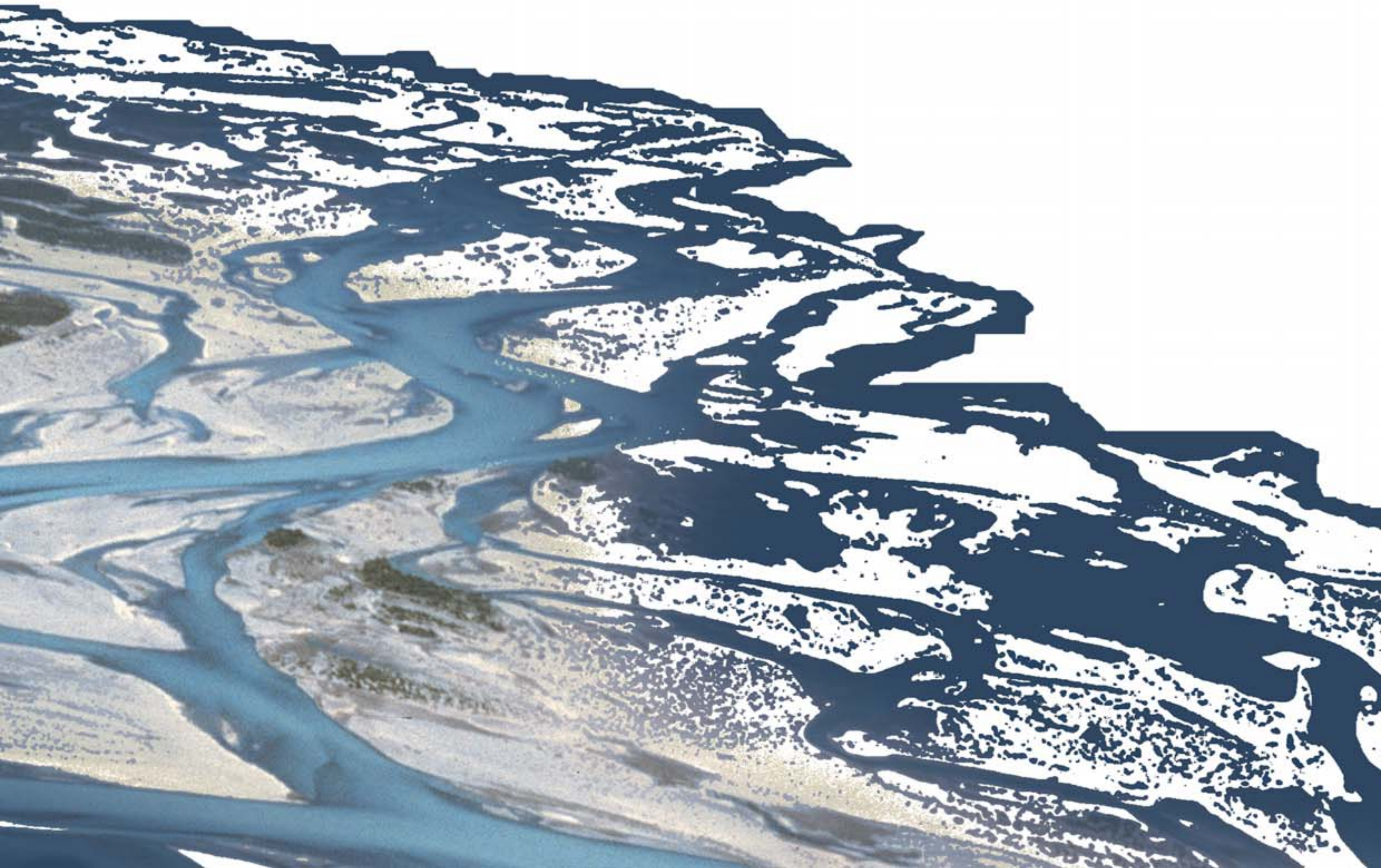
BASEMENT

BASIC SIMULATION ENVIRONMENT
FOR SIMULATION OF ENVIRONMENTAL FLOW
AND NATURAL HAZRAD SIMULATION

SYSTEM MANUALS

VERSION 3.0
September 2019



Preamble

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For Website:

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BASIC SIMULATION ENVIRONMENT
FOR SIMULATION OF ENVIRONMENTAL FLOW
AND NATURAL HAZRAD SIMULATION

REFERENCE MANUAL

VERSION 3.0
September 2019

BASEMENT

Contents

1

Mathematical Models

1.1 Hydrodynamics

1.1.1 Introduction

Mathematical models of the so-called *shallow water* type govern a wide variety of physical phenomena. Especially the one-dimensional (1D) de Saint-Venant equations (SVE) or two-dimensional (2D) shallow water equations (SWE) are of practical interest with regard to water flows with a free surface under the influence of gravity. Applications of the models include e.g.:

- River hydrodynamics
- Propagation of flood waves
- Dam break waves
- Flooding and inundation
- Ecological assessment based on flow quantities

The 2D SWE are based on the following set of hypotheses:

- the water is assumed to be incompressible; i.e. the water density ρ is constant
- the vertical acceleration of the water particles are assumed to be small compared to the longitudinal component of the acceleration. As a consequence the pressure distribution is hydrostatic;
- the bottom slope is small enough for the longitudinal coordinate to coincide with the horizontal axis;
- the flow regime is turbulent. As a consequence the head loss, mainly due to friction against the bottom, is proportional to the square of the flow velocity.

1.1.2 Governing Equations

The governing equations are obtained under shallow water conditions imposing mass conservation for the fluid and solid phases and the momentum principle to a flow in an open channel with a fixed bottom.

Introducing a Cartesian reference system (x, y, z) in which the z axis is vertical and the $x - y$ plane is horizontal with respect to gravity g , the system of governing equations can be written as

$$\begin{cases} \frac{\partial h}{\partial t} + \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = S_h \\ \frac{\partial q_x}{\partial t} + \frac{\partial}{\partial x}\left(\frac{q_x^2}{h} + \frac{1}{2}gh^2\right) + \frac{\partial}{\partial y}\left(\frac{q_x q_y}{h}\right) + gh\left(S_{bx} + S_{fx}\right) = 0 \\ \frac{\partial q_y}{\partial t} + \frac{\partial}{\partial x}\left(\frac{q_y q_x}{h}\right) + \frac{\partial}{\partial y}\left(\frac{q_y^2}{h} + \frac{1}{2}gh^2\right) + gh(S_{by} + S_{fy}) = 0 \; , \end{cases} \tag{1.1}$$

where:

h	$[m]$	water depth
g	$[m/s^2]$	gravity acceleration
u (v)	$[m/s]$	depth averaged velocity in x (y) direction
q_x (q_y)	$[m^2/s]$	discharge per unit width in x (y) direction
S_h	$[m/s]$	lateral inflow/outflow discharge per unit width
S_{fx} (S_{fy})	$[-]$	friction terms in x (y) direction .

The bed slope source terms

$$S_{bx}, S_{by}$$

are evaluated as follows:

$$S_{bx} = -\frac{\partial z_B}{\partial x} \quad ; \quad S_{by} = -\frac{\partial z_B}{\partial y} \tag{1.2}$$

1.1.3 Closure relations

In order to solve system (eq. 1.1) we need to specify the closure relations for the friction terms S_{fx}, S_{fy} and the value of lateral inflow/outflow discharge per unit width S_h.

1.1.3.1 Friction terms

The governing equations (eq. 1.1) have been derived under the hypothesis of turbulent flow, hence the friction terms can be assumed proportional to the square of the depth-averaged velocity and can be written as:

Several formulae are available for S_f. All these formulae use hypothesis (H3) of a turbulent flow regime, hence the assumption that the slope of the energy line is proportional to the square of the flow velocity u. The most frequently used laws are

Adopting a quadratic friction law, the friction term is proportional to the square of the depth-averaged velocity and can be written as:

$$S_{fx} = \frac{u|\vec{u}|}{gc_f^2 h} \quad ; \quad S_{fy} = \frac{v|\vec{u}|}{gc_f^2 h} \tag{1.3}$$

where g is the gravity acceleration, u and v are the depth averaged velocities in x and y direction, $|\vec{u}| = \sqrt{u^2 + v^2}$ is the magnitude of the velocity vector and c_f is the dimensionless friction coefficient.

Several formulae are available for the dimensionless friction coefficient c_f. Here it is quantified using both a power or a logarithmic for which are described in the next sections.

1.1.3.1.1 Power Law

The Manning-Strickler power law is widely used in practice and it requires that either the Strickler's k_{str} $[m^{1/3}/s]$ or the Manning's n coefficients ($k_{str} = n^{-1}$) is specified.

In this case the dimensionless friction coefficient c_f is calculated as

$$c_f = \frac{k_{str} h^{1/6}}{\sqrt{g}} \tag{1.4}$$

1.1.3.1.2 Logarithmic Law

The following approaches are implemented to determine the friction coefficient c_f:

Chézy:

$$\begin{aligned} c_f &= 5.75 \log \left(12 \frac{R}{K_s}\right) && \text{for} \quad R > K_s \\ c_f &= 5.75 \log (12) && \text{for} \quad R < K_s \;, \end{aligned} \tag{1.5}$$

where K_s $[m]$ is the bed roughness height which is commonly taken to be proportional to a representative sediment size d_x. For rivers, K_s can be assumed $K_s = n_k d_{90}$ where $n_k = 2 \div 3$.

Bezzola:

In this closure relation, proposed by Bezzola (2002), c_f is given as a function of the roughness sublayer height y_R $[m]$ (usually for rivers $y_R \approx 1.0 d_{90}$ is a good approximation). This approach is also valid for small values of the relative submergence h/y_r Bezzola (2002).

$$\begin{cases} c_f = 2.5\sqrt{1 - \dfrac{y_R}{h}} \, \ln\left(10.9 \dfrac{R}{y_R}\right), & \text{for} \quad \dfrac{h}{y_R} > 2 \\ c_f = 1.25\sqrt{\dfrac{h}{y_R}} \, \ln\left(10.9 \dfrac{R}{y_R}\right), & \text{for} \quad 0.5 \le \dfrac{h}{y_R} \le 2 \\ c_f = 1.5, & \text{for} \quad \dfrac{h}{y_R} < 0.5 \end{cases} \tag{1.6}$$

1.1.3.2 Lateral inflow/outflow

S_h is used to represent additional sources of water like rainfall and springs or water abstraction (sink) and are allocated on a set of elements defined by regions. The external

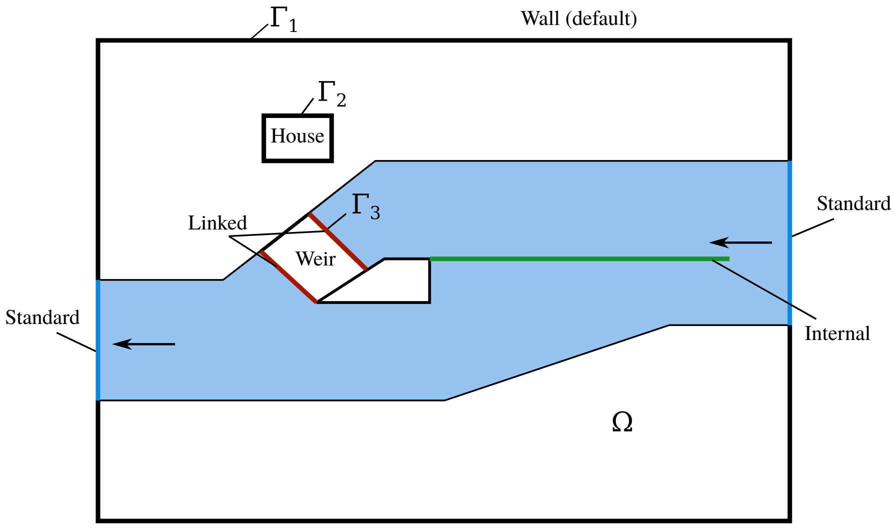

Figure 1.1 *Modeling domain and types of boundary conditions available. The flow is from right to left and a side weir (green line) divides the channel into a lower and an upper channel through the weir.* External *boundary conditions must be provided at* Γ_1, Γ_2 *and* Γ_3 *while* internal *boundary conditions can be specified in any place within* Ω

source can be specified as total discharge $[m^3/s]$ or distributed over time $[mm/h]$. Different approaches are used to manage the behaviour of the external sources:

- Exact: The specified water volume is added or extracted (non conservative)
- Available: The specified water volume to extract is limited by the available water volume in the elements (conservative)
- Infinity: All available water will be abstracted (conservative)

Addition of water always follows the “Exact” behaviour as there is no upper limit. The abstraction of water could also follow the “Exact” behaviour but the simulation might end abruptly if the available water volume is smaller than the volume prescribed. Therefore, the “Available” behaviour aims to avoid this situation. The “Infinity” behaviour abstracts all available water volume.

1.1.4 Boundary Conditions

After the specification of the *closure relations* there are now three equations and three unknowns, namely h, q_x and q_y. In principle, given initial and boundary conditions, one should be able to solve system (eq. 1.1) for h, q_x and q_y as functions of space x, y, and time t. Given the modeling domain described in Figure 1.1, boundary conditions are required at the domain boundary Γ and optionally can be specified within the interior domain Ω.

Therefore, two different types of boundary conditions can be defined:

- External boundary conditions: located at the domain boundary Γ_i
- Internal boundary conditions: located inside the domain Ω

External boundaries (at Γ) represent the limits of the computational domain possibly including also buildings, weirs or structures for water intake (see Figure 1.1).

1.1.4.1 External boundary conditions

At the external boundaries two different types of boundary conditions can be specified: wall or flow boundaries. Flow boundary conditions allow the flow to enter or leave the domain while wall boundary conditions express no mass flux over the boundary. By default, the external boundaries are set as wall.

1.1.4.1.1 Wall boundary

The *Wall* or *reflective* boundary consider the boundary at Γ_i and suppose it physically consists of a fixed, reflective impermeable wall. Then the physical situation is modelled imposing that:

$$\rho \vec{u} \cdot \vec{n} = 0 \quad ; \quad \frac{\partial \vec{u}}{\partial \vec{n}} = 0 \tag{1.7}$$

Where $\vec{n}$ is the outward directed unit vector perpendicular to the wall and $\vec{u} = (u, v)^T$ is the velocity vector. The static pressure is assumed to be zero.

1.1.4.1.2 Flow Boundaries

The *Flow* boundary conditions are defined as *inflow* if they let water entering or as *outflow* if they let water leaving the domain. Flow boundaries are further distinguished into *Standard* and *Linked*. The former are applied on the boundary domain Γ, while the latter establish a *link* between two portions of the domain.

Standard

Inflow boundaries:

This boundary requires the specification of a value for the total volume discharge Q, $[m^3/s]$, which is then divided by the length of the boundary Γ and projected orthogonally to the boundary to obtain the values of q_x and q_y. In case of supercritical flow the following possibilities to specify the value of the water depth h are possible:

- Uniform: h is calculated assuming that local uniform flow conditions. The calculation proceeds as follows:

$$h = \sqrt[3]{\frac{(Q/b)^2}{g c_f^2 s}} \tag{1.8}$$

  where c_f is the Chézy coefficient, b is the entire length of the boundary Γ and s is the value of the local bed slope that must be specified.

- Explicit: In this case the flow depth h is calculated as follows:

$$h = \sqrt[3]{\frac{(Q/b)^2}{g Fr^2}} \tag{1.9}$$

where b is the entire length of the boundary and Fr is the value of the local Froude number that must be specified

- zhydrograph: The water surface elevation (wse) at the boundary must be specified by the user. The depth is calculated as:

$$h = wse - z_B \tag{1.10}$$

where z_B is the bottom elevation at the boundary. The flow velocity at the boundary is set to zero.

Outflow boundaries:

At the outflow boundaries a value for the water depth h must be specified. These are the possible options:

- Uniform: the water depth h is calculated using equation (eq. 1.8) specifying a value for the total discharge Q and a local bed slope s. Uniform flow is calculated based on given slope and cell state at boundary (eq. 1.8).

- Weir: This boundary establishes a relation between the approaching discharge q constant and the water depth using the Poleni weir formula:

$$q = \frac{2}{3}\mu\sqrt{2g(h_{up} - w)^3} \tag{1.11}$$

where h_{up} is the water depth of the approaching flow and w is the weir elevation. The Poleni factor μ can be either set as constant ($\mu = 0.75$ by default) or dynamically evaluated as:

$$\mu = \frac{0.611}{a}\frac{0.75}{b}\frac{h_{up} - z_w}{w} \tag{1.12}$$

where a and b must be specified by the user (default values are $a = 0.611$ and $b = 0.075$).

- h-Q relation: The water surface elevation is determined as a function of the discharge, thus a h-Q relation has to be specified.

- zhydrograph: The water surface elevation (wse) at the boundary must be specified by the user. The depth is calculated as:

$$h = wse - z_B \tag{1.13}$$

where z_B is the bottom elevation at the boundary. The flow velocity is calculated with the Riemann solver (Hllc).

- Zero gradient (scientific use only): Transmissive, or transparent boundaries allow the passage of waves without any effect on them. This is mathematically obtained imposing over the entire length of the boundary that:

$$\rho\vec{u} \cdot \vec{n} = \text{const} \quad ; \quad \frac{\partial \vec{u}}{\partial \vec{n}} = 0 \tag{1.14}$$

In this case there is no need to specify further parameters.

Note: This is boundary condition should ***not*** *be used for practical problems and is intended for scientific use only.*

Linked

This type of boundaries establish a *link* between within a certain region of the domain where equations are not solved. Once this domain portion is identified the two boundaries, between which the link is established, must be specified. Let us call them Γ_{in} and Γ_{out}. Then, one inflow boundary condition must be specified at Γ_{in} and one outflow boundary condition at Γ_{out} while in the remaining boundaries wall conditions are automatically assigned. Not necessarily, Γ_{in} and Γ_{out} must have the same number of elements.

Linked boundaries can describe a $h - Q$ relation or a weir, i.e.:

- Weir: Similar to the standard weir boundary, the weir height w has to be specified. No kinetic energy is considered.
- $h - Q$ relation: The flux is calculated given a h-Q relation (see description of the h-Q relation for standard boundaries).
- 2 ways $h - Q$ relation: The internal boundary works as dynamic wall that is controlled by water surface elevation thresholds. If the upper water surface elevation threshold is reached, the internal boundary is removed until the water level reaches the lower water surface elevation, where the wall is re-established.

1.1.4.2 Internal boundary conditions

The internal boundary condition allows a direct cell-cell relation due to the exact same number of elements on the left and on right side of the boundary. Internal boundary conditions can be used to specify internal walls, dynamic walls or an h-Q relation.

- Wall: The wall conditions (eq. 1.7) are applied on both sides of the internal boundary.
- Dynamic Wall: The wall conditions are applied on the internal boundary until reaching a threshold value (time or water depth) after which the wall is removed.
- $h - Q$ relation: A $h - Q$ relation is applied on one side of the internal boundary, while on the other side, wall conditions apply (unidirectional flow).

1.1.5 Flood tracking

The flood tracking aims at extracting the flood arrival time, the maximum water depth, flow velocity and specific discharge along the numerical simulation and over a selected domain area. The area is defined by a regiondef and is required to be flooded (wet cells). The flood tracking provides outputs within a tracking time step defined by the user.

1.2 Morphodynamics

1.2.1 Introduction

Morphodynamic models provide scientific frameworks for advancing our understanding of river systems. The research on involved topics is an important and socially relevant undertaking regarding our environment. Nowadays numerical models are used for different purposes, from answering questions about basic morphodynamic research to managing complex river engineering problems. Due to increasing computer power and the development of advanced numerical techniques, morphodynamic models are now more and more used to predict the bed patterns evolution to a broad spectrum of spatial and temporal scales. The development and the success of application of such models are based upon a wide range of disciplines from applied mathematics for the numerical solution of the equations to geomorphology for the physical interpretation of the results.

Applications of morphodynamic models include:

- Damming of river basins
- Morphological changes due to width changes (e.g. River widenings)
- Effects of sediment mining
- River straightening

1.2.2 Governing Equation

The governing equations are obtained under shallow water conditions imposing mass conservation for the fluid and solid phases and the momentum principle to a flow in an open channel with a cohesionless bottom. Introducing a Cartesian reference system $(x; y; z)$ in which the z axis is vertical and the $x - y$ plane is horizontal, the system of governing equations is described by the system of equations (eq. 1.1) for hydrodynamics coupled with one equation for the conservation of the total sediment mass (the Exner equation (Exner, 1925)), i.e.:

$$(1-p)\frac{\partial z_B}{\partial t} + \frac{\partial q_{B_x}}{\partial x} + \frac{\partial q_{B_y}}{\partial y} - Sl_b = 0 \tag{1.15}$$

where p is the porosity, Sl_b is the source term specifying local input or output of sediment material (e.g. slope collapse or excavation) per unit width and q_{B_x} and q_{B_y} are the specific bed load flux in x and y direction, respectively. The Exner equation describes the bed evolution due to erosion or deposition processes, which results in changes of the bed level z_B.

1.2.3 Closure relations

In order to solve system (eq. 1.1) and equation (eq. 1.31) we need to specify the closure relations. For the friction terms S_{fx}, S_{fy} and the value of lateral inflow/outflow discharge per unit width S_h we can use the relations already introduced in the Hydrodynamic part (Section 1.1.3). For the Exner equation we need relations quantifying the bedload discharges.

1.2.3.1 Bedload sediment transport: Fundamentals

The key dimensionless parameter quantifying sediment mobility is the Shields parameter defined as:

$$\theta = \frac{\tau_b}{(\rho_s - \rho) g d} \tag{1.16}$$

where τ_b is the bottom shear stress (drag force acting on the particle), d is the sediment diameter, ρ and ρ_s are the water and sediment density, respectively. The Shields parameter can be interpreted as the ratio scaling the impelling force of flow drag acting on a particle to the Coulomb force resisting motion acting on the same particle. The bed shear stress is usually estimated by a closure condition using an empirical or semi-empirical formula. Here we use the quadratic friction law which relates the depth-averaged velocities to the bed shear stress as follows:

$$\tau_{bx} = \rho \frac{|\vec{u}| u}{c_f^2} \quad ; \quad \tau_{by} = \rho \frac{|\vec{u}| v}{c_f^2} \tag{1.17}$$

where τ_b is the bottom shear stress and ρ_s and ρ are the density of sediments and water,respectively.

1.2.3.1.1 Threshold conditions for sediment movement

When a granular bed is subjected to a turbulent flow, it is found that virtually no motion of the grains is observed below a critical value (θ_{cr}) of the Shileds parameter. According to the Shileds' theory Shields (1936), θ_{cr} can be expressed as a function of the Reynolds number $Re^* = \frac{d u_*}{\nu}$. Alternatively, the diagram of incipient motion (see Figure 1.2) can be plot as a function of the dimensionless grain diameter D^* ($\theta_{cr} = f(D^*)$), where

$$D^* = d \left[\frac{g(s-1)}{\nu^2} \right]^{1/3}$$

.

The curve representing the particle incipient motion ($\theta = \theta_{cr}$) can be divided into three parts in the log-log graph:

- for $D^* \leq 3$, can be approximated by a linear segment;
- for $3 \leq D^* \leq 100$ this is represented by a curve with a relative minimum;
- for \$D^* > 100 \$ by a constant trend.

An approximation of the original Shields diagram was proposed by van Rijn (1984):

$$\begin{array}{lll}
\theta_{cr} = 0.24(D^*)^{-1} & \text{for} & 1 \leq D^* \leq 4 \\
\theta_{cr} = 0.14(D^*)^{-0.64} & \text{for} & 4 < D^* \leq 10 \\
\theta_{cr} = 0.04(D^*)^{-0.1} & \text{for} & 10 < D^* \leq 20 \\
\theta_{cr} = 0.013(D^*)^{0.29} & \text{for} & 20 < D^* \leq 150 \\
\theta_{cr} = 0.055 & \text{for} & D^* > 150
\end{array} \tag{1.18}$$

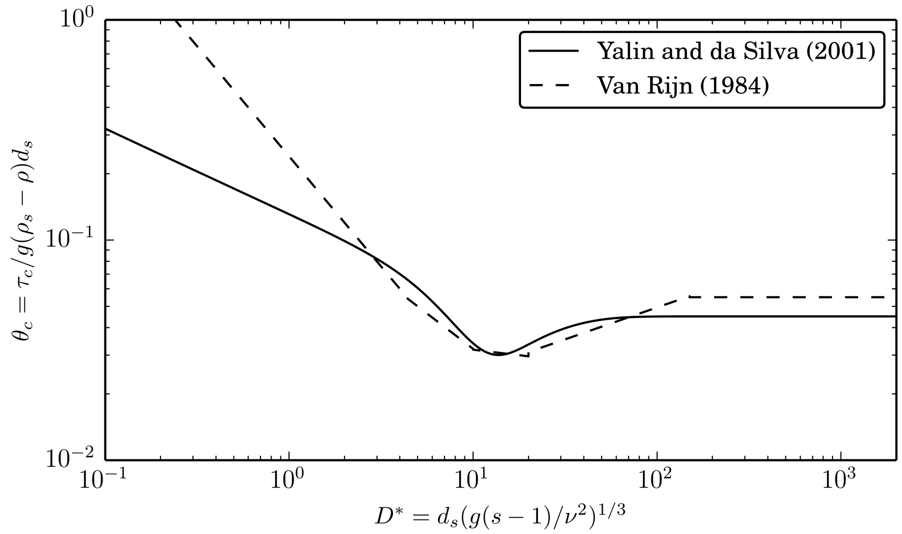

Figure 1.2 *Modified Shields diagram for initiation of sediment motion*

Another explicit formulation of the Shields curve was proposed by Yalin and Silva (2001). It reads

$$\theta_{cr} = 0.13D^{*-0.392}\exp(-0.015D^*) + 0.045\left(1 - \exp(-0.068D^*)\right) \tag{1.19}$$

1.2.3.1.2 Influence of Local Slope on Incipient Motion

The threshold condition for incipient motion of grains developed by Shields is valid for almost horizontal bed. In case of sloped bed in flow direction or transverse to it, the stability of grains is either increased or reduced due to the gravity. The critical shear stress value can be adapted consequently to account for the influence of local slopes. One approach is to multiply the critical shear stress for almost horizontal bed with correction factors for the local bed slope in the flow direction and transverse to it. The corrected critical bed shear stress becomes:

$$k_\beta k_\delta \theta_{cr} \tag{1.20}$$

The correction factors $k_\beta k_\delta$ are calculated as suggested by van Rijn (1989):

$$\begin{aligned} k_\beta &= \begin{cases} \dfrac{sin(\gamma - \beta)}{sin\gamma} & \text{if slope} < 0 \\ \dfrac{sin(\gamma + \beta)}{sin\gamma} & \text{if slope} > 0 \end{cases} \\ k_\delta &= \left\{ cos\delta\sqrt{1 - \dfrac{tan^2\delta}{tan^2\gamma}} \right. \end{aligned} \tag{1.21}$$

where β is the angle between the horizontal and the bed along flow direction, δ is the slope angle transversal to the flow direction and γ is the angle of repose of the sediment material.

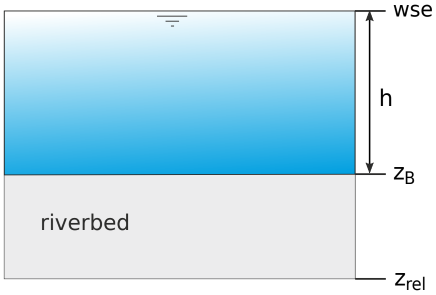

Figure 1.3 *Fixed bed concept and definition*

1.2.3.1.3 Fixed bed concept

Morphodynamic simulations generate deposition and erosion patterns of the riverbed. Erosion processes, if not limited, can proceed indefinitely in the vertical direction. This limit can be imposed by defining a non-erodible fixed bed elevation z_{rel}, below which the river bed is considered as *fixed*. This threshold also determines the amount of sediment available for transport (see Figure 1.3). The fixed bed elevation is defined relative to the initial bottom elevation z_B with $z_{rel} \leq 0$.

The accuracy of the fixed bed correction is guaranteed by defining the maximal overshoot below the fix bed elevation and the maximal number of iterations required for the correction.

1.2.3.2 Closure relations for Bed Load Transport

Let us now introduce the dimensionless bed load transport rate Φ also known as the Einstein bed load number, first introduced by Hans Albert Einstein in 1950, and given by

$$\Phi = \frac{q_B}{\sqrt{(s-1)gd^3}} \tag{1.22}$$

where $s = \rho_s/\rho$.

It is common practice to quantify bedload transport empirically relating Φ with either the Shields stress θ or the excess of the Shields stress θ above some appropriately defined "critical" Shields stress $(\theta - \theta_{cr}$. θ_{cr} is defined so as to fit experimenatl or field data and provide a threshold for which the bedload transport rate is too low to be of interest.

In what follows we describe the bedload transport formulas that are implemented to calculate the transport capacity $q_B = |\vec{q_B}|$ where $\vec{q_B} = (q_{B_x}, q_{B_y})$. The Shields parameter, takes the following form:

$$\theta = \frac{h\sqrt{S_{fx}^2 + S_{fy}^2}}{(s-1)} \tag{1.23}$$

and the specific bed load flux has the same direction as the water flow.

1.2.3.2.1 Meyer-Peter and Müller (MPM)

The bed load transport formula of Meyer-Peter and Müller (Meyer-Peter and Müller, 1948) reads as:

$$\Phi_B = \alpha(\theta - \theta_{cr})^m \tag{1.24}$$

Herein, α denotes the bed load coefficient, m the bed load exponent. In the original form of the formula $\alpha = 8$ and $m = 3/2$.

Meyer-Peter and Müller observed in their experiments that the first grains moved already for $\theta_{cr} = 0.03$. But as their experiments took place with steady conditions they used a value for which already 50% of the grains where moving. They proposed the value of $\theta_{cr} = 0.047$. The formula of Meyer-Peter and Müller is applicable in particular for coarse sand and gravel with grain diameters larger than 1 mm (Malcherek, 2001).

The bed load coefficient α, the exponent m and the critical Shields parameter θ_{cr} can be adapted by the user in the MPM-like formula.

1.2.3.2.2 Grass formula

The Grass formula (Grass, 1981) does not require the evaluation of the Shields stress:

$$\Phi_B = \alpha(\theta - \theta_{cr})^m \tag{1.25}$$

where $\alpha \in [0, 1]$ is a dimensional constant that encompasses the effects of grain size and kinematic viscosity and is usually determined from experimental data and m being chosen in the range $[1 - 4]$. The two-dimensional projection of (eq. 1.33) is obtained as follows:

$$q_{Bx} = \alpha \frac{q_x |\vec{q}|^{m-1}}{h^m} \qquad , \qquad q_{By} = \alpha \frac{q_y |\vec{q}|^{m-1}}{h^m} \ .$$

The coefficient α characterizes the interaction between the sediment and the fluid phase. The smallest α the weaker the interaction.

1.2.3.2.3 Engelund and Hansen

Engelund and Hansen (1972) proposed a transport formula for uniform bed material taking into account at the same time the presence of both bed- and suspended-load

This formula is commonly used as a bulk load formula and reads

$$\Phi_B = 0.05\sqrt{(s-1)g}\, c_f^2 \tag{1.26}$$

This formula formula does not consider the critical shear stress as threshold condition for incipient motion.

1.2.3.3 Correction of Bed Load Direction

The 2D projection of the solid discharge along x and y is obtained through standard procedures, that iare mostly based on empirical basis and which account for the downward effect of gravity on sediment particles due to local bed slope and the presence of spiral flow motion in curved riches.

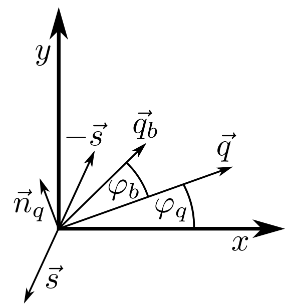

Figure 1.4 *Bed load transport deviation angle φ_b from the flow direction $\vec{q}$ due to the lateral bed slope $\vec{s}$ (Vonwiller, 2017)*

1.2.3.3.1 Lateral Bed Slope

Empirical bed load formulas were originally derived for situations where bed slope equals flow direction. However, in case of lateral bed slope with respect to flow direction, the bed load direction differs from the flow direction due to gravity acting on the bed material. Figure 1.4 illustrates the deviation of the bed load transport direction due to lateral bed slope in a Cartesian coordinate system.

The bed load direction is corrected for lateral bed slope based on the following approach (e.g. see Ikeda (1982) and Talmon et al. (1995)):

$$\tan\varphi_b = \left(\frac{-r}{\theta}\right)\vec{s}\cdot\ \vec{n}_q \quad \text{for} \quad \vec{s}\cdot\vec{n}_q < 0 \tag{1.27}$$

$$r = N_l \theta_{cr}^{1/2} \tag{1.28}$$

where φ_b = bed load direction with respect to the flow vector $\vec{q}$, N_l = lateral transport factor ($0.75 \leq N_l \leq 2.63$), $\vec{s} = \left(\frac{\partial z_B}{\partial x}, \frac{\partial z_B}{\partial y}\right)$ bed slope (positive uphill, negative downhill), $\vec{n}_q$ = unit vector perpendicular to $\vec{q}$ pointing in downhill direction ($\vec{s}\cdot\vec{n}_q < 0$) , θ = effective dimensionless shear stress and θ_{cr} = critical dimensionless shear stress of sediment.

The direction of the bed load transport under the influence of lateral bed slope is written as:

$$\frac{q_{B_y}}{q_{B_x}} = \tan(\varphi_b + \varphi_q) \tag{1.29}$$

1.2.3.3.2 Curvature Effect

Curvature in rivers may cause deviation of the bed load direction from the depth averaged flow direction. Due to three dimensional spiral flow motion, the bed load direction tends to point towards the inner side of the curve, while the flow direction points towards the outer side (Figure 1.5). This curvature effect is taken into account according to an approach proposed by Engelund (1974) , where the deviation angle φ_c of the bottom shear stress $\vec{\tau}_b$ (positive counterclockwise and vice versa) from the main flow direction is determined as

$$\tan\varphi_c = \frac{|\vec{\tau}_{bn}|}{|\vec{\tau}_{bs}|} = -N_* \frac{h}{R} \tag{1.30}$$

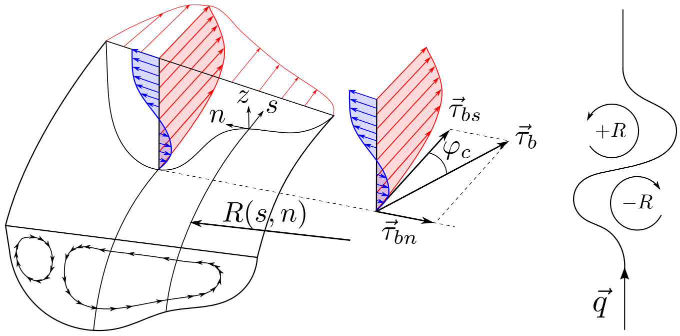

Figure 1.5 *Effect of spiral motion in river bend on bed shear stress $\vec{\tau}_b$ with deviation angle from main flow direction φ_c (Vonwiller, 2017)*

where $\vec{\tau}_{bn}$ and $\vec{\tau}_{bs}$ are the bed shear stress normal to and in the flow direction respectively, h denotes the water depth, N_* is a curvature factor, and R denotes the radius of the river bend (positive for curvature in counterclockwise direction and vice versa).

Note that the curvature factor N_* mainly depends on bed roughness. Therefore, $N_* \approx 7$ for natural streams (Engelund, 1974), and values up $N_* \approx 11$ for laboratory channels (Rozovskii, 1961).

1.2.4 Uniform Sediment Transport

1.2.4.1 Governing Equation

The conservation of bed material is ensured by the Exner equation (eq. 1.31), named after the Austrian sedimentologist Felix M. Exner (Exner, 1925). The Exner equation allows to describe the bed evolution due to erosion or deposition, which results in the elevation change of the actual bed level z_B:

$$(1-p)\frac{\partial z_B}{\partial t} + \frac{\partial q_{B_x}}{\partial x} + \frac{\partial q_{B_y}}{\partial y} - Sl_b = 0 \tag{1.31}$$

where p is the porosity, Sl_b is the source term per unit width specifying local input or output of sediment material (e.g. slope collapse or excavation) and $\vec{q}_B = \begin{pmatrix} q_{B_x} \\ q_{B_y} \end{pmatrix}$ is the specific bed load flux.

The Exner equation is solved in a decoupled way, meaning that the shallow water equations and the Exner equation are solved in sequence. This approach makes the assumption that the bed load flux is much slower than the water flow velocity (Soares-Frazão and Zech, 2011).

1.2.4.2 Closures for Bed Load Transport

The following section describes the bedload transport formulas that are implemented to calculate the transport capacity $q_b = |\vec{q}_b|$. The specific bed load flux has the same direction as the water flow. For practical purposes, the bed load transport formula can be calibrated by a pre-factor.

1.2.4.2.1 Meyer-Peter and Müller (MPM)

The bed load transport formula of Meyer-Peter and Müller (Meyer-Peter and Müller, 1948) is written as follows:

$$q_B = \alpha(\theta - \theta_{cr})^m \sqrt{(s-1)gd^3} \tag{1.32}$$

Herein, α denotes the bed load coefficient (originally $\alpha = 8$), m the bed load exponent (originally $m = 1.5$), q_B is the specific bed load transport rate, θ is the dimensionless bed shear stress (Shields parameter), θ_{cr} is the critical dimensionless bed shear stress, d is the grain diameter, $s = \rho_s/\rho$ and g stands for the gravitational acceleration. Meyer-Peter and Müller observed in their experiments that the first grains moved already for $\theta_{cr} = 0.03$. But as their experiments took place with steady conditions they used a value for which already 50% of the grains where moving. They proposed the value of $\theta_{cr} = 0.047$. The formula of Meyer-Peter and Müller is applicable in particular for coarse sand and gravel with grain diameters larger than 1 mm (Malcherek, 2001).

The bed load coefficient α, the exponent m and the critical Shields parameter θ_{cr} can be adapted by the user in the MPM-like formula.

1.2.4.2.2 Grass formula

The Grass model (Grass, 1981) proposes a simple bedload transport formula, where q_b is a function of the flow velocity and a dimensional constant α.

$$q_B = \alpha(u - u_c)^m \tag{1.33}$$

With u_c the critical velocity. The exponent m is usually set to m = 3. The threshold condition for incipient motion of grains is set to zero, meaning that the bedload transport and the fluid motion start simultaneously. The coefficient α characterizes the interaction between the bed and the fluid. If $\alpha = 0$, no sediment transport occurs. If $\alpha = 1$ the interaction between the bed and fluid is the largest.

1.2.4.2.3 Engelund and Hansen

Engelund and Hansen (1972) proposed a bedload transport formula for uniform bed material:

$$q_B = 0.05\sqrt{(s-1)g}\, c_f^2 \theta^{2.5} d^{1.5} \tag{1.34}$$

where d denotes the median sediment size of the bed material and θ the Shields parameter. The Engelund and Hansen formula for bed load transport does not consider the critical shear stress as threshold condition for incipient motion.

1.2.4.3 External Sources Terms

The source term Sl_b represents additional sediment mass input or output (sink) that occurs locally on the computational domain on a set of elements defined by regions. The source can be specified as total volume flux including porosity $[m^3/s]$. Different approaches are used to manage the behaviour of the external sources in case of a negative flux (sink):

- Exact: The specified sediment volume is added or extracted (non conservative)
- Available: The specified sediment volume to extract is limited by the defined fixed bed elevation of the elements (conservative)
- Infinity: All available sediment will be abstracted (conservative)

Addition of sediment always follows the “Exact” behaviour as there is no upper limit. The abstraction of sediment could also follow the “Exact” behaviour but the simulation might end abruptly if the available sediment volume is smaller than the volume abstracted. Therefore, the “Available” behaviour aims to avoid this situation. The “Infinity” behaviour abstracts all available sediment volume.

1.2.5 Boundary Conditions

After the specification of the *closure relations* for the sediment transport, the system of governing equations (eq. 1.1) and (eq. 1.31) can be solved within the modeling domain described in Figure 1.1, provided boundary conditions (morphologic boundary conditions) are specified at the domain boundary Γ. For the sediment transport only *external boundaries* that allow sediment flowing into or out of the domain can be specified. A morphologic boundary condition can ‘sit’ on a hydraulic boundary condition. In case no hydraulic boundary condition is specified, the boundary will behave as a wall and sediment transport will not occur.

1.2.5.1 Upstream boundary condition

The bed load input type is given by the upstream boundary condition. Three types of upstream boundary condition are available:

- Sediment discharge: based on a sediment hydrograph describing the bed load inflow as function of time (constant or variable). The bed load is defined as a volumetric flow rate $Q_b = \dfrac{\mu_s}{\rho_s}$ $[m^3/s]$, where μ_s is the sediment mass flow rate $[kg/s]$ and ρ_s the sediment density $[kg/m^3]$. Notice that the porosity is not considered in the bed load input and is specified separately as own parameter value.
- Transport capacity: the sediment inflow is defined by calculating the equilibrium transport capacity according to the hydraulic state at the boundary. The bed load is defined as a compact volumetric flow rate (without porosity) Q_b $[m^3/s]$.
- Equilibrium: this upstream boundary condition called IOup grants a constant bed load inflow. The same amount of sediment leaving the first computational cell in flow direction enters the cell from the upstream boundary. This leads to a constant bed elevation at the boundary condition.

For the sediment discharge and transport capacity boundary condition types, the specific sediment discharge q_b is distinguished by three weighting schemes:

1. Geometrical weighting with respect to the total nodestring length L_n.

$$q_b = \frac{Q_b}{L_n} \qquad \left[\frac{m^3}{s \cdot m}\right] \tag{1.35}$$

2. Wetted area weighting

$$q_b = \frac{Q_b}{A_{w,tot}} \cdot h \qquad \left[\frac{m^3}{s \cdot m^2}\right] \tag{1.36}$$

3. Conveyance weighting

$$q_b = \frac{Q_b}{K_{tot}} h \sqrt{c_f h} \qquad \left[\frac{m^3}{s \cdot m}\right] \tag{1.37}$$

with $K_{tot} = A_{w,tot}\sqrt{c_f h}$ the total conveyance and c_f the friction coefficient.

1.2.5.2 Downstream boundary condition

Two types of downstream boundary condition are available:

- Equilibrium: all sediment entering the last computational cell will leave the cell over the downstream boundary.
- Check-dam: the equilibrium downstream boundary condition is activated only if the bed level reaches a threshold value. Before reaching the threshold value, a wall type boundary is assumed.

2

Numerical Models

2.1 General View

The governing equations of hydro- and morphodynamics are conservation laws expressing conservation of mass and momentum. The aim of the numerical simulation is to solve these equations over the computational domain and for a given time. The computational domain is discretized by a computational mesh (Figure 2.1) consisting of elements (often having triangular shape) and conservation equations are applied on each domain element. In order to numerically solve the conservation equations, the mathematical model is approximated by numerical schemes, i.e. the numerical approximation consists of the spatial and temporal discretization of the conservation equations including an algorithm that solves the discretized equations.

The conservation equations can be formulated either in integral or differential form. Different numerical schemes exist to discretise the equations:

- Finite difference: The discrete values are considered as point values defined at mesh points
- Finite element: The discrete values are determined in terms of the nodal values of the mesh
- Finite volume: The discrete values are averaged over finites volumes of the mesh

In BASEMENT, the spatial discretisation of the domain is based on an unstructured mesh made of triangular elements. For the conservation equations, the spatial discretisation follows the finite volume scheme, while for the temporal discretisation an explicit first order Euler scheme is used. The numerical model processes the hydro- and morphodynamic equations in a decoupled way (Figure 2.2).

The discretization and the solution method for the hydro- and morphodynamic equations will be presented in the following sections.

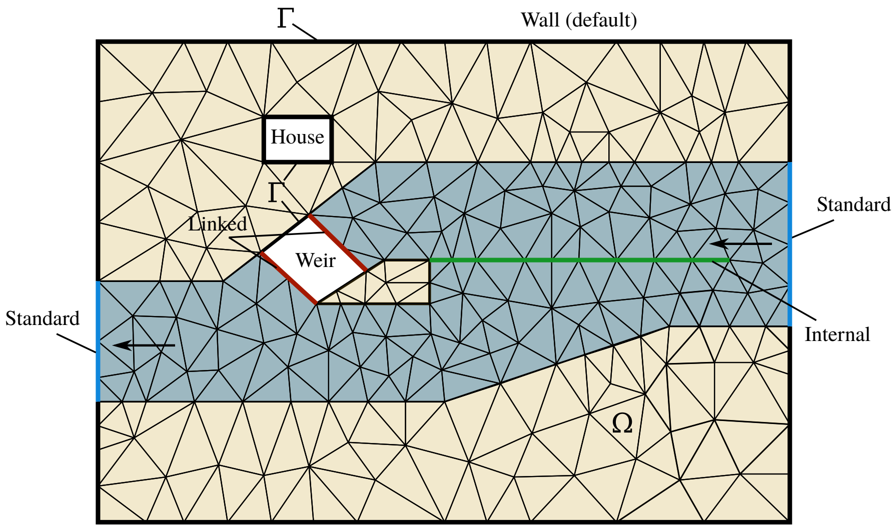

Figure 2.1 *Modeling domain, types of boundary conditions and computational mesh. The flow is from right to left and a side weir (green line) divides the channel into a lower and an upper channel through the weir.* External *boundary conditions must be provided at* Γ_1*,* Γ_2 *and* Γ_3 *while* internal *boundary conditions can be specified in any place within* Ω

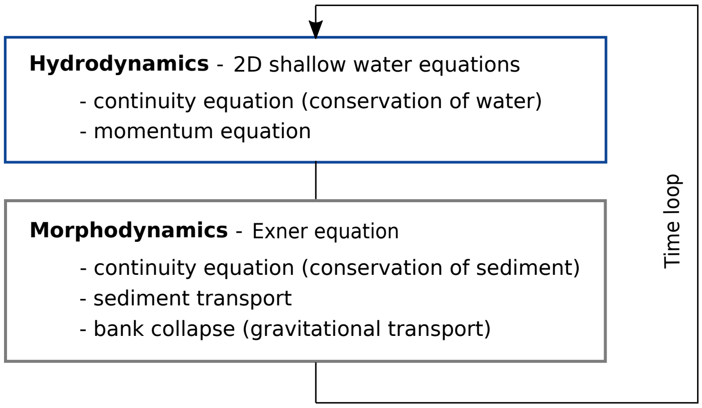

Figure 2.2 *Overview of the numerical model*

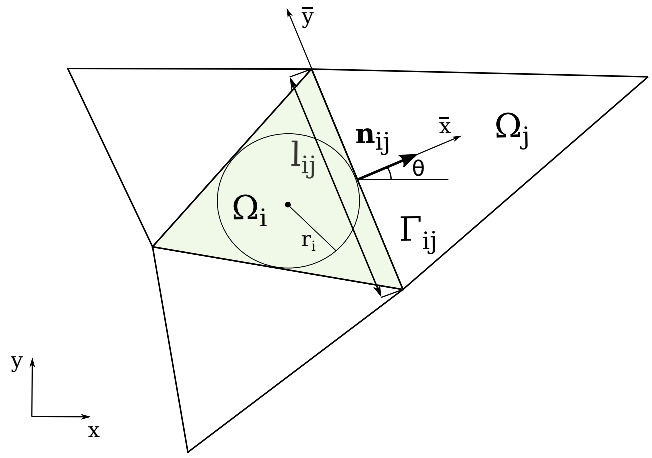

Figure 2.3 *Element (shaded triangle) of unstructured triangular mesh and used notation*

2.2 Discretization

The problem is discretised adopting a finite volume approach over unstructured triangular meshes. A conforming triangulation T_Ω of the computational domain $\Omega \subset \mathbb{R}^2$ by elements Ω_i such that $T_\Omega = \bigcup \Omega_i$, is assumed. Hereafter we will use the following notation: given a finite volume Ω_i, $j = 1, 2, 3$ is the set of indexes such that Ω_j is a neighbour of Ω_i; Γ_{ij} is the common edge of two neighbour cells Ω_i and Ω_j, and l_{ij} its length. $\mathbf{n}_{ij} = (n_{ij,x}, n_{ij,y})$ is the unit vector which is normal to the edge Γ_{ij} and points toward the cell Ω_j (see Figure 2.3). Data are represented by cell averages $\boldsymbol{U}_i^n$ and the numerical solution sought at time $t^{n+1} = t^n + \Delta t$, is denoted by $\boldsymbol{U}_i^{n+1}$.

2.3 Numerical solution of Hydrodynamics

2.3.1 Vectorial form of the governing equations

For numerical convenience, the system of governing equations (eq. 1.1) is rewritten in vectorial form in terms of the water surface elevation $H = h + z_B$. It now reads:

$$\frac{\partial \boldsymbol{U}}{\partial t} + \frac{\partial \boldsymbol{F}_x}{\partial x} + \frac{\partial \boldsymbol{F}_y}{\partial y} = \boldsymbol{S} \tag{2.1}$$

where the vector of unknowns is

$$\boldsymbol{U} = \begin{pmatrix} H \\ q_x \\ q_y \end{pmatrix} \tag{2.2}$$

the vector fluxes are

$$\boldsymbol{F}_x = \begin{pmatrix} q_x \\ uq_x + \frac{1}{2}g(H^2 - 2Hz_b) \\ uq_y \end{pmatrix} \quad ; \quad \boldsymbol{F}_y = \begin{pmatrix} q_y \\ vq_x \\ vq_y + \frac{1}{2}g(H^2 - 2Hz_b) \end{pmatrix} \tag{2.3}$$

and the vector of source terms is

$$\boldsymbol{S} = \begin{pmatrix} S_h \\ gHS_x \\ gHS_y \end{pmatrix} \ . \tag{2.4}$$

The motivation of using H instead of h lies in the fact that it is easier to develop numerical schemes which preserve depth positivity and satisfy the well-balanced property.

2.3.2 Spatial discretisation

In order to discretise the system of governing equations, the domain is meshed by a set of triangular elements. The spatial discretization of the conservation equations is carried out by the finite volume method, where the differential equations are integrated over the single elements, i.e. control volumes. The water surface elevation is defined at the element center and is equally distributed over the element.

By integrating the governing system of equations eq. 2.1 in the control volume $V = [\Omega_i] \times [t^n, t^{n+1}]$, we obtain

$$\boldsymbol{U}_i^{n+1} = \boldsymbol{U}_i^n - \frac{\Delta t}{|\Omega_i|} \sum_{j=1}^{3} l_{ij} \left[\boldsymbol{F}_{ij}\right] + \Delta t \boldsymbol{S}_i \ . \tag{2.5}$$

2.3.3 Flux estimation

2.3.3.1 Rotational invariance of the shallow water equations

The flux $\boldsymbol{F}_{ij}$ are evaluated taking advantage of the rotational invariance property of the shallow water equations. According to this property the two-dimensional homogeneous shallow water equations satisfy the following equality (Toro, 2009):

$$\boldsymbol{n}_{ij} \cdot [\boldsymbol{F}_x(\boldsymbol{U}), \boldsymbol{F}_y(\boldsymbol{U})] = \boldsymbol{T}^{-1}(\theta)\boldsymbol{F}_x[\boldsymbol{T}(\theta)\boldsymbol{U}] \tag{2.6}$$

where θ is the angle between the vector $\boldsymbol{n}_{ij}$ and x-axis, measured counter clockwise from the x-axis (see Figure 2.3) and

$$\boldsymbol{T}(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} \tag{2.7}$$

being

$\boldsymbol{T}^{-1}(\theta) =$ inverse of $\boldsymbol{T}(\theta)$.

2.3.3.2 Computation of the flux

The flux $\boldsymbol{F}_{ij}$ is obtained at every edge of the finite volume mesh, as the solution of the one-dimensional projected Riemann problem along the normal direction of the two conservation laws eq. 2.1. The computational steps can be summarized as follows:

- First, the vector of conserved variables $\boldsymbol{U}$ is transformed into the local coordinate system $(\bar{x}, \bar{y})$ (see Figure 2.3) at the edge with the operation $\boldsymbol{T}(\theta)\boldsymbol{U}$.
- A one-dimensional, local Riemann problem is formulated and solved in the normal direction of the edge. From this calculation the new flux vector over the edge $\boldsymbol{F}[\boldsymbol{T}(\theta)\boldsymbol{U}]$ is defined.
- The flux vector, formulated in the local coordinate system is transformed back to the global coordinates (Cartesian) with $\boldsymbol{T}^{-1}\boldsymbol{F}[\boldsymbol{T}(\theta)\boldsymbol{U}]$. The sum of the fluxes of all edges of an element gives the total fluxes in the x- and y directions.

The fluxes are calculated in the normal direction of the element edges. The normal direction of the edge is defined positive from element i (L) to element j regarding the edge direction.

2.3.3.3 The HLLC approximated Rieman solver

The HLLC approximate Riemann solver (Toro, 1994) is a modified HLL (Harten, Lax and van Leer) approximate Riemann solver that includes the shear wave.

The numerical flux at the cell interface is computed as follows:

$$\boldsymbol{F}_{ij}^{HLLC} = \begin{cases} \boldsymbol{F}_i & if \quad 0 \leq S_i, \\ \boldsymbol{F}_{*i} = \boldsymbol{F}_i + S_i(\boldsymbol{U}_{*L} - \boldsymbol{U}_i) & if \quad S_i \leq 0 \leq S_*, \\ \boldsymbol{F}_{*j} = \boldsymbol{F}_j + S_j(\boldsymbol{U}_{*R} - \boldsymbol{U}_j) & if \quad S_* \leq 0 \leq S_j, \\ \boldsymbol{F}_j & if \quad 0 \geq S_j. \end{cases} \tag{2.8}$$

The wave speed velocities are estimated as:

$$S_i = u_i - \sqrt{gh_i}\xi_i \; ; \; S_j = u_j + \sqrt{gh_j}\xi_j \tag{2.9}$$

where $\xi_{K=(i,j)}$ is defined as:

$$\xi_K = \begin{cases} \sqrt{\frac{1}{2}\left[\frac{(h_* + h_K)h_*}{h_K^2}\right]} & if \quad h_* > h_K, \\ 1 & if \quad h_* \leq h_K. \end{cases} \tag{2.10}$$

with h_*, an estimate for the exact solution of the water depth in the star region obtained using the depth positivity condition. It reads as

$$h_* = \frac{1}{2}(h_L + h_R) - \frac{1}{4}(u_R - u_L)(h_L - h_R)/(\sqrt{gh_L} + \sqrt{gh_R}) \tag{2.11}$$

In case of dry-bed conditions, the wave speeds are estimated as the exact dry front speed, i.e.:

$$S_i = \begin{cases} u_i - 2\sqrt{gh_i} & if \quad h_i = 0, \\ \text{usual estimate} & if \quad h_i > 0, \end{cases}$$
$$S_j = \begin{cases} u_j + 2\sqrt{gh_j} & if \quad h_j = 0, \\ \text{usual estimate} & if \quad h_j > 0. \end{cases} \tag{2.12}$$

And the middle estimated wave speed S_* corresponds to the front wave speed in case of dry-bed problem.

The expression of the states $\boldsymbol{U}_{*i}, \boldsymbol{U}_{*j}$ and the middle wave speed S_* can be found in the book of Toro (2009).

2.3.4 Numerical stability

Numerical stability is assured by choosing the time step Δt for time integration such that it obeys the Courant-Friedrichs-Lewy (CFL) condition. In 2-D the Courant number (CFL) can be defined as follows:

$$CFL = \frac{(\sqrt{u^2 + v^2} + c)\Delta t}{r_i} \tag{2.13}$$

where r_i is the radius of the inscribed circle that defines the element center (Figure 2.3), u, v are the corresponding velocities of the element and $c = \sqrt{gh}$. The HLLC scheme is stable for

$$0 < CFL \leq 1 \tag{2.14}$$

2.3.5 Discretisation of Source terms

2.3.5.1 Bed slope source term

The bed slope source term (eq. 1.2) is discretized using the robust modified-state approach proposed by Duran et al. (2013). The discretization presents a motionless steady states-preserving scheme:

$$\boldsymbol{S}_{b,i} = \sum_{j=1}^{m} l_{ij} \boldsymbol{S}_{b,ij} = \sum_{j=1}^{m} l_{ij} \begin{pmatrix} 0 \\ gH^*_{ij}(z_i - \bar{z}_{ij})\vec{\boldsymbol{n}}_{ij} \end{pmatrix} \tag{2.15}$$

where $\bar{z}_{ij} = \check{z}_{ij} - \Delta_{ij}$ with $\check{z}_{ij} = \max(z_{bi}, z_{bj})$ the maximum bed elevation between cells i and j and $\Delta_{ij} = \max(0, \check{z}_{ij} - H_i)$. H^*_{ij} is the approximated value of the water surface elevation H at the cell interface Γ_{ij}.

2.3.5.2 Friction source term

We handle the inhomogeneous character of system eq. 1.1 due to the presence of frictional source terms by adopting a robust splitting technique Toro (2001). We initially consider the initial value problem (IVP)

$$\left.\begin{array}{llll} PDE: & \mathcal{A}(\boldsymbol{U}) & = & \mathcal{S}(\boldsymbol{U}) \\ IC: & \boldsymbol{U}(x,y,0) & = & \boldsymbol{U}_i^n \end{array}\right\}\text{IVP}\ .$$

where $\mathcal{A}$ represents the advective operator

$$\mathcal{A}(\boldsymbol{U}) = \frac{\partial \boldsymbol{U}}{\partial t} + \frac{\partial \boldsymbol{F}_x}{\partial x} + \frac{\partial \boldsymbol{F}_y}{\partial y} = \mathbf{0}\ ,$$

and $\mathcal{S}$ represents the frictional source term operator.

The numerical solution is then obtained by subsequently integrating *two* initial value problems (IVPs):

$$\left.\begin{array}{llll} ODEs: & \frac{\mathrm{d}\boldsymbol{U}}{\mathrm{d}t} & = & \mathcal{S}(\mathbf{U}) \\ ICs: & \boldsymbol{U}(x,y,0) & = & \boldsymbol{U}_i^n \end{array}\right\} \stackrel{\Delta t}{\Longrightarrow} \overline{\boldsymbol{U}}_i \quad \text{IVP1}\ ,$$

$$\left.\begin{array}{llll} PDEs: & \mathcal{A}(\mathbf{U}) & = & 0 \\ ICs: & \boldsymbol{U}(x,y,0) & = & \overline{\boldsymbol{U}}_i \end{array}\right\} \stackrel{\Delta t}{\Longrightarrow} \boldsymbol{U}_i^{n+1} \quad \text{IVP2}\ ,$$

The initial condition (IC) for IVP1 is $\mathbf{U}_i^n$, corresponding to the initial condition of the full problem IVP. The solution of IVP1 is obtained solving a system of ordinary differential equations (ODEs) after integration by a time step Δt and is denoted by $\overline{\boldsymbol{U}}_i$. IVP2 is then integrated by a time step Δt, with initial condition given by the solution of IVP1 $\overline{\boldsymbol{U}}_i$. The solution of IVP2 $\boldsymbol{U}_i^{n+1}$ is obtained solving an hyperbolic homogeneous system of partial differential equations (PDEs) and represents the approximate solution of the full problem IVP.\ Since we adopt an implicit second-order Runge-Kutta method for solving the ODEs systems IVP1 and an explicit finite volume method for solving IVP2, the integration time step Δt is determined accordingly with the CFL stability condition for IVP2.

2.3.5.3 External Source Term

An external source is defined as specific mass flux δ (m/s), uniformly distributed over a number of elements of the domain with a specific surface area. The external source can either be specified as discharge (m^3/s) or precipitation intensity (mm/h) for a specific region of the domain. The external source value is divided among the cells composing the region and converted to cell specific mass flux δ_i. The volume allocated is characterized by different behaviors:

$$\begin{array}{llll} \text{Exact:} & S_{h,i} = \delta_i & & \\ \text{Available:} & S_{h,i} = \delta_i & \text{if} & \delta_i \cdot \Delta t > 0 \\ & S_{h,i} = \max(\delta_i, -h_i) & \text{if} & \delta_i \cdot \Delta t < 0 \\ \text{Infinity:} & S_{h,i} = \delta_i & \text{if} & \delta_i \cdot \Delta t > 0 \\ & S_{h,i} = -h_i & \text{if} & \delta_i \cdot \Delta t < 0 \end{array} \tag{2.16}$$

Where h_i is the water depth of the element i. The external source volume is added to the initial water volume.

$$h_i^{t+1} = h_i^t + S_{h,i} \cdot \Delta t \tag{2.17}$$

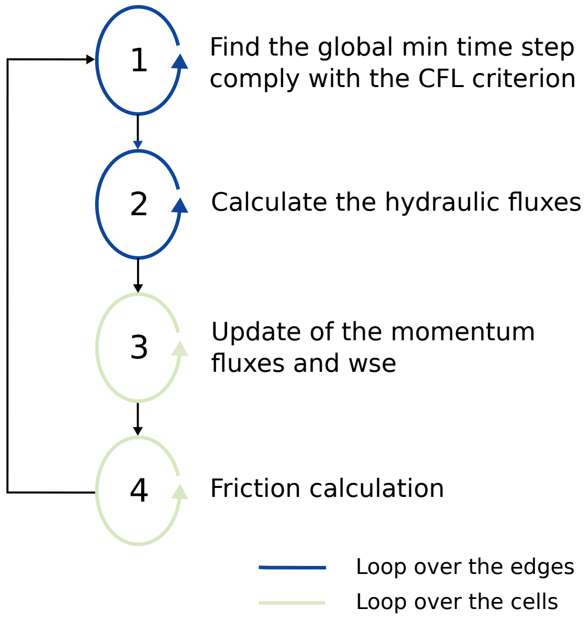

Figure 2.4 *Numerical solution procedure of hydrodynamic simulation for each time step* Δt

2.3.6 Solution procedure

The numerical solution procedure of BASEMENT explains how the discretised shallow water equation (eq. 1.1) is solved inside a defined time step Δt through a sequence of loops over the edges or cells (Figure 2.4).

First, a global minimum time step Δt should be defined. Then, the hydraulic fluxes (liquid mass, x-momentun and y-momentum) are calculated with a HLLC Riemann solver at the element edges according to the initial states of the left and right cells (Section 2.3.3). Subsequently, the hydraulic state variables i.e. cell centered quantities are updated and finally, the friction (source term) is calculated using an implicit scheme, thus looping twice over the cell.

2.4 Numerical solution of Morphodynamics

2.4.1 Numerical solution of the Exner equation

2.4.1.1 Fundamentals

The Exner equation assures that sediment mass is conserved in the bed and is used to model the riverbed time evolution. The rate of sediment transport is determined using a closure equation. The cell centered finite volume approach is used to discretise the Exner equation and in particular the HLL approximate Riemann solver with a wave speed estimator defined in Soares-Frazão and Zech (2011) is adopted. The shallow water and the Exner equations create a system of equations that is solved in a decoupled way (Figure 2.2).

2.4.1.2 Spatial discretization

In order to discretise the the Exner equation, we use the same unstructured mesh adopted for the hydrodynamic part and the same finite volume approach. As a consequence, the bed level z_B is defined at the element center and is equally distributed over the element.

By integrating the Exner equation in the control volume $V = [\Omega_i] \times [t^n, t^{n+1}]$, we obtain

$$z_{Bi}^{n+1} = z_{Bi}^n - \frac{\Delta t}{|\Omega_i|} \sum_{j=1}^{3} \left[q_{Bij \cdot l_{ij}} \right] + \Delta t \boldsymbol{S}_i \ . \tag{2.18}$$

The calculation of the sediment flux at the cell interface proceeds as follows:

1. loop over the cells and calculate:

    1. correction terms for the bed-load vector directions (if selected by the user), therefore:

    - calculation of the local bed slope, for the lateral-transport correction (see section Section 1.2.3.3.1)
    - calculation of the local curvature of the flow field, for the spiral flow correction (see section Section 1.2.3.3.2)

2. loop over the cell interfaces and:

    1. calculate the flux projection along the normal vector ($n_{ij,x}$,$n_{ij,y}$ of edge Γ_{ij}, i.e.: $q_{Bi,n} = q_{Bi,x} \cdot n_{ij,x} + q_{Bi,y} \cdot n_{ij,x}$ and $q_{Bj,n} = q_{Bj,x} \cdot n_{ij,x} + q_{Bj,y} \cdot n_{ij,x}$ with j=1,2,3
    2. compute the flux at the interface using the approximate HLL Riemann solver at the interface

    - Evaluate the wave speeds at the interface. this is obtained following the approach proposed by Soares-Frazão and Zech (2011), for which the wave speeds can be calculated as an approximation of the smallest eigenvalue of the system of governing equations, i.e. Shallow water and Exner. They read:

$$\lambda_1 = 1/2(u_n - c - \sqrt{(u_n - c)^2 + 4a_2c^2}) \tag{2.19}$$

$$\lambda_2 = 1/2(u_n - c + \sqrt{(u_n - c)^2 + 4a_2c^2}) \tag{2.20}$$

where $u_n = u \cdot n_{ij,x} + v \cdot n_{ij,y}$, $c = \sqrt{gh}$ and $a_2 = \dfrac{\partial q_{b,n}}{\partial q_n}$ which is the derivative of the bed load discharge in the normal flow direction with respect to the hydraulic flux direction. Then the speeds estimate are

$$S^- = \min(\lambda_{1,L}, \lambda_{1,R}) \tag{2.21}$$

and

$$S^+ = \max(\lambda_{2,L}, \lambda_{2,R}) \tag{2.22}$$

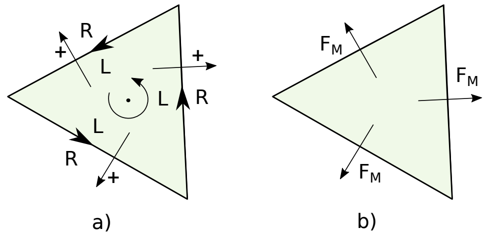

Figure 2.5 *a) Sign convention for the edge direction: counterclockwise b) Positive morphological flux direction at edges: from left (L) to right (R)*

- Flux calculation:

$$q_{Bij}^{HLL} = \begin{cases} q_{Bi,n} & if \quad S^- \geq 0, \\ \dfrac{q_{Bi,n}S^+ - q_{Bj,n}S^- + S^-S^+\left(z_{Bj} - z_{Bi}\right)}{S^+ - S^-} & if \quad S^- < 0 < S^+, \\ q_{Bj,n} & if \quad S^+ \leq 0. \end{cases}$$

The convention for the positive bed load flux direction is the same as for the hydrodynamic flux and is presented on Figure 2.5

2.4.1.3 Discretization of External Source Term

The source term Sl_b describes a local input or removal of sediment mass into a river.

An external source is defined as specific mass flux δ (m/s), uniformly distributed over a number of elements of the domain (region) with a specific surface area. The external source can be specified as the total volume flux (m^3/s) for a specific region of the domain. The external source value is divided among the cells composing the region and converted to cell specific mass flux δ_i. The volume allocated is characterized by different behaviors:

$$\begin{array}{lll} \text{Exact:} & S_{b,i} = \delta_i & \\ \text{Available:} & S_{b,i} = \delta_i & \text{if} \quad \delta_i \cdot \Delta t > 0 \\ & S_{b,i} = \max(\delta_i, -(z_{Fix} - z_i) & \text{if} \quad \delta_i \cdot \Delta t < 0 \\ \text{Infinity:} & S_{b,i} = \delta_i & \text{if} \quad \delta_i \cdot \Delta t > 0 \\ & S_{b,i} = -(z_{Fix} - z_i) & \text{if} \quad \delta_i \cdot \Delta t < 0 \end{array} \tag{2.23}$$

Where z_i is the bottom elevation and z_{Fix} the fixed bed elevation of the element i. The external source volume is added to the initial bottom elevation of element i.

$$z_i^{t+1} = z_i^t + S_{b,i} \cdot \Delta t \tag{2.24}$$

2.4.2 Solution procedure

The numerical solution procedure of BASEMENT explains how the discretised Exner equation (eq. 1.31) is solved through a sequence of loops over the edges or cells (Figure 2.6).

In the numerical simulation, the hydrodynamic and morphodynamic simulations are performed in a decoupled way. The morphodynamic simulation is executed after the

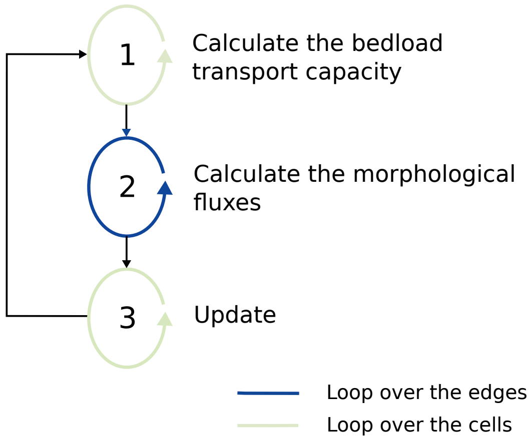

Figure 2.6 *Numerical solution procedure of morphodynamic simulation for each time step* Δt

hydrodynamic simulation, using the hydraulic fluxes to calculate the morphological fluxes. This approach assumes that the sediment transport is much slower than the water velocity, which is an accurate assumption for the numerical modelling of slow flood with morphological changes occurring over a long period (Soares-Frazão and Zech, 2011). The numerical solution procedure of Figure 2.6 is performed after the step 4 of Figure 2.4 inside the same time step Δt.

The numerical solution of the Exner equation starts with a loop over the cells in order to find the bedload transport capacity q_b with a potential correction due to a curvature effect or lateral bed slope. Then, the morphological fluxes F_M are calculated at the element edges and finally, the bed elevation z_b is updated over the cells.

3

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APPENDIX

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1

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the interface presents a list of user commands or options, such as a
menu, a prominent item in the list meets this criterion.

1. Source Code.

The "source code" for a work means the preferred form of the work
for making modifications to it. "Object code" means any non-source
form of a work.

A "Standard Interface" means an interface that either is an official
standard defined by a recognized standards body, or, in the case of
interfaces specified for a particular programming language, one that
is widely used among developers working in that language.

The "System Libraries" of an executable work include anything, other
than the work as a whole, that (a) is included in the normal form of
packaging a Major Component, but which is not part of that Major
Component, and (b) serves only to enable use of the work with that
Major Component, or to implement a Standard Interface for which an
implementation is available to the public in source code form. A
"Major Component", in this context, means a major essential component
(kernel, window system, and so on) of the specific operating system
(if any) on which the executable work runs, or a compiler used to
produce the work, or an object code interpreter used to run it.

The "Corresponding Source" for a work in object code form means all
the source code needed to generate, install, and (for an executable
work) run the object code and to modify the work, including scripts to
control those activities. However, it does not include the work's
System Libraries, or general-purpose tools or generally available free
programs which are used unmodified in performing those activities but
which are not part of the work. For example, Corresponding Source
includes interface definition files associated with source files for

the work, and the source code for shared libraries and dynamically linked subprograms that the work is specifically designed to require, such as by intimate data communication or control flow between those subprograms and other parts of the work.

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6. Conveying Non-Source Forms.

You may convey a covered work in object code form under the terms
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in one of these ways:

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  Corresponding Source fixed on a durable physical medium
  customarily used for software interchange.

  b) Convey the object code in, or embodied in, a physical product
  (including a physical distribution medium), accompanied by a
  written offer, valid for at least three years and valid for as
  long as you offer spare parts or customer support for that product
  model, to give anyone who possesses the object code either (1) a
  copy of the Corresponding Source for all the software in the
  product that is covered by this License, on a durable physical
  medium customarily used for software interchange, for a price no
  more than your reasonable cost of physically performing this
  conveying of source, or (2) access to copy the
  Corresponding Source from a network server at no charge.

  c) Convey individual copies of the object code with a copy of the
  written offer to provide the Corresponding Source.  This
  alternative is allowed only occasionally and noncommercially, and
  only if you received the object code with such an offer, in accord
  with subsection 6b.

  d) Convey the object code by offering access from a designated
  place (gratis or for a charge), and offer equivalent access to the
  Corresponding Source in the same way through the same place at no
  further charge.  You need not require recipients to copy the
  Corresponding Source along with the object code.  If the place to
  copy the object code is a network server, the Corresponding Source
  may be on a different server (operated by you or a third party)
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  that supports equivalent copying facilities, provided you maintain
  clear directions next to the object code saying where to find the
  Corresponding Source.  Regardless of what server hosts the
  Corresponding Source, you remain obligated to ensure that it is
  available for as long as needed to satisfy these requirements.

  e) Convey the object code using peer-to-peer transmission, provided
  you inform other peers where the object code and Corresponding
  Source of the work are being offered to the general public at no
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from the Corresponding Source as a System Library, need not be
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GCC Runtime Library Exception

Version 3.1, 31 March 2009

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