumetric concentration, and S_s is the source term per unit width, specifying local input or output of the specie s.

1.3.2.2 External Sources Terms

The source term S_s conveys an input or output (sink) that occurs locally on the computational domain over elements limited by regions. It can be specified as total volumetric flux $[m^3/s]$. Different approaches define the behaviour of external sources if it imposes negative fluxes (sink):

- Exact: The specified volume is added or extracted (non conservative)
- Available: The specified volume to extract is limited by the available tracer volume (conservative)
- Infinity: All available tracer volume will be extracted (conservative)

Additionally, there is the option of forcing a target concentration ϕ_s^f homogeneously across all cells defined by the region. In such case the value of q_s is set directly as $h\phi_s^f$ (non conservative).

1.3.2.3 Boundary Conditions

The system of governing equations (eq. 1.1) and (eq. 1.44) can be solved within the modeling domain described in Figure 1.1, provided that boundary conditions (hydrodynamical and tracer boundary conditions) are specified at the domain boundary Γ. For the tracer transport only *external boundaries* that allow tracer flowing into or out of the domain can be specified. A tracer boundary condition should be co-located with a hydraulic boundary condition. Otherwise, the boundary will behave as a wall and tracer transport will not occur.

1.3.2.4 Upstream Boundary Condition

Two upstream boundary conditions are available:

- *discharge_in* and *discharge_in_warea*: based on a tracer discharge inflow, either as a constant or a function of time, the prescribed volumetric flow rate Q_s $[m^3/s]$ is imposed at the boundary. The volumetric flow rate is either distributed using a geometrical weighting or a wet-area weighting, and the specific tracer discharge q_s is thus given by:

1. Geometrical weighting with respect to the total nodestring length L_n.

$$q_s = \frac{Q_s}{L_n} \qquad \left[\frac{m^3}{s \cdot m}\right]$$

2. Wetted-area weighting

$$q_s = \frac{Q_s}{A_{w,tot}} \cdot h \qquad \left[\frac{m^3}{s \cdot m}\right]$$

- *concentration_in*: the tracer inflow is defined by forcing a target tracer concentration ϕ_s^f. The volumetric flow rate is thus given by the target concentration paired with the total hydrodynamic mass flux q through that boundary as $q_s = q\phi_s^f$.

Note: If the hydrodynamical mass flow at the boundary is not inward directed then no tracer flux is imposed.

1.3.2.5 Downstream Boundary Condition

One downstream boundary condition is available:

- *zero_gradient_out*: this downstream boundary allows the free outflow of any tracer quantities in the flow. **Note:** If this condition is not prescribed, a wall condition is assumed and the tracer quantities will be retained (no outflow).

2

Numerical Models

2.1 General View

The governing equations of hydro- and morphodynamics are conservation laws expressing conservation of mass and momentum. The aim of the numerical simulation is to solve these equations over the computational domain and for a given time. The computational domain is discretized by a computational mesh (Figure 2.1) consisting of elements (often having triangular shape) and conservation equations are applied on each domain element. In order to numerically solve the conservation equations, the mathematical model is approximated by numerical schemes, i.e. the numerical approximation consists of the spatial and temporal discretization of the conservation equations including an algorithm that solves the discretized equations.

The conservation equations can be formulated either in integral or differential form. Different numerical schemes exist to discretise the equations:

- Finite difference: The discrete values are considered as point values defined at mesh points
- Finite element: The discrete values are determined in terms of the nodal values of the mesh
- Finite volume: The discrete values are averaged over finites volumes of the mesh

In BASEMENT, the spatial discretisation of the domain is based on an unstructured mesh made of triangular elements. For the conservation equations, the spatial discretisation follows the finite volume scheme, while for the temporal discretisation an explicit first order Euler scheme is used. The numerical model processes the hydro- and morphodynamic equations in a decoupled way (Figure 2.2).

The discretization and the solution method for the hydro- and morphodynamic equations will be presented in the following sections.

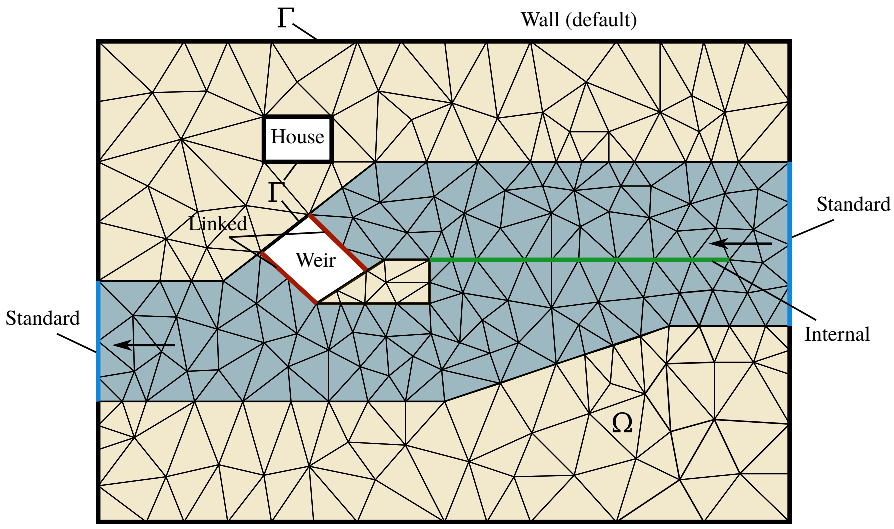

Figure 2.1 *Modeling domain, types of boundary conditions and computational mesh. The flow is from right to left and a side weir (green line) divides the channel into a lower and an upper channel through the weir.* External *boundary conditions must be provided at* Γ_1*,* Γ_2 *and* Γ_3 *while* internal *boundary conditions can be specified in any place within* Ω

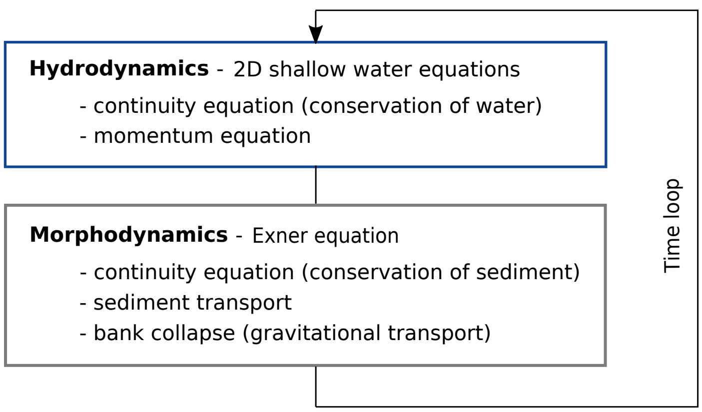

Figure 2.2 *Overview of the numerical model*

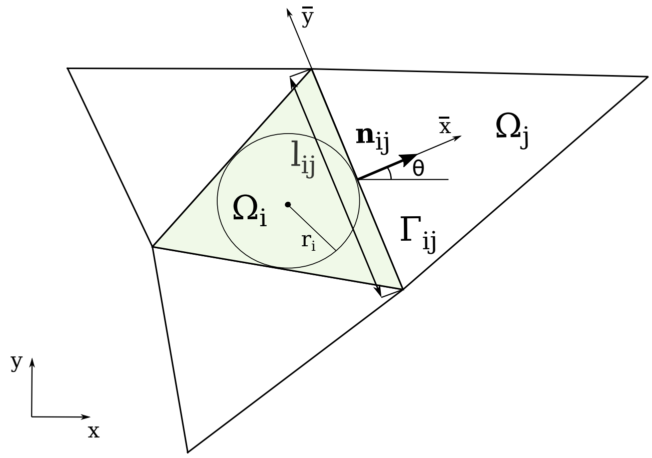

Figure 2.3 *Element (shaded triangle) of unstructured triangular mesh and used notation*

2.2 Discretization

The problem is discretised adopting a finite volume approach over unstructured triangular meshes. A conforming triangulation T_Ω of the computational domain $\Omega \subset \mathbb{R}^2$ by elements Ω_i such that $T_\Omega = \bigcup \Omega_i$, is assumed. Hereafter we will use the following notation: given a finite volume Ω_i, $j = 1, 2, 3$ is the set of indexes such that Ω_j is a neighbour of Ω_i; Γ_{ij} is the common edge of two neighbour cells Ω_i and Ω_j, and l_{ij} its length. $\mathbf{n}_{ij} = (n_{ij,x}, n_{ij,y})$ is the unit vector which is normal to the edge Γ_{ij} and points toward the cell Ω_j (see Figure 2.3). Data are represented by cell averages $\boldsymbol{U}_i^n$ and the numerical solution sought at time $t^{n+1} = t^n + \Delta t$, is denoted by $\boldsymbol{U}_i^{n+1}$.

2.3 Numerical solution of Hydrodynamics

2.3.1 Vectorial Form of the Governing Equations

For numerical convenience, the system of governing equations (eq. 1.1) is rewritten in vectorial form in terms of the water surface elevation $H = h + z_B$. It now reads:

$$\frac{\partial \boldsymbol{U}}{\partial t} + \frac{\partial \boldsymbol{F}_x}{\partial x} + \frac{\partial \boldsymbol{F}_y}{\partial y} = \boldsymbol{S} \tag{2.1}$$

where the vector of unknowns is

$$\boldsymbol{U} = \begin{pmatrix} H \\ q_x \\ q_y \end{pmatrix} \tag{2.2}$$

the vector fluxes are

$$\boldsymbol{F}_x = \begin{pmatrix} q_x \\ uq_x + \frac{1}{2}g(H^2 - 2Hz_b) \\ uq_y \end{pmatrix} \quad ; \quad \boldsymbol{F}_y = \begin{pmatrix} q_y \\ vq_x \\ vq_y + \frac{1}{2}g(H^2 - 2Hz_b) \end{pmatrix} \tag{2.3}$$

and the vector of source terms is

$$\boldsymbol{S} = \begin{pmatrix} S_h \\ gHS_x \\ gHS_y \end{pmatrix} \ . \tag{2.4}$$

The motivation of using H instead of h lies in the fact that it is easier to develop numerical schemes which preserve depth positivity and satisfy the well-balanced property.

2.3.2 Spatial Discretisation

In order to discretise the system of governing equations, the domain is meshed by a set of triangular elements. The spatial discretization of the conservation equations is carried out by the finite volume method, where the differential equations are integrated over the single elements, i.e. control volumes. The water surface elevation is defined at the element center and is equally distributed over the element.

By integrating the governing system of equations eq. 2.1 in the control volume $V = [\Omega_i] \times [t^n, t^{n+1}]$, we obtain

$$\boldsymbol{U}_i^{n+1} = \boldsymbol{U}_i^n - \frac{\Delta t}{|\Omega_i|} \sum_{j=1}^{3} l_{ij} \left[\boldsymbol{F}_{ij}\right] + \Delta t \boldsymbol{S}_i \ . \tag{2.5}$$

2.3.3 Flux Estimation

2.3.3.1 Rotational Invariance of the Shallow Water Equations

The flux $\boldsymbol{F}_{ij}$ are evaluated taking advantage of the rotational invariance property of the shallow water equations. According to this property the two-dimensional homogeneous shallow water equations satisfy the following equality (Toro, 2009):

$$\boldsymbol{n}_{ij} \cdot [\boldsymbol{F}_x(\boldsymbol{U}), \boldsymbol{F}_y(\boldsymbol{U})] = \boldsymbol{T}^{-1}(\theta)\boldsymbol{F}_x[\boldsymbol{T}(\theta)\boldsymbol{U}] \tag{2.6}$$

where θ is the angle between the vector $\boldsymbol{n}_{ij}$ and x-axis, measured counter clockwise from the x-axis (see Figure 2.3) and

$$\boldsymbol{T}(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} \tag{2.7}$$

being

$\boldsymbol{T}^{-1}(\theta) =$ inverse of $\boldsymbol{T}(\theta)$.

2.3.3.2 Computation of the Flux

The flux $\boldsymbol{F}_{ij}$ is obtained at every edge of the finite volume mesh, as the solution of the one-dimensional projected Riemann problem along the normal direction of the two conservation laws eq. 2.1. The computational steps can be summarized as follows:

- First, the vector of conserved variables $\boldsymbol{U}$ is transformed into the local coordinate system $(\bar{x}, \bar{y})$ (see Figure 2.3) at the edge with the operation $\boldsymbol{T}(\theta)\boldsymbol{U}$.
- A one-dimensional, local Riemann problem is formulated and solved in the normal direction of the edge. From this calculation the new flux vector over the edge $\boldsymbol{F}[\boldsymbol{T}(\theta)\boldsymbol{U}]$ is defined.
- The flux vector, formulated in the local coordinate system is transformed back to the global coordinates (Cartesian) with $\boldsymbol{T}^{-1}\boldsymbol{F}[\boldsymbol{T}(\theta)\boldsymbol{U}]$. The sum of the fluxes of all edges of an element gives the total fluxes in the x- and y directions.

The fluxes are calculated in the normal direction of the element edges. The normal direction of the edge is defined positive from element i (L) to element j regarding the edge direction.

2.3.3.3 The HLLC approximated Rieman Solver

The HLLC approximate Riemann solver (Toro, 1994) is a modified HLL (Harten, Lax and van Leer) approximate Riemann solver that includes the shear wave.

The numerical flux at the cell interface is computed as follows:

$$\boldsymbol{F}_{ij}^{HLLC} = \begin{cases} \boldsymbol{F}_i & if \quad 0 \leq S_i, \\ \boldsymbol{F}_{*i} = \boldsymbol{F}_i + S_i(\boldsymbol{U}_{*L} - \boldsymbol{U}_i) & if \quad S_i \leq 0 \leq S_*, \\ \boldsymbol{F}_{*j} = \boldsymbol{F}_j + S_j(\boldsymbol{U}_{*R} - \boldsymbol{U}_j) & if \quad S_* \leq 0 \leq S_j, \\ \boldsymbol{F}_j & if \quad 0 \geq S_j. \end{cases} \tag{2.8}$$

The wave speed velocities are estimated as:

$$S_i = u_i - \sqrt{gh_i}\xi_i \; ; \; S_j = u_j + \sqrt{gh_j}\xi_j \tag{2.9}$$

where $\xi_{K=(i,j)}$ is defined as:

$$\xi_K = \begin{cases} \sqrt{\frac{1}{2}\left[\frac{(h_*+h_K)h_*}{h_K^2}\right]} & if \quad h_* > h_K, \\ 1 & if \quad h_* \leq h_K. \end{cases} \tag{2.10}$$

with h_*, an estimate for the exact solution of the water depth in the star region obtained using the depth positivity condition. It reads as

$$h_* = \frac{1}{2}(h_L + h_R) - \frac{1}{4}(u_R - u_L)(h_L - h_R)/(\sqrt{gh_L} + \sqrt{gh_R}) \tag{2.11}$$

In case of dry-bed conditions, the wave speeds are estimated as the exact dry front speed, i.e.:

$$S_i = \begin{cases} u_i - 2\sqrt{gh_i} & if \quad h_i = 0, \\ \text{usual estimate} & if \quad h_i > 0, \end{cases}$$
$$S_j = \begin{cases} u_j + 2\sqrt{gh_j} & if \quad h_j = 0, \\ \text{usual estimate} & if \quad h_j > 0. \end{cases} \tag{2.12}$$

And the middle estimated wave speed S_* corresponds to the front wave speed in case of dry-bed problem.

The expression of the states $\boldsymbol{U}_{*i}, \boldsymbol{U}_{*j}$ and the middle wave speed S_* can be found in the book of Toro (2009).

2.3.4 Numerical Stability

Numerical stability is assured by choosing the time step Δt for time integration such that it obeys the Courant-Friedrichs-Lewy (CFL) condition. In 2-D the Courant number (CFL) can be defined as follows:

$$CFL = \frac{(\sqrt{u^2 + v^2} + c)\Delta t}{r_i} \tag{2.13}$$

where r_i is the radius of the inscribed circle that defines the element center (Figure 2.3), u, v are the corresponding velocities of the element and $c = \sqrt{gh}$. The HLLC scheme is stable for

$$0 < CFL \leq 1 \tag{2.14}$$

2.3.5 Discretisation of Source terms

2.3.5.1 Bed Slope Source Term

The bed slope source term (eq. 1.2) is discretized using the robust modified-state approach proposed by Duran et al. (2013). The discretization presents a motionless steady states-preserving scheme:

$$\boldsymbol{S}_{b,i} = \sum_{j=1}^{m} l_{ij} \boldsymbol{S}_{b,ij} = \sum_{j=1}^{m} l_{ij} \begin{pmatrix} 0 \\ gH^*_{ij}(z_i - \bar{z}_{ij})\vec{\boldsymbol{n}}_{ij} \end{pmatrix} \tag{2.15}$$

where $\bar{z}_{ij} = \check{z}_{ij} - \Delta_{ij}$ with $\check{z}_{ij} = \max(z_{bi}, z_{bj})$ the maximum bed elevation between cells i and j and $\Delta_{ij} = \max(0, \check{z}_{ij} - H_i)$. H^*_{ij} is the approximated value of the water surface elevation H at the cell interface Γ_{ij}.

2.3.5.2 Friction Source Term

We handle the inhomogeneous character of system eq. 1.1 due to the presence of frictional source terms by adopting a robust splitting technique Toro (2001). We initially consider the initial value problem (IVP)

$$\left.\begin{array}{llll} PDE: & \mathcal{A}(\boldsymbol{U}) & = & \mathcal{S}(\boldsymbol{U}) \\ IC: & \boldsymbol{U}(x,y,0) & = & \boldsymbol{U}_i^n \end{array}\right\}\text{IVP} .$$

where $\mathcal{A}$ represents the advective operator

$$\mathcal{A}(\boldsymbol{U}) = \frac{\partial \boldsymbol{U}}{\partial t} + \frac{\partial \boldsymbol{F}_x}{\partial x} + \frac{\partial \boldsymbol{F}_y}{\partial y} = \boldsymbol{0} ,$$

and $\mathcal{S}$ represents the frictional source term operator.

The numerical solution is then obtained by subsequently integrating *two* initial value problems (IVPs):

$$\left.\begin{array}{llll} ODEs: & \frac{\mathrm{d}\boldsymbol{U}}{\mathrm{d}t} & = & \mathcal{S}(\mathbf{U}) \\ ICs: & \boldsymbol{U}(x,y,0) & = & \boldsymbol{U}_i^n \end{array}\right\} \stackrel{\Delta t}{\Longrightarrow} \overline{\boldsymbol{U}}_i \quad \text{IVP1} ,$$

$$\left.\begin{array}{llll} PDEs: & \mathcal{A}(\mathbf{U}) & = & 0 \\ ICs: & \boldsymbol{U}(x,y,0) & = & \overline{\boldsymbol{U}}_i \end{array}\right\} \stackrel{\Delta t}{\Longrightarrow} \boldsymbol{U}_i^{n+1} \quad \text{IVP2} ,$$

The initial condition (IC) for IVP1 is $\mathbf{U}_i^n$, corresponding to the initial condition of the full problem IVP. The solution of IVP1 is obtained solving a system of ordinary differential equations (ODEs) after integration by a time step Δt and is denoted by $\overline{\boldsymbol{U}}_i$. IVP2 is then integrated by a time step Δt, with initial condition given by the solution of IVP1 $\overline{\boldsymbol{U}}_i$. The solution of IVP2 $\boldsymbol{U}_i^{n+1}$ is obtained solving an hyperbolic homogeneous system of partial differential equations (PDEs) and represents the approximate solution of the full problem IVP.\ Since we adopt an implicit second-order Runge-Kutta method for solving the ODEs systems IVP1 and an explicit finite volume method for solving IVP2, the integration time step Δt is determined accordingly with the CFL stability condition for IVP2.

2.3.5.3 External Source Term

An external source is defined as specific mass flux δ (m/s), uniformly distributed over a number of elements of the domain with a specific surface area. The external source can either be specified as discharge (m^3/s) or precipitation intensity (mm/h) for a specific region of the domain. The external source value is divided among the cells composing the region and converted to cell specific mass flux δ_i. The volume allocated is characterized by different behaviors:

$$\begin{array}{llll} \text{Exact:} & S_{h,i} = \delta_i & & \\ \text{Available:} & S_{h,i} = \delta_i & \text{if} & \delta_i \cdot \Delta t > 0 \\ & S_{h,i} = \max(\delta_i, -h_i) & \text{if} & \delta_i \cdot \Delta t < 0 \\ \text{Infinity:} & S_{h,i} = \delta_i & \text{if} & \delta_i \cdot \Delta t > 0 \\ & S_{h,i} = -h_i & \text{if} & \delta_i \cdot \Delta t < 0 \end{array} \tag{2.16}$$

Where h_i is the water depth of the element i. The external source volume is added to the initial water volume.

$$h_i^{t+1} = h_i^t + S_{h,i} \cdot \Delta t \tag{2.17}$$

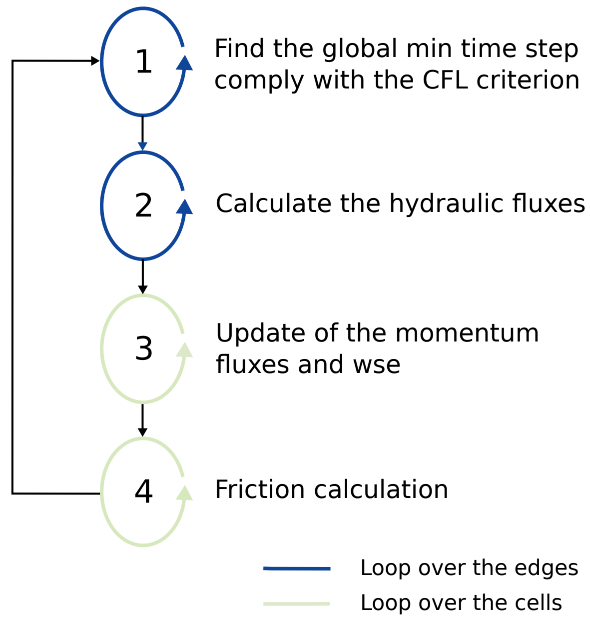

Figure 2.4 *Numerical solution procedure of hydrodynamic simulation for each time step* Δt

2.3.6 Solution Procedure

The numerical solution procedure of BASEMENT explains how the discretised shallow water equation (eq. 1.1) is solved inside a defined time step Δt through a sequence of loops over the edges or cells (Figure 2.4).

First, a global minimum time step Δt should be defined. Then, the hydraulic fluxes (liquid mass, x-momentun and y-momentum) are calculated with a HLLC Riemann solver at the element edges according to the initial states of the left and right cells (Section 2.3.3). Subsequently, the hydraulic state variables i.e. cell centered quantities are updated and finally, the friction (source term) is calculated using an implicit scheme, thus looping twice over the cell.

2.4 Numerical solution of Morphodynamics

2.4.1 Numerical solution of the Exner equation

2.4.1.1 Fundamentals

The Exner equation assures that sediment mass is conserved in the bed and is used to model the riverbed time evolution. The rate of sediment transport is determined using a closure equation. The cell centered finite volume approach is used to discretise the Exner equation and in particular the HLL approximate Riemann solver with a wave speed estimator defined in Soares-Frazão and Zech (2011) is adopted. The shallow water and the Exner equations create a system of equations that is solved in a decoupled way (Figure 2.2). This approach makes the assumption that the bed load flux is much slower than the water flow velocity (Soares-Frazão and Zech, 2011).

2.4.1.2 Spatial discretization

In order to discretise the the Exner equation, we use the same unstructured mesh adopted for the hydrodynamic part and the same finite volume approach. As a consequence, the bed level z_B is defined at the element center and is equally distributed over the element.

By integrating the Exner equation in the control volume $V = [\Omega_i] \times [t^n, t^{n+1}]$, we obtain

$$z_{Bi}^{n+1} = z_{Bi}^{n} - \frac{\Delta t}{|\Omega_i|} \sum_{j=1}^{3} \left[q_{Bij \cdot l_{ij}} \right] + \Delta t \boldsymbol{S}_i \ . \tag{2.18}$$

The calculation of the sediment flux at the cell interface proceeds as follows:

1. loop over the cells and calculate:

    1. correction terms for the bed-load vector directions (if selected by the user), therefore:

    - calculation of the local bed slope, for the lateral-transport correction (see section Section 1.2.2.4.1)
    - calculation of the local curvature of the flow field, for the spiral flow correction (see section Section 1.2.2.4.2)

2. loop over the cell interfaces and:

    1. calculate the flux projection along the normal vector ($n_{ij,x}$,$n_{ij,y}$ of edge Γ_{ij}, i.e.: $q_{Bi,n} = q_{Bi,x} \cdot n_{ij,x} + q_{Bi,y} \cdot n_{ij,x}$ and $q_{Bj,n} = q_{Bj,x} \cdot n_{ij,x} + q_{Bj,y} \cdot n_{ij,x}$ with j=1,2,3
    2. compute the flux at the interface using the approximate HLL Riemann solver at the interface

    - Evaluate the wave speeds at the interface. this is obtained following the approach proposed by Soares-Frazão and Zech (2011), for which the wave speeds can be calculated as an approximation of the smallest eigenvalue of the system of governing equations, i.e. Shallow water and Exner. They read:

$$\lambda_1 = 1/2(u_n - c - \sqrt{(u_n - c)^2 + 4a_2c^2}) \tag{2.19}$$

$$\lambda_2 = 1/2(u_n - c + \sqrt{(u_n - c)^2 + 4a_2c^2}) \tag{2.20}$$

where $u_n = u \cdot n_{ij,x} + v \cdot n_{ij,y}$, $c = \sqrt{gh}$ and $a_2 = \dfrac{\partial q_{b,n}}{\partial q_n}$ which is the derivative of the bed load discharge in the normal flow direction with respect to the hydraulic flux direction. Then the speeds estimate are

$$S^- = \min(\lambda_{1,L}, \lambda_{1,R}) \tag{2.21}$$

and

$$S^+ = \max(\lambda_{2,L}, \lambda_{2,R}) \tag{2.22}$$

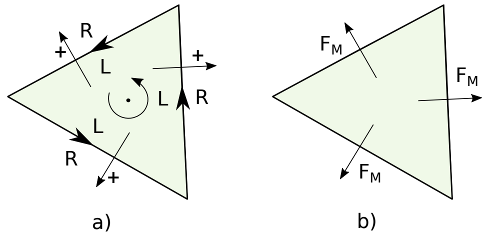

Figure 2.5 *a) Sign convention for the edge direction: counterclockwise b) Positive morphological flux direction at edges: from left (L) to right (R)*

- Flux calculation:

$$q_{Bij}^{HLL} = \begin{cases} q_{Bi,n} & if \quad S^{-} \geq 0, \\ \dfrac{q_{Bi,n}S^{+} - q_{Bj,n}S^{-} + S^{-}S^{+}\left(z_{Bj} - z_{Bi}\right)}{S^{+} - S^{-}} & if \quad S^{-} < 0 < S^{+}, \\ q_{Bj,n} & if \quad S^{+} \leq 0. \end{cases}$$

The convention for the positive bed load flux direction is the same as for the hydrodynamic flux and is presented on Figure 2.5

2.4.1.3 Discretization of External Source Term

The source term Sl_b describes a local input or removal of sediment mass into a river.

An external source is defined as specific mass flux δ (m/s), uniformly distributed over a number of elements of the domain (region) with a specific surface area. The external source can be specified as the total volume flux (m^3/s) for a specific region of the domain. The external source value is divided among the cells composing the region and converted to cell specific mass flux δ_i. The volume allocated is characterized by different behaviors:

$$\begin{array}{llll} \text{Exact:} & S_{b,i} = \delta_i & & \\ \text{Available:} & S_{b,i} = \delta_i & \text{if} & \delta_i \cdot \Delta t > 0 \\ & S_{b,i} = \max(\delta_i, -(z_{Fix} - z_i) & \text{if} & \delta_i \cdot \Delta t < 0 \\ \text{Infinity:} & S_{b,i} = \delta_i & \text{if} & \delta_i \cdot \Delta t > 0 \\ & S_{b,i} = -(z_{Fix} - z_i) & \text{if} & \delta_i \cdot \Delta t < 0 \end{array} \tag{2.23}$$

Where z_i is the bottom elevation and z_{Fix} the fixed bed elevation of the element i. The external source volume is added to the initial bottom elevation of element i.

$$z_i^{t+1} = z_i^t + S_{b,i} \cdot \Delta t \tag{2.24}$$

2.4.2 Solution procedure

The numerical solution procedure of BASEMENT explains how the discretised Exner equation (eq. 1.15) is solved through a sequence of loops over the edges or cells (Figure 2.6).

In the numerical simulation, the hydrodynamic and morphodynamic simulations are performed in a decoupled way. The morphodynamic simulation is executed after the

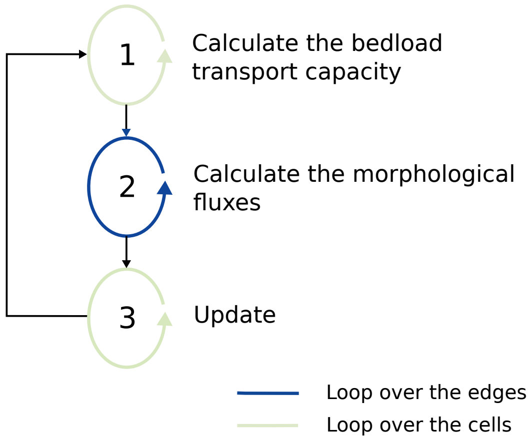

Figure 2.6 *Numerical solution procedure of morphodynamic simulation for each time step* Δt

hydrodynamic simulation, using the hydraulic fluxes to calculate the morphological fluxes. This approach assumes that the sediment transport is much slower than the water velocity, which is an accurate assumption for the numerical modelling of slow flood with morphological changes occurring over a long period (Soares-Frazão and Zech, 2011). The numerical solution procedure of Figure 2.6 is performed after the step 4 of Figure 2.4 inside the same time step Δt.

The numerical solution of the Exner equation starts with a loop over the cells in order to find the bedload transport capacity q_b with a potential correction due to a curvature effect or lateral bed slope. Then, the morphological fluxes F_M are calculated at the element edges and finally, the bed elevation z_b is updated over the cells.

2.5 Numerical solution of Passive Tracers

2.5.1 Numerical solution of the scalar advection equation

2.5.1.1 Fundamentals and spatial discretization

The scalar advection equation assures that scalar masses are conserved in the flow column. The same cell cell-centered finite volume approach used for the SWE is employed in scalar advection, namely the HLLC approximate Riemann solver. In order to discretize the scalar advection equation, the same unstructured mesh adopted for the hydrodynamic and morphodynamic parts is used, as is the same finite volume approach. As a consequence, the scalar concentration ϕ_s is defined at the element center and is equally distributed over the element.

By integrating the scalar advection equation in the control volume $V = [\Omega_i] \times [t^n, t^{n+1}]$, we obtain

$$q_{Si}^{n+1} = q_{Si}^{n} - \frac{\Delta t}{|\Omega_i|} \sum_{j=1}^{3} [f_{Sij} \cdot l_{ij}] + \Delta t \boldsymbol{S}_{s_i} \; . \tag{2.25}$$

where f_{Sij} is the intercell scalar flux. The calculation of the scalar fluxes at the cell interface proceeds as follows:

1. loop over the cell interfaces and compute the flux at the interface using a simplified approximate HLLC Riemann solver at each interface:
   - Retrieve the hydrodynamic fluxes $\boldsymbol{F}_{ij}$ at the interface between cells i and j and extract the fist component of the flux vector (mass fluxes) to the variable q_m.
   - For each specie, perform the flux calculation through a simplified version of the HLLC solver for solute transport:

   $$f_{Sij} = \frac{q_{Si}}{h} q_m \;\; \text{if} \;\; q_m > 0$$
   $$f_{Sij} = \frac{q_{Sj}}{h} q_m \;\; \text{if} \;\; q_m < 0$$

   this approach significantly reduces the numerical effort involved in the computation of the numerical fluxes for each species without sacrificing accuracy.

2. loop over the cells and update the conserved quantities with the fluxes at each of its interfaces:
   - Retrieve the hydrodynamic fluxes f_{Sij} at each of the three interfaces.
   - For each specie, perform the update as prescribed in eq. 2.25.

2.5.1.2 Discretization of External Source Term

The source term S_s describes a local input or removal of scalar mass into a river. An external source is defined as specific mass flux δ (m/s), uniformly distributed over a number of elements of the domain (region) with a specific surface area. The external source can be specified as the total volume flux (m^3/s) for a specific region of the domain. The external source value is divided among the cells composing the region and converted to cell specific mass flux δ_i. The volume allocated is characterized by different behaviors:

$$\begin{array}{lll} \text{Exact:} & S_{S,i} = \delta_i & \\ \text{Available:} & S_{S,i} = \delta_i & \text{if} \quad \delta_S \cdot \Delta t > 0 \\ & S_{S,i} = \max(\delta_i, -q_{Si}) & \text{if} \quad \delta_i \cdot \Delta t < 0 \\ \text{Infinity:} & S_{S,i} = \delta_i & \text{if} \quad \delta_i \cdot \Delta t > 0 \\ & S_{S,i} = -q_{Si} & \text{if} \quad \delta_i \cdot \Delta t < 0 \end{array}$$

The external source volume is then added to the initial bottom elevation of element i through a first-order Euler approach

$$q_{Si}^{t+1} = q_{Si}^{t} + S_{S,i} \cdot \Delta t$$

3

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"recipients" may be individuals or organizations.

To "modify" a work means to copy from or adapt all or part of the work
in a fashion requiring copyright permission, other than the making of an
exact copy.  The resulting work is called a "modified version" of the
earlier work or a work "based on" the earlier work.

A "covered work" means either the unmodified Program or a work based
on the Program.

To "propagate" a work means to do anything with it that, without
permission, would make you directly or secondarily liable for
infringement under applicable copyright law, except executing it on a
computer or modifying a private copy.  Propagation includes copying,
distribution (with or without modification), making available to the
public, and in some countries other activities as well.

To "convey" a work means any kind of propagation that enables other
parties to make or receive copies.  Mere interaction with a user through
a computer network, with no transfer of a copy, is not conveying.

An interactive user interface displays "Appropriate Legal Notices"
to the extent that it includes a convenient and prominently visible
feature that (1) displays an appropriate copyright notice, and (2)
tells the user that there is no warranty for the work (except to the
extent that warranties are provided), that licensees may convey the
work under this License, and how to view a copy of this License.  If
the interface presents a list of user commands or options, such as a
menu, a prominent item in the list meets this criterion.

1. Source Code.

The "source code" for a work means the preferred form of the work
for making modifications to it.  "Object code" means any non-source
form of a work.

A "Standard Interface" means an interface that either is an official
standard defined by a recognized standards body, or, in the case of
interfaces specified for a particular programming language, one that
is widely used among developers working in that language.

The "System Libraries" of an executable work include anything, other
than the work as a whole, that (a) is included in the normal form of
packaging a Major Component, but which is not part of that Major
Component, and (b) serves only to enable use of the work with that
Major Component, or to implement a Standard Interface for which an
implementation is available to the public in source code form.  A
"Major Component", in this context, means a major essential component
(kernel, window system, and so on) of the specific operating system
(if any) on which the executable work runs, or a compiler used to
produce the work, or an object code interpreter used to run it.

The "Corresponding Source" for a work in object code form means all
the source code needed to generate, install, and (for an executable
work) run the object code and to modify the work, including scripts to
control those activities.  However, it does not include the work's
System Libraries, or general-purpose tools or generally available free
programs which are used unmodified in performing those activities but
which are not part of the work.  For example, Corresponding Source
includes interface definition files associated with source files for
the work, and the source code for shared libraries and dynamically
linked subprograms that the work is specifically designed to require,
such as by intimate data communication or control flow between those
subprograms and other parts of the work.

The Corresponding Source need not include anything that users
can regenerate automatically from other parts of the Corresponding
Source.
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The Corresponding Source for a work in source code form is that
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You may convey a work based on the Program, or the modifications to
produce it from the Program, in the form of source code under the
terms of section 4, provided that you also meet all of these conditions:

  a) The work must carry prominent notices stating that you modified
  it, and giving a relevant date.

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  released under this License and any conditions added under section
  7.  This requirement modifies the requirement in section 4 to
  "keep intact all notices".
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  c) You must license the entire work, as a whole, under this
  License to anyone who comes into possession of a copy.  This
  License will therefore apply, along with any applicable section 7
  additional terms, to the whole of the work, and all its parts,
  regardless of how they are packaged.  This License gives no
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  invalidate such permission if you have separately received it.

  d) If the work has interactive user interfaces, each must display
  Appropriate Legal Notices; however, if the Program has interactive
  interfaces that do not display Appropriate Legal Notices, your
  work need not make them do so.

A compilation of a covered work with other separate and independent
works, which are not by their nature extensions of the covered work,
and which are not combined with it such as to form a larger program,
in or on a volume of a storage or distribution medium, is called an
"aggregate" if the compilation and its resulting copyright are not
used to limit the access or legal rights of the compilation's users
beyond what the individual works permit.  Inclusion of a covered work
in an aggregate does not cause this License to apply to the other
parts of the aggregate.

6. Conveying Non-Source Forms.

You may convey a covered work in object code form under the terms
of sections 4 and 5, provided that you also convey the
machine-readable Corresponding Source under the terms of this License,
in one of these ways:

  a) Convey the object code in, or embodied in, a physical product
  (including a physical distribution medium), accompanied by the
  Corresponding Source fixed on a durable physical medium
  customarily used for software interchange.

  b) Convey the object code in, or embodied in, a physical product
  (including a physical distribution medium), accompanied by a
  written offer, valid for at least three years and valid for as
  long as you offer spare parts or customer support for that product
  model, to give anyone who possesses the object code either (1) a
  copy of the Corresponding Source for all the software in the
  product that is covered by this License, on a durable physical
  medium customarily used for software interchange, for a price no
  more than your reasonable cost of physically performing this
  conveying of source, or (2) access to copy the
  Corresponding Source from a network server at no charge.

  c) Convey individual copies of the object code with a copy of the
  written offer to provide the Corresponding Source.  This
  alternative is allowed only occasionally and noncommercially, and
  only if you received the object code with such an offer, in accord
  with subsection 6b.

  d) Convey the object code by offering access from a designated
  place (gratis or for a charge), and offer equivalent access to the
  Corresponding Source in the same way through the same place at no
  further charge.  You need not require recipients to copy the
  Corresponding Source along with the object code.  If the place to
  copy the object code is a network server, the Corresponding Source
  may be on a different server (operated by you or a third party)
  that supports equivalent copying facilities, provided you maintain
  clear directions next to the object code saying where to find the
  Corresponding Source.  Regardless of what server hosts the
  Corresponding Source, you remain obligated to ensure that it is
  available for as long as needed to satisfy these requirements.

  e) Convey the object code using peer-to-peer transmission, provided
  you inform other peers where the object code and Corresponding
  Source of the work are being offered to the general public at no
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  charge under subsection 6d.

A separable portion of the object code, whose source code is excluded
from the Corresponding Source as a System Library, need not be
included in conveying the object code work.

A "User Product" is either (1) a "consumer product", which means any
tangible personal property which is normally used for personal, family,
or household purposes, or (2) anything designed or sold for incorporation
into a dwelling.  In determining whether a product is a consumer product,
doubtful cases shall be resolved in favor of coverage.  For a particular
product received by a particular user, "normally used" refers to a
typical or common use of that class of product, regardless of the status
of the particular user or of the way in which the particular user
actually uses, or expects or is expected to use, the product.  A product
is a consumer product regardless of whether the product has substantial
commercial, industrial or non-consumer uses, unless such uses represent
the only significant mode of use of the product.

"Installation Information" for a User Product means any methods,
procedures, authorization keys, or other information required to install
and execute modified versions of a covered work in that User Product from
a modified version of its Corresponding Source.  The information must
suffice to ensure that the continued functioning of the modified object
code is in no case prevented or interfered with solely because
modification has been made.

If you convey an object code work under this section in, or with, or
specifically for use in, a User Product, and the conveying occurs as
part of a transaction in which the right of possession and use of the
User Product is transferred to the recipient in perpetuity or for a
fixed term (regardless of how the transaction is characterized), the
Corresponding Source conveyed under this section must be accompanied
by the Installation Information.  But this requirement does not apply
if neither you nor any third party retains the ability to install
modified object code on the User Product (for example, the work has
been installed in ROM).

The requirement to provide Installation Information does not include a
requirement to continue to provide support service, warranty, or updates
for a work that has been modified or installed by the recipient, or for
the User Product in which it has been modified or installed.  Access to a
network may be denied when the modification itself materially and
adversely affects the operation of the network or violates the rules and
protocols for communication across the network.

Corresponding Source conveyed, and Installation Information provided,
in accord with this section must be in a format that is publicly
documented (and with an implementation available to the public in
source code form), and must require no special password or key for
unpacking, reading or copying.

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a) You must cause the modified files to carry prominent notices stating that you changed the files and the date of any change.

b) You must cause any work that you distribute or publish, that in whole or in part contains or is derived from the Program or any part thereof, to be licensed as a whole at no charge to all third parties under the terms of this License.

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    c) If the modified program normally reads commands interactively
    when run, you must cause it, when started running for such
    interactive use in the most ordinary way, to print or display an
    announcement including an appropriate copyright notice and a
    notice that there is no warranty (or else, saying that you provide
    a warranty) and that users may redistribute the program under
    these conditions, and telling the user how to view a copy of this
    License.  (Exception: if the Program itself is interactive but
    does not normally print such an announcement, your work based on
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These requirements apply to the modified work as a whole.  If
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    source code, which must be distributed under the terms of Sections
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    cost of physically performing source distribution, a complete
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    distributed under the terms of Sections 1 and 2 above on a medium
    customarily used for software interchange; or,

    c) Accompany it with the information you received as to the offer
    to distribute corresponding source code.  (This alternative is
    allowed only for noncommercial distribution and only if you
    received the program in object code or executable form with such
    an offer, in accord with Subsection b above.)

The source code for a work means the preferred form of the work for
making modifications to it.  For an executable work, complete source
code means all the source code for all modules it contains, plus any
associated interface definition files, plus the scripts used to
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