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Power System Models, Objectives and Constraints in Optimal Power Flow Calculations

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1 Introduction

The electric power system is defined as the system with the goal to generate and to transmit the electric power user via the transmission system to the electric power end users. In order to achieve a practically useful power system engineers have come up with principal components and characteristics of the power system: These components are pieces of hardware like high voltage transmission lines, underground cables, high voltage transformers, nuclear, hydro or hydro-thermal power stations, compensators, etc.

Electric power can be provided in direct or alternating current form, however, in the beginning of the 20th century alternating current has been determined as the more economical and technically feasible solution. An electric power system where the sources are alternating currents is principally of a dynamic nature, i.e. all current and dependent quantities like voltage and electric power itself are varying over time. A physical law exists among these three quantities:

$$Power(t) = Voltage(t) * Current(t) \quad (1)$$

In (1) the variable (t) refers to time.

From a control point of view it is the electric power consumption of end users which is hard to control: Today in most civilized countries the end user consumes the electric power with almost no restrictions. Fortunately, due to a regular daily behavior of the businesses and households electric power consumption follows more or less regular patterns. Due to the regular behavior it is possible to use some kind of prediction method to get good approximative values for power consumption at any future time.

Electric power is of such a nature that whatever electric power is needed must be generated by generators at any time t. This generated power includes the power losses in the resistances of the electrical transmission network.

Thus the approximate total sum of generated power is always defined for any given total end user power consumption. Since the total generation can be produced by a high number of generators, a power system could physically be operated in a huge number of states. However, many of them are not tolerable due to physical or operational constraints. Examples are extreme material usage due to e.g. excessive power flow through a cable, excessive voltage at an electrical busbar, etc.

Quality standards have quickly been identified and standards have been formulated which are valid world-wide with minor variations. These standards say that the power system operation must be **safe, reliable and economical**. Safety relates mainly to life threatening aspects for people involved in the usage of the electric power. Reliability standards determine the total time per year during which an electric power end user can be without electric power. Today, in the western world, the standard for electric power end user availability at any time during the year is very high at almost 100%. Since the generation and transport of power is very costly and since this high cost must mainly be paid by the electric power consumers the importance of economical aspects related to all aspects of power system operation are obvious.

This very complex dynamic behavior, the wide geographical distribution of the power system and also the high cost and economical risk of planning, building and maintaining power system components and to achieve the above mentioned power system goals have led to a mostly regional organization of the power system. The power system apparatus and power system operation for these regional areas ('the power system control areas') are coordinated by electric utilities which are mostly privately held or mixed privately held / state owned companies in the western countries.

With the size of the interconnected power systems getting larger and larger in the early 50's manual ad hoc operation of the power system even with local control mechanisms has become harder and harder and high level, often hierarchical control systems have been introduced into the power system to have an automatic or semi-automatic operation of parts of the power system satisfying new standards.

Since almost all major power system control areas are interconnected with mainly high voltage transmission lines operating rule standards have been established in Western Europe (UCPTE) and also in the U.S.A. around 1960. The standards are mainly related to the quality of the power system frequency (Europe: 50 Hz, U.S.A.: 60 Hz) and also to the control of power or energy contract based inter-utility transfers.

This automatic high level control mechanism is called Load Frequency Control (LFC) and has quickly been established as a standard control mechanism to hold both the frequency and also the inter-utility power transfers within given tolerances. LFC establishes some standards for the cooperation of even foreign electric utilities but still allows each utility to operate and maintain its power system part autonomously.

The LFC concept, respecting autonomous power system area control is accepted today by all participating utilities. It is within this framework where the individual utility must satisfy the goals of the power system: Safe, reliable and economic operation.

The above keyword 'safety' meaning, has been discussed before. 'Reliability' and 'Econonomy' can be realized or enforced by computer controlled operation. It is obvious that the result of some optimization could bring the utility nearer to achieve the above goals. The Optimal Power Flow (OPF) is one very important computer tool to help the operator achieve this reliable and economic power system operation for which the utility has autonomous responsibility. However, the OPF is only one of several tools within a real-time based hierarchical computer assisted control system as will be briefly discussed in the next section.

Much literature can be found about the characteristics and modelling of the power system. A good overview of the data acquisition and the generation control system is given in [1] and [2]. These two papers give the reader a brief overview of how real-time data is transferred from the power system to the computer where it is both displayed and used as input for many kinds of algorithms.

2 The role of the optimal power flow (OPF) computation within the overall power system control

Due to the complexity and very high degree of freedom of power system operation hierarchical models have been established which are valid for different time frames. Usually, the output of the higher level model based computations can be taken as input for the next lower level model, etc.

Within each model type different optimization problems can be formulated. The distinction from model type to model type is both the time frame for which the models are valid and also the resulting solution algorithms.

By decoupling the power system into hierarchical models for different time frames global mathematical optimality is lost. However, practical experience has shown that the chosen time based hierarchical characterization leads to quite a practicable approach which is probably not too far from a global optimum.

From an optimization point of view two different steady state power system models exist:
Category 1: In this category the models are derived in such a way to be applicable to power system simulation from approx. 24 hours to 10 years from actual time. Algorithms based on category 1 models are executed in a real-time environment at a rate of about one hour or slower. Within this time frame the goal is to determine the least expensive subset of power generators for each discrete time step based on uncertain and predicted total system load determined by some method which constitutes the only hard equality constraint per discrete time step. Many other inequality type constraints for the individual generation units are incorporated. The objective function is usually to minimize the sum of the cost of all generators for all discrete time steps.

The main mathematical solution problem comes from the mixed discrete, continuous variable problem formulation and also from the fact that inequality constraints for variable changes from time step at time t to those from time step $t + \Delta t$ exist.

The problem area of category 1 is often called Optimal Unit Commitment (UC). Different problem statements, mainly depending on the chosen time horizon and the discrete time steps are possible, however, all have in common, that the power transmission system, i.e. the transmission lines, the transformers, etc. are **not modelled**. In summary, all computations within this category 1 have in common

- that the total end user power consumption is predicted by some method and is given from now until some given future time in discrete intervals.
- that the power system active power transmission losses for each network state in the future are zero or simply predicted by some heuristic approaches and
- that no power transmission based power system limits are violated.

Category 2: Model validity: 5 minutes to 1 hour from actual time into the future. All dynamic quantities are assumed to be in sinusoidal wave form with constant frequency and constant wave peak amplitude (a network state called 'steady state') which allows the application of complex numbers to the solution of the problem. The goal is to determine the optimal scheduled values or control set points for a set of generators and other non-generator controls such that the power system with dynamically varying end user power consumption is continuously operated in or towards a reliable and economic network state.

The main algorithmic problem is given by the need to incorporate the model of the power transmission system with elements like transmission lines, underground cables, transformers, shunts and associated operational constraints. Also, simple models for the generators and loads must be used. This is in contrast to the models used in category 1 where the generation and load are the only power system elements and variables respectively.

The aspect of load behavior uncertainty in algorithms of this category 2 is much smaller than in the algorithms of category 1, due to the relatively short period in the future for which the load must be predicted. Today, due to the complexity given by the need to model the operational constraints of the power transmission system and also due to the additional complexity of the immediate application of the optimization result as control means, many algorithms of this category 2 neglect the varying load behavior at all. In order to compensate for this, a different, less accurate power system model which incorporates the changing load behavior is used in some utilities and applied in real-time more often, e.g. at intervals of 1 minute or even faster.

The OPF which is the main theme of this paper, today must be put within category 2. It is the function in an Energy Management System that schedules the power system controls in some optimal way being at the same time constrained by the power flow network model and power system operating limits.

The modelling challenges are mainly coming from two problem areas:

- Due to the necessity to model the transmission system and its operating constraints for sinusoidal waves with constant frequency and amplitude (which means usage of complex variables) the dimension of the mathematical formulation is enormous. This and the non-linearity of the underlying power system model results in an optimization problem statement whose solution is not at all straightforward.
- The fact that the optimization output should be applicable to the controllers of the power system in real-time mandates very fast and robust algorithms solving a problem based on models which represent the real power system behavior as closely as possible.

In this paper an OPF problem statement is derived independent on the actual computer solution and the fact that the result must be transferred in real-time to the controllers. Thus emphasis is given to the facts that the OPF formulation

- is derived in a form to get a model of very high quality of the power system for a steady state power system;
- is precise in mathematical terms;
- satisfies most constraints given from an operator controlled real-time application of the OPF output.

This is in constrast to the actual solution processes for the OPF problem formulation which are not presented in this paper. However, it must always be kept in mind, that the solution process itself can lead to necessary simplifications of the original problem formulation. Also it is very hard if not impossible to prove that a closed form mathematical solution process with an optimal solution exists for the given OPF problem formulation. It is known that at least for certain defined subproblems given in the following sections, optimal solutions and clear, straighforward solution algorithms exist (see paper by H. Glavitsch).

During the past 20 years much literature has been written about OPF problem formulations and related solutions algorithms. In the appendix literature is cited which is mainly related to the modelling of the power system in connection with the OPF (see [3] ... [7]). Most of these papers have detailed literature references which are not repeated in this paper.

3 The power flow model as equality constraint set of the OPF

3.1 Basic model assumptions

3.1.1 Power system loads

The OPF is based on a power system model for category 2, see the preceding section. The end user power consumption is varying permanently over time and this has to be (or should be) considered in any power system model, especially when applying the model based algorithmic output back into the power system via control mechanisms. As mentioned in the preceding section the validity for the OPF model can be found in the 5 minute to approx. 1 hour time frame. The change of load within this time frame can result in different power system states at time t and time t+5 Min., t+15 Min. or t+1 hour which again could lead to quite different optimal controller settings. Two possibilities exist to handle this situation, knowing that the OPF is executed only once during the chosen time interval:

- Either the utility moves the controller setpoints only once at every discrete time interval, i.e. after the OPF algorithm output is obtained, or
- an additional mechanism with the incorporation of measured or accurately predicted end user power consumption is used to adapt or recompute the optimal controller settings at faster intervals than the main OPF execution rates.

The first approach leads to a non-optimal or only near-optimal network operation. The second approach seems to be better from an optimal operation point of view, however, a more complicated, probably hierarchical algorithmic usage is needed.

In conclusion electric end user power consumption is discretized at pretermined intervals. The individual end user power consumption at a discrete time step is called a **load** in OPF models and computations.

3.1.2 Power system model: Steady state and symmetric power system operation

All electric quantities power system like current, power and voltages are quantities varying over time. For the power system model it is assumed that

- All currents, powers and voltages are quantities with sinusoidal wave form with constant amplitude. This behavior is called steady state power system operation and leads to the very important fact that the power system states can be modelled with complex variables. The complex variables for voltages, currents and powers are transformations of the corresponding steady state power system quantities.
- The power flow model of the three-phase power system assumes so-called (phase-) symmetric power system operation. Without going into details this assumption allows the modelling of the power transmission system with electric two-ports.

Note that both above assumptions are only models of the real power system. Thus model errors (errors with respect to the real-world power system) are introduced. However, practical experience shows that this assumption is valid for a wide variety of cases within category 2.

3.1.3 Power system model: Generation, load and the transmission system

The main components of the power system which must be modelled under the assumption of known or predicted loads at geographical locations and under steady state and symmetric power system operation are the following:

Overhead transmission lines, underground cables, transformers, shunt elements. Each of these passive power system transmission elements is modelled as a two-port mathematical element, situated between electrical nodes i and j (shunt elements are only associated with one electrical node i). Thus the power system model is composed of many passive elements placed between nodes i and j ($i \neq j$) yielding a network of branch elements. The special properties of this passive network are summarized as follows:

An **electrical node** (called from now on 'node') is connected via passive elements only to about 2 or 3 other nodes (on the average). The resulting connectivity matrix is very sparse: Matrix element (i,j) is non-zero only if there is a connection between i and j.

Since the power system is divided into power system control areas, each operated by an electric utility, each utility can model its own control area with high accuracy.

Often the individual utilities split the model of their control area into submodels to reduce the size and the complexity of the power system model. This splitting is possible especially in the lower high voltage levels (e.g. below 110 kV or 60 kV), due to the organization of the electric transmission system: Often the lower voltage network parts are only connected at one point via a transformer to the highest voltage level network parts. Thus a split into parts at these transformers is possible and does not lead to very high modelling inaccuracies.

At the highest voltage level, however, other circumstances exist: Here, power system control areas of different utilities are interconnected. The modelling problem is obvious: Due to the high degree of connectivity of highest voltage networks, each individual utility should also model at least parts of the highest voltage levels and associated network elements of neighboring utilities. However, especially in the network parts representing the highest voltage levels, ownership and mostly profit-based economic reasons often prevent one utility to know the exact data of the network components of the neighboring utilities.

Thus assumptions about passive network elements of mainly the highest voltage levels of the neighboring or even more distant electric utilities must be made by each individual utility. It must be emphasized that modelling only the data of the highest voltage levels of the own control area usually leads to very inaccurate simulation results which might not be useful in practice. This modelling problem has been recognized by the utilities and data exchange for the most sensitive power system elements and their status (on-off) has been established. Thus for the power system model of the transmission network operated by a utility, it can be assumed that the model comprises at least parts of the neighboring utilities. Some theoretical approaches which determine some criteria for the neecessary size of neighboring areas exist today, although not in perfect form. Often they are based on heuristic assumptions.

For power system models (category 2) it must be assumed that the chosen network comprises an expanded geographical area and that the most important power system elements of the own area and neighboring areas are modelled with high accuracy. In this text, the term 'network' refers to this expanded power system network, at least for highest voltage levels networks.

Depending on the size of the utility and the data-exchange related cooperation between neighboring utilities, networks can have varying sizes, starting at about 100 nodes up to 5000 or more nodes with about 150 to more than 10000 passive network elements, each modelled by two-ports, derived in the next subsection.

Generators: They are modelled individually, i.e. at their geographical location at a node of the electrical network. For OPF models it can be assumed that it is known which ones of the individual, geographically distributed generators are 'on' and which ones are 'off'. Only those generators which have a status of 'on' can deliver power into the network and are important in the category 2 models. Certain new modelling aspects are introduced per generator as compared to the category 1 model, such as constraints on the upper and lower reactive generator power (to be defined later in this paper) and upper and lower voltage magnitude levels.

Loads: It is assumed that loads are modelled individually at their geographical location at a node of the modelled network. If passive elements for subareas of the power system control area are not modelled or if they are split apart to be treated in separate models (discussion see above), the individual loads of such an area are assumed to be collected together at one precisely known node. Also, it is assumed that the power values for all the loads are known, either because they are measured precisely or because they are predicted by some method.

DC-Lines: DC-Lines and associated control equipment represent important power system elements in certain power system control areas. A detailed model description is not given in this paper. However, a simple model for a DC line with given MW flow at each end is a generator with given MW. The generator model for OPF computations is described in detail in this paper.

3.2 Power flow: Mathematical model of the power system for steady state simulation

3.2.1 Passive power system elements

Branches

Branches are passive network elements which can all be modelled by the same type of two-port equation, given below. Branch elements always refer to two different nodes i and j:

For each branch-element a relationship between the current, the voltage and the line parameters, all in complex quantities, exists:

$$\left[\begin{array}{c} \underline{I}_{el-i-j_i} \\ \underline{I}_{el-i-j_j} \end{array}\right] = \left[\begin{array}{cc} \underline{y}_{ii}^{el-i-j} & \underline{y}_{ij}^{el-i-j} \\ \underline{y}_{ji}^{el-i-j} & \underline{y}_{jj}^{el-i-j} \end{array}\right] \left[\begin{array}{c} \underline{V}_i \\ \underline{V}_j \end{array}\right] \tag{2}$$

(2) is the two-port equation for a branch in the model for category 2. The meaning of the variables is summarized in the appendix of this paper.

Branch elements can be subdivided into two major subcategories: **Lines and transformers**:

Lines can be categorized into two main classes:

- Overhead transmission lines and
- underground cables.

The four complex matrix terms of (2) for a line are computed with the line parameters (variables with capital Y) as follows:

$$\begin{aligned} \underline{y}_{ii}^{el-i-j:Line} &= \underline{Y}_{iio}^{el-i-j} + \underline{Y}_{ij}^{el-i-j} \\ \underline{y}_{ij}^{el-i-j:Line} &= -\underline{Y}_{ij}^{el-i-j} \\ \underline{y}_{ji}^{el-i-j:Line} &= -\underline{Y}_{ij}^{el-i-j} \\ \underline{y}_{jj}^{el-i-j:Line} &= \underline{Y}_{jjo}^{el-i-j} + \underline{Y}_{ij}^{el-i-j} \end{aligned} \tag{3}$$

Transformers are passive network elements which allow the transformation of one voltage to another (or some even to two or even three other voltage levels). Transformers represent a (rather small) subset of the branch-elements defined with (2).

The four complex matrix terms of (2) for a transformer are computed as follows:

$$\begin{aligned} \underline{y}_{ii}^{el-i-j:Trafo} &= \underline{Y}_{iio}^{el-i-j} + \underline{Y}_{ij}^{el-i-j} \\ \underline{y}_{ij}^{el-i-j:Trafo} &= -\underline{t}^{el-i-j}\underline{Y}_{ij}^{el-i-j} \\ \underline{y}_{ji}^{el-i-j:Trafo} &= -(\underline{t}^{el-i-j})^{*}\underline{Y}_{ij}^{el-i-j} \\ \underline{y}_{jj}^{el-i-j:Trafo} &= (|\underline{t}^{el-i-j}|)^{2}(\underline{Y}_{jjo}^{el-i-j} + \underline{Y}_{ij}^{el-i-j}) \end{aligned} \tag{4}$$

The elements denoted by capital Y in (3) and (4) can be assumed to be numerically known at precise values for each line or transformer from node i to another node j (although this is not perfectly true due to data uncertainties in the range of plus or minus 10 %). This data uncertainty comes from the fact that the variables are dependent on many effects which are hard to quantify precisely. For example, a complex geometry of the transmission line tower, earth resistance dependency on ground condition which again is weather dependent, geometric construction of the transformer, etc.

The complex transformer-related variable $\underline{t}^{el-i-j}$ (t: tap) represents the measure for varying the transformer two-port parameters which in the power system allows to have a variable voltage relationship of side i to side j of the transformer.

In practice, $\underline{t}$ represents a discrete control of the power system. However, in OPF algorithms, $\underline{t}$ is taken as a continuous variable within upper and lower bounds and is set to the next practically possible discrete step after the optimization.

Shunts

Shunts are passive network elements which can all be modelled by the same type of equation, given below. Shunt-elements always refer two one node i:

For each shunt-element a relationship between the current, the voltage and the shunt parameters, all in complex quantities, exists:

$$\left[\ \underline{I}_{el-i-o}\ \right] = \left[\ \underline{y}_{ii}^{el-i-o}\ \right]\left[\ \underline{V}_{i}\ \right] \tag{5}$$

(5) is the equation for a shunt in the model for category 2.

The complex network element of (5) for a shunt is computed as follows:

$$\underline{y}_{ii}^{el-i-o:Shunt} = s^{el-i-o}\underline{Y}_{iio}^{el-i-o} \tag{6}$$

The element with capital Y in (6) can be assumed to be numerically known at precise values for each shunt at node i. However, also here, data uncertainties exist for the same reasons as given for branch elements.

This data uncertainty, however, is **not considered** in most OPF models. Thus all variables written with capital Y can be assumed to be precisely known.

The real variable s^{el-i-o} represents the measure for varying the shunt admittance. Practically this must be seen as a measure indicating how many individual shunts of a shunt bank at a node i must be switched in and how many are in an 'out'-status. Thus, in practice, s represents a discrete control of the power system. However, in OPF algorithms, s is taken as a continuous variable within upper and lower bounds and is set to the next practically possible discrete step only after the optimization.

3.2.2 Kirchhoff-law: All currents at a node must add up to zero

A fundamental law - one of the Kirchhoff laws - says that all currents flowing into an electrical node must sum up to exactly zero. This together with conventions on the direction of currents is represented in Fig. 1.

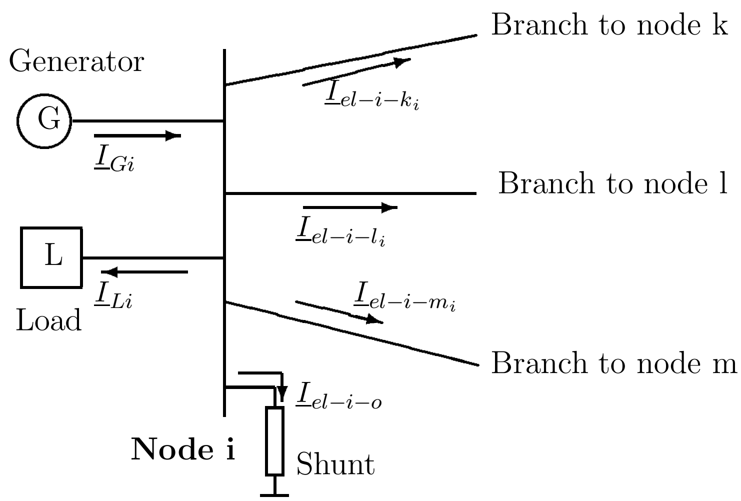


Abbildung 1: Currents at a node i: Conventions

Note the conventions for the generator currents $\underline{I}_{Gi}$, going into the node, the load currents $\underline{I}_{Li}$, the shunt currents $\underline{I}_{el-i-o}$ and the branch currents for a branch between nodes i and j computed at node i $\underline{I}_{el-i-j_i}$ leaving the node i.

For these currents the Kirchhoff Law is as follows:

$$\underline{I}_{Gi} - \underline{I}_{Li} - \sum_{j=1}^{N} \underline{I}_{el-i-j_i} - \underline{I}_{el-i-o} = 0 \;\; ; \text{i} = 1 \text{ ... N} \tag{7}$$

In both (7) and Fig. 1 and also throughout the rest of this paper the following assumptions are made:

- N represents the total number of electrical nodes

- All generators at a node i are summarized in one generator ($\underline{I}_{Gi}$).
- All loads at a node i are summarized in one load ($\underline{I}_{Li}$).
- All shunts at a node i are summarized in one shunt ($\underline{I}_{el-i-o}$).
- All parallel branches from node i to node j are summarized into 1 branch element with one two-port equation ($\underline{I}_{el-i-j_i}$ and $\underline{I}_{el-i-j_j}$).
- If there is no generator at a node i then $\underline{I}_{Gi} = 0$.
- If there is no load at a node i then $\underline{I}_{Li} = 0$.
- If there is no shunt at a node i then $\underline{I}_{el-i-o} = 0$.
- If there is no branch-type connection between nodes i and j then $\underline{I}_{el-i-j_i} = 0$ and $\underline{I}_{el-i-j_j} = 0$.

With (2), (5) and (7) a mathematical relationship between the nodal voltage related variables $\underline{V}$ (node-related) and the element related current variable $\underline{I}_{Gi}$ (generator-related), $\underline{I}_{Li}$ (load-related), $\underline{I}_{el-i-o}$ (shunt-related) and $\underline{I}_{el-i-j_i}$ (branch-related) is given.

3.2.3 Power - voltage - current - relationship

For every network element (generator, load, branch, shunt) relationship between the complex power ($\underline{S}$), the complex element-related voltage ($\underline{V}$) and the complex element-related current ($\underline{I}$) is valid:

Generators

$$\underline{S}_{Gi} = \underline{V}_i \underline{I}_{Gi}^* \quad ; \text{i} = 1 \dots \text{N} \tag{8}$$

Loads

$$\underline{S}_{Li} = \underline{V}_i \underline{I}_{Li}^* \quad ; \text{i} = 1 \dots \text{N} \tag{9}$$

Branch-Elements

$$\underline{S}_{el-i-j_i} = \underline{V}_i \underline{I}_{el-i-j_i}^* \quad ; \text{i-j} = \text{all branch elements} \tag{10}$$

$$\underline{S}_{el-i-j_j} = \underline{V}_j \underline{I}_{el-i-j_j}^* \quad ; \text{i-j} = \text{all branch elements} \tag{11}$$

Shunts

$$\underline{S}_{el-i-o} = \underline{V}_i \underline{I}_{el-i-o}^* \quad ; \text{i} = 1 \dots \text{N} \tag{12}$$

I.e. the complex power of an element is always the product of the complex voltage times the conjugate complex current of the corresponding element.

3.3 Mathematical formulation of the various equality constraint sets

3.3.1 General characteristics

The equations (2) to (12) written for all corresponding network elements, represent the set of equality constraints for the power flow model. Its characteristics are:

- All variables are complex with the exception of the shunt taps.
- There is an enormous number of variables and equality constraints which define non-linear relations among the variables.
- The connectivity of the power system elements (the branch elements) determines the number of terms in the equality constraints, especially in (7).

3.3.2 Classification of the variables and equality constraints

Since this set of equations represents a subset of the constraint set a classification of the already used variables in subsets is advantageous:

The variables of the OPF problem will be called $\underline{x}$ from now on. Similarly behaving variables will receive the same index and belong to the same group.

Subsets of variables can be characterized as follows (Sets of variables are put into a bracket like { .. }).

- $\{\underline{x}_{A1}\} = \{\underline{Y}_{ii}^{el-i-j}, \underline{Y}_{ij}^{el-i-j}, \underline{Y}_{ji}^{el-i-j}, \underline{Y}_{jj}^{el-i-j}, \underline{Y}_{iio}^{el-i-o}\}$ (numerically given parameters of all passive network elements)
- $\{\underline{x}_{A2}\} = \{\underline{t}^{el-i-j}, s^{el-i-o}\}$ (transformer and shunt related variables)
- $\{\underline{x}_{B}\} = \{\underline{I}_{el-i-j_i}, \underline{I}_{el-i-j_j}, \underline{I}_{el-i-o}\}$ (branch and shunt related current variables)
- $\{\underline{x}_{C}\} = \{\underline{I}_{Gi}, \underline{I}_{Li}\}$ (generator and load related current variables)
- $\{\underline{x}_{D}\} = \{\underline{V}_{i}\}$ (nodal voltage variables)
- $\{\underline{x}_{E}\} = \{\underline{S}_{Gi}, \underline{S}_{Li}\}$ (generator and load related complex power variables)
- $\{\underline{x}_{F}\} = \{\underline{S}_{el-i-j_i}, \underline{S}_{el-i-j_j}, \underline{S}_{el-i-o}\}$ (branch and shunt related complex power variables)

Note that all of the above variable sets are complex, except s^{el-i-o} related to shunt elements.

Some of the variable sets created above can be further categorized: There is one variable set whose elements can be assumed to be numerically known at fixed, given values:

- $\{\underline{x}_{A1}\}$: Numerically given network parameters of the passive elements.

With this variable classification the equality constraints (2) to (12) can also be classified into four categories:

(2) and (5) are characterized as follows:

$$\mathbf{g}_B = \underline{\mathbf{x}}_B - \mathbf{f_B}(\underline{x}_{A2}, \underline{x}_D) = \mathbf{0} \qquad (13)$$

(7) is characterized as follows:

$$\mathbf{g}_A = \mathbf{f_A}(\underline{x}_B, \underline{x}_C) = \mathbf{0} \tag{14}$$

(8) and (9) are characterized as follows:

$$\mathbf{g}_E = \underline{\mathbf{x}}_\mathbf{E} - \mathbf{f_E}(\underline{x}_C, \underline{x}_D) = \mathbf{0} \tag{15}$$

(10), (11) and (12) are characterized as follows:

$$\mathbf{g}_F = \underline{\mathbf{x}}_\mathbf{F} - \mathbf{f_F}(\underline{x}_B, \underline{x}_D) = \mathbf{0} \tag{16}$$

(13) .. (16) represent the sets of equality constraints which must be satisfied for any numeric power flow solution. They show certain properties:

- Each equality constraint set has its own properties. These properties have consequences in the OPF solution process: The Jacobian submatrices, representing the first partial derivatives of the equations with respect to all variables will have special and different properties like sparse matrices, block-diagonal matrices, symmetric matrices, etc..

- The sets of equality constraints $\mathbf{g}_E$ and $\mathbf{g}_F$ could be eliminated from a mathematical solution process if the set of free variables $\{\underline{x}_E\}$ and $\{\underline{x}_F\}$ do not appear in any other equality **and/or** inequality constraint set. Note that as shown later in this paper, the latter is the case, thus $\{\underline{x}_E\}$ and $\{\underline{x}_F\}$ cannot be eliminated from the equality constraint set.

- $\{\underline{x}_B\}$ of the equation set $\mathbf{g}_B$ could immediately be replaced into $\mathbf{g}_A$ and $\mathbf{g}_F$, thus eliminating the variables $\{\underline{x}_B\}$ in the equality constraints and reducing the number of equality constraints. However, for certain mathematical formulations and because of OPF solution based reasons, it is better not to eliminate the variables at this stage.

(13) .. (16) can be summarized into one compact equality constraint set as follows:

$$\mathbf{g}(\underline{\mathbf{x}}) = \mathbf{0} \tag{17}$$

with $\underline{\mathbf{x}}^T = (\underline{\mathbf{x}}_{A2}^T, \underline{\mathbf{x}}_B^T, \underline{\mathbf{x}}_C^T, \underline{\mathbf{x}}_D^T, \underline{\mathbf{x}}_E^T, \underline{\mathbf{x}}_F^T)$ and $\mathbf{g} = (\mathbf{g}_A, \mathbf{g}_B, \mathbf{g}_E, \mathbf{g}_F)$.

4 Mathematical formulation of operational constraints

4.1 Introduction

In the preceding section a power system model and needed equality constraints have been formulated. Satisfying these equality constraints with any numerical set of variables means that the physical characteristics of the power system for a model of category 2 are satisfied. The problem is that many of these physically possible states do not make operational sense or are not operationally possible. Thus in order to model the power system behavior more realistically additional constraints have to be formulated. Different types of operational constraints can be formulated:

- Physical damage to network equipment must be prevented since power system equipment is often very expensive and hard to repair.
- Laws dictate mandatory standards to be satisfied by all utilities, e.g. the voltage amplitude at a node must be within certain upper and lower limits, or: a power end user must have power available at almost 100% during the year, or: if any network element is unvoluntarily outaged the power system must be brought back to an acceptable network state within a given time period.
- Physically given limits for power system sources (any generator has its upper power limit).
- Power consumption at certain discrete time steps force the power to flow from the generators via the transmission system to predefined geographical places, i.e. the power consumed by loads at certain nodes at certain times is given and must be considered.
- Contracts among utilites determine precisely at what times how much power must be imported or exported from one utility to another. This imposes limitations on the power system operation and also on its model.
- Operational limits being computed by other power system problem analysis areas like network stability determine e.g. the maximum allowable complex voltage angle shift from node i to another node j.
- Human operators cannot implement more than say 10 % of all possible controls manually within a give short time period.

These examples show that a huge number of operational constraints exist which must be translated into mathematical constraint types.

These mathematical constraints are derived in the following subsections. Three main constraint groups are identified: The **transmission constraints**, representing all operational limits of the actual network. The **contingency constraints** are related to all operational aspects if any network element is outaged as compared to the actual network and its associated network state. In the third constraint group the **operational policy based constraints** are formulated. They represent e.g. limits of human operator based system control.

The correct mathematical representation of the three constraint groups is emphasized in the following subsections, although this might lead to problems which are not solvable today with classical optimization tools.

4.2 Transmission constraints

Transmission constraints are always related to the actual network, i.e. to a network with given branch connectivity.

4.2.1 Given complex loads

As already discussed in a preceding section individual loads cannot be influenced by operating policies and must be satisfied at any time by the corresponding generation. Thus for any given discrete time step where the individual load values $\underline{S}^0_{Li}$ are either measured or predicted with some method, the following must be valid:

$$\underline{S}_{Li} = \underline{S}^o_{Li} \quad ; \mathrm{i} = 1 \; .. \; \mathrm{N} \tag{18}$$

An upper index ^{o} means a numerically given value (complex if underlined). This equality constraint set deals with a subset of the variable set $\mathbf{x}_E$.

4.2.2 Branch current magnitude - maximum limit

The maximum current magnitude values for transmission branches, i.e. lines and transformers, are given due to limitation of the branch material. Excessive currents would damage the transmission elements.

$$\begin{aligned} |\underline{I}_{el-i-j_i}| &\leq I^{max}_{el-i-j} \quad ; \text{i-j: all branches} \\ |\underline{I}_{el-i-j_j}| &\leq I^{max}_{el-i-j} \quad ; \text{i-j: all branches} \end{aligned} \tag{19}$$

An upper index ^{max} means a numerically given maximum limit value. An upper index ^{min} means a numerically given minimum limit value. The symbol $|(.)|$ refers to the absolute value of the variable in (.). This inequality constraint set deals with the variable set $\mathbf{x}_B$.

4.2.3 Branch MVA-power - maximum limit

The same reason as the one for branch maximum current limit, discussed before, is valid.

$$\begin{aligned} |\underline{S}_{el-i-j_i}| &\leq S^{max}_{el-i-j} \quad ; \text{i-j: all branches} \\ |\underline{S}_{el-i-j_j}| &\leq S^{max}_{el-i-j} \quad ; \text{i-j: all branches} \end{aligned} \tag{20}$$

This inequality constraint set deals with the variable set $\mathbf{x}_F$.

Either (19) or (20) or both must be formulated for each branch of the network.

4.2.4 Lower and upper nodal voltage magnitude limits

These limits are often given by very strict standards. Too high or too low voltages could cause problems with respect to end user power apparatus damage or instability in the power system. This could lead to unwanted and economically expensive partial unavailability of power for end users.

$$V^{min}_i \leq |\underline{V}_i| \leq V^{max}_i \quad ; \mathrm{i} = 1 \; .. \; \mathrm{N} \tag{21}$$

This inequality constraint set deals with the variable set $\mathbf{x}_D$.

(21) is valid for every node of the network. There are nodes (often a subset of the generator nodes) where the upper and lower voltage limits are identical, i.e. the voltage magnitude of this node is numerically given.

4.2.5 Lower and upper generator active power limits

The active power of a generator i is defined to be the real part of the complex variable $\underline{S}_{Gi}$. This quantity is physically limited in each generator.

$$P_{Gi}^{min} \leq Real(\underline{S}_{Gi}) \leq P_{Gi}^{max} \quad ; \text{i} = 1 \text{ .. N} \tag{22}$$

This inequality constraint set deals with the variable set $\mathbf{x}_E$.

(22) must be formulated for every generator. Often the lower limit is zero.

4.2.6 Lower and upper generator reactive power limits

The reactive power of a generator i is defined to be the imaginary part of the complex variable $\underline{S}_{Gi}$. It is an important measure of voltage magnitude quality, e.g. a low voltage indicates a local shortage of reactive power.

$$Q_{Gi}^{min} \leq Imag(\underline{S}_{Gi}) \leq Q_{Gi}^{max} \quad ; \text{i} = 1 \text{ .. N} \tag{23}$$

This inequality constraint set deals with the variable set $\mathbf{x}_E$.

(23) must be formulated for every generator. The upper and lower reactive power limits are often not given as numeric values but as functions of the active generator power:

$$\begin{aligned} Q_{Gi}^{min} &= f_{min_i}(Real(\underline{S}_{Gi})) \\ Q_{Gi}^{max} &= f_{max_i}(Real(\underline{S}_{Gi})) \end{aligned} \tag{24}$$

4.2.7 Upper and lower transformer tap magnitude limit

These limits come from the fact that the range of a tap is limited by the physical construction of each transformer. Thus clear limits exist for each transformer.

$$t_{el-i-j}^{min} \leq |\underline{t}_{el-i-j}| \leq t_{el-i-j}^{max} \quad ; \text{i-j: all transformer branches} \tag{25}$$

This inequality constraint set deals with a subset of the variable set $\mathbf{x}_{A2}$.

4.2.8 Upper and lower transformer tap angle limit

The same reasons as discussed for the transformer tap magnitude limits are valid here.

$$\begin{aligned} \delta t_{el-i-j}^{min} \quad &\leq \angle(\underline{t}_{el-i-j}) \leq \delta t_{el-i-j}^{max} \\ &; \text{i-j: all transformer branches} \end{aligned} \tag{26}$$

Note that the symbol $\angle$ refers to the angle of the following complex variable. This inequality constraint set deals with a subset of the variable set $\mathbf{x}_{A2}$.

In the power system mainly two types of transformers can be found with variable tap position:

- In-phase tap changing transformer: The transformer can vary or regulate the voltage magnitude
- Phase shifting transformer: The transformer can vary or regulate the voltage angle.

For an in-phase tap changing transformer usually a constraint of type (25), for a phase shifting transformer a constraint of type (26) must be formulated.

4.2.9 Upper and lower shunt tap limit

This limit has to be understood as follows: In practice a limited number of discrete shunt elements is available at some electrical nodes. These individual shunt elements can either be switched in or out. Thus the upper shunt tap limit corresponds to the state where all shunts are switched in and the lower limit to the lowest number of possible switched-in shunts.

$$s_{el-i-o}^{min} \leq s^{el-i-o} \leq s_{el-i-o}^{max} \quad \text{; i-o: all shunt elements} \tag{27}$$

This inequality constraint set deals with a subset of the variable set $\mathbf{x}_{A2}$.

4.2.10 Upper and lower limits on branch voltage angle

This type of constraint must be formulated if some stability based criterion is formulated for the branch angle. Violating some of the constraints can cause severe dynamic stability problems which could lead to power outage and severe economic penalties.

$$\begin{aligned} \delta_{el-i-j}^{min} \quad & \leq |\angle(\underline{V}_i) - \angle(\underline{V}_j)| \leq \delta_{el-i-j}^{max} \\ & \text{; i-j: all critical branch elements} \end{aligned} \tag{28}$$

This inequality constraint set deals with a subset of the variable set $\mathbf{x}_D$ and must be formulated for every branch determined by some stability criterion.

4.2.11 Minimum generator active power spinning reserve

$$P_{reserve}^{min} \leq \sum_{i=1}^{N} (P_{Gi}^{max} - Real(\underline{S}_{Gi})) \tag{29}$$

This inequality constraint is necessary in order to force each power system control area to have a certain amount of total active generator power available for unforseen cases. An example is the generator outage in a neighbouring power system control area: Here the power system control areas with intact power generation automatically help to provide the power for a certain time after the outage.

4.2.12 Upper and lower limits on total active power of a given set of branches

$$\begin{aligned} P_{BS_k}^{min} \quad & \leq |\textstyle\sum_{(i-j)\in\{BS_k\}} Real(\underline{S}_{el-i-j_i})| \leq P_{BS_k}^{max} \\ & \text{; } BS_k\text{: set of branches with limited total active power} \end{aligned} \tag{30}$$

This inequality constraint set deals with a subset of the variable set $\mathbf{x}_D$ and must be formulated for every branch set for which a constraint on the sum of the active power is valid. Often the upper and lower limits for this total branch set flow is identical and numerically given. This is the case if the network comprises the network data of more than one utility and if the utilities have contracts for power transfer from one into the other power control area.

4.2.13 Control variable time-related movement limits

In a power system the operator can only control certain quantities. These quantities are called controls and represent a physical apparatus to implement any operationally feasible and acceptable network state. Each control has an associated control variable. The most important control variables are the following:

- Generator active power control P_{Gi}: This control variable is the real part of the variable $\underline{S}_{Gi}$.

$$P_{Gi} = Real(\underline{S}_{Gi}) \;\; ; \text{i: all generators} \tag{31}$$

- Generator reactive power control Q_{Gi}: This control variable is the imaginary part of the variable $\underline{S}_{Gi}$.

$$Q_{Gi} = Imag(\underline{S}_{Gi}) \;\; ; \text{i: all generators} \tag{32}$$

- Generator voltage magnitude control V_{Gi}: This control variable is the voltage magnitude $|\underline{V}|_i$ only of the generator nodes.

$$V_{Gi} = |\underline{V}_i| \;\; ; \text{i: all generators} \tag{33}$$

- Phase shifter transformer tap position control δt_{el-i-j}: This control variable is the angle of the complex tap of a transformer $\underline{t}_{el-i-j}$.

$$\delta t_{el-i-j} = \angle(\underline{t}^{el-i-j}) \;\; ; \text{i-j: all phase shifter transformers} \tag{34}$$

- In-phase transformer tap position control t_{el-i-j}: This control variable is the magnitude of the transformer tap $\underline{t}_{el-i-j}$.

$$t_{el-i-j} = |\underline{t}^{el-i-j}| \;\; ; \text{i-j: all in-phase transformers} \tag{35}$$

- Shunt value control: This setpoint is usually the real-number shunt tap variable s_{el-i-o} itself.

$$s_{el-i-o} = s^{el-i-o} \;\; ; \text{i: all shunts} \tag{36}$$

- Active power interchange transaction control: A utility connected to neighbouring utilities can buy or sell active power via the tie-lines which connect the areas. The control variable is called $P_{interchange-k-l}$ (The index k refers to area k and the index l to a neighbouring area l):

$$\begin{aligned} P_{interchange-k-l} \;\; &= \textstyle\sum_{(i-j)\in\{k-l\}} Real(\underline{S}_{el-i-j_i}) \\ &\; ; \text{k-l: branches connecting utility l to utility k} \end{aligned} \tag{37}$$

The above mentioned variables on the left hand side of the equality constraints are called control variables and represent variables with real values which can be directly influenced by the power system operator.

The mathematical representation of control variables as used in this paper is $\mathbf{u}$. They can always be derived from variables of the vector $\mathbf{x}$ as shown with (31) .. (37) and are summarized as follows:

$$\mathbf{u} = \mathbf{g}_u(\underline{\mathbf{x}}_{\mathbf{A2}}, \underline{\mathbf{x}}_{\mathbf{E}}, \underline{\mathbf{x}}_{\mathbf{D}}) = \mathbf{g}_u(\underline{\mathbf{x}}) \tag{38}$$

It is important to understand that the power system cannot realize the state of the desired values at the time t when the control variables and the associated physical controls are changed by the operator: Due to the dynamic nature of the power system there is always a time delay from changing the control setpoint values to the time when the power system actually shows the desired values. Also, it does not make sense to change the controls to values which cannot be realized in the power system at all or only after quite a long time.

Thus there is an additional set of inequalities, representing the maximum difference between actual power system state and the state where the power system can be moved from this state within the time frame for which the OPF result is valid. This time frame corresponds often to the discrete time interval in which one OPF calculation is done, see sections 2 and 3 of this paper.

These additional lower and upper control variable constraints are formulated as follows in general form:

$$\mathbf{u}^{min} \leq \mathbf{u} \leq \mathbf{u}^{max} \tag{39}$$

In (39), the limit vectors are derived both from the values of the actual network state and the ability to move controls within a given time period. They can be assumed to be numerically given.

4.2.14 Summary: Transmission constraints

Using the variable characterization of the preceding section the additional equality (18) and inequality constraints (19) .. (30) can be setup as follows:

(18)	$\{\mathbf{g}_{E1}(\underline{\mathbf{x}}_{E1}) = 0\}$
(38)	$\{\mathbf{g}_u(\underline{\mathbf{x}}_{\mathbf{A2}}, \underline{\mathbf{x}}_{\mathbf{E}}, \underline{\mathbf{x}}_{\mathbf{D}}) - \mathbf{u} = \mathbf{0}\}$
(19)	$\{\mathbf{h}_B(\underline{\mathbf{x}}_B) \leq 0\}$
(20), (22), (23), (30)	$\{\mathbf{h}_E(\underline{\mathbf{x}}_E) \leq 0\}$
(21), (28)	$\{\mathbf{h}_D(\underline{\mathbf{x}}_D) \leq 0\}$
(25), (26), (27)	$\{\mathbf{h}_A(\underline{\mathbf{x}}_{A2}) \leq 0\}$
(39)	$\{\mathbf{h}_u(\mathbf{u}) \leq \mathbf{0}\}$

All of the above equality constraints and those of (17) and all inequality constraints above can be summarized in compact form as follows:

$$\mathbf{g}(\underline{\mathbf{x}}, \mathbf{u}) = 0 \tag{40}$$

$$\mathbf{h}(\underline{\mathbf{x}}, \mathbf{u}) \leq 0 \tag{41}$$

Both (40) and also (41) are functions of mixed complex and real variables. However it is always possible to formulate the problem with real variables by using the simple rectangular or polar coordinate formulation.

From a mathematical point of view these equality and inequality constraints are relatively simple functions of related complex OPF variables. At this point the choice of the variable coordinate system for the OPF variables $\underline{\mathbf{x}}$ has a direct influence on the complexity of the real-variable formulation of the constraints:

When choosing the **polar coordinate system** then all angle related inequality constraints ($\angle(.)$) (like (26), (28), (34)) and all magnitude related functions ($|(.)|$) (like (19), (20), (21), (25), (27), (30), (33), (35) and (37)) are very simple 1:1 functions (meaning $x_{magnitude_i}$ or x_{angle_i}) of the corresponding polar magnitude and polar angle related variables.

These functions are more complicated if using the **rectangular coordinate system**: Here the functions for magnitude related functions take the form $\sqrt{(x_{real_i} + x_{imag_i})}$ and for angle related functions $\arctan \frac{x_{imag_i}}{x_{real_i}}$.

For functions where the real or imaginary part of a complex variable is desired (like (22), (23), (31) and (32)), the polar coordinate formulation is more complex and takes the form $cos(x_{angle_i}) \cdot x_{magnitude_i}$ for 'Real'- type functions and for 'Imag' - type functions $sin(x_{angle_i}) \cdot x_{magnitude_i}$.

In rectangular coordinates the corresponding functions are simple: For 'Real'- type functions x_{real_i} and for 'Imag' - type functions x_{imag_i}.

(40) and (41) represent the set of equality and inequality constraints. Both together are called the **transmission constraints** within the complete OPF formulation.

Note that for each discrete time step for which an OPF is valid the transmission constraints can be slightly different with respect to the symbolic formulation, but significantly different with respect to the numerically given limit and load values.

4.3 Contingency constraints

In the preceding subsection all equality and inequality constraints refer to the actual network state. This means that the OPF transmission constraints are based on a given network structure with known branch, shunt, and generator statuses. It has been assumed that all these elements are in a 'switched-in' status. The network where all elements are in this 'switched-in' status is called the 'actual network'.

In addition to satisfying the transmission constraints for this 'actual network', law or economic reasons force utilities not only to satisfy all transmission constraints of the 'actual network', but also to satisfy the so-called 'n-1-security' based constraints which can be defined as follows:

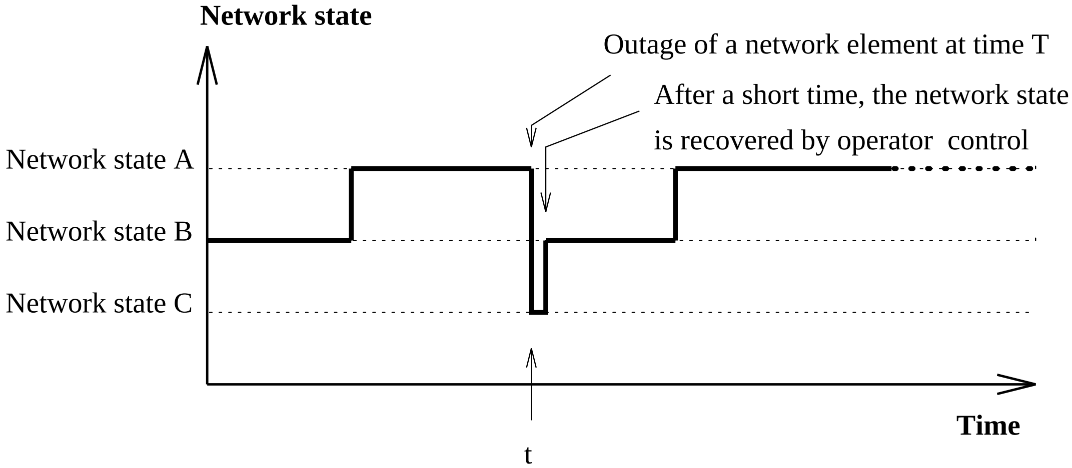
The state of the 'actual network' is defined as 'n-1-secure' with post-contingency rescheduling at any time T if

1. all operational constraints of the 'actual network' are satisfied, i.e. the **transmission constraints and if**

2. any one of the branches, shunts or generators is outaged (representing a **contingency**), the new network state must be such that within a given short time period τ_{short}, the 'new actual network' can be shifted to a transmission constrained state by control means.

Since this control variable movement occurs immediately after the contingency happens it is called **post-contingency control movement**.

Figure 2 shows typical network state changes caused by the outage on an element:



Comments:

network state changing over time

Network state A: "n-1" secure network state with post-contingency control movements

Network state B: Transmission-constrained network state

Network state C: State with violated transmission constraints

Abbildung 2: Network state changes due to the outage of a network element at time t

Many utilities, e.g. at the east coast in the U.S.A., choose the time for recovering the network state after the contingency has occured to a transmission constrained network state, i.e. τ_{short} as zero. This means that the utility wants to operate its actual network in such a state that if any contingency occurs the new network state after the contingency is immediately such that no transmission constraints are violated.

It is important to note that it can be assumed that the control variables $\mathbf{u}$ do not change from the network state immediately before to immediately after the contingency occurs. Only the OPF variables $\underline{\mathbf{x}}$ change immediately due to the physical behavior of the power system. Thus a contingency can immediately lead to operational constraint violations which must be corrected by moving the control variables $\mathbf{u}$ within a short time τ_{short}.

A dynamic simulation not being the objective, the recovery time effect is translated into a maximum control variable move after the outage occurs (the so-called post-contingeny control variable movement):

It is known how long it takes to move a certain power system control by a certain amount, e.g. the active power of a generator from a generation of 100 MW to 130 MW. Thus it is assumed that the maximum possible and operationally acceptable movements of all control variables are known. The resulting new inequality constraint set representing the maximum allowable upward and downward movements of all control variables per outage is as follows:

First the maximum movements for all control variables are computed numerically. Note that different values can be chosen per contingency i:

$$\mathbf{\Delta u}^{(i)^{max}} = \tau_{short} \cdot \Delta\mathbf{u/sec.}^{(i)^{max}} \tag{42}$$

The scalar value τ_{short} is given (in seconds) and the vector $\Delta\mathbf{u/sec.}^{(i)^{max}}$ represents the numerically given maximum possible movements per second for each control variable.

$$\mathbf{u} - \mathbf{\Delta u}^{(i)^{max}} \leq \mathbf{u^{(i)}} \leq \mathbf{u} + \mathbf{\Delta u}^{(i)^{max}} \quad \text{i: all possible contingencies} \tag{43}$$

In (43) the vector $\mathbf{u^{(i)}}$ refers to the control variable set valid at time $t+\tau_{short}$, assuming that the outage of network element i has occured at time t. I.e. for every possibly outage network element i such a vector is created, representing the new state at time $t+\tau_{short}$. The size of the vector $\mathbf{u^{(i)}}$ per outage element i is the same as the size of $\mathbf{u}$, with the exception of a control variable related to an outaged network element.

Since the new network state after the outage must satisfy the transmission constraints the model must satisfy the same equality and inequality constraints like in the 'actual network' state (see (40) and (41)). Thus for each outage case i the following equality and inequality constraints have to be formulated with the new outage state related variable set as defined above:

$$\mathbf{g^{(i)}}(\underline{\mathbf{x}}^{\mathbf{(i)}}, \mathbf{u^{(i)}}) = \mathbf{0} \quad ; \text{ i: all possible contingencies} \tag{44}$$

$$\mathbf{h^{(i)}}(\underline{\mathbf{x}}^{\mathbf{(i)}}, \mathbf{u^{(i)}}) \leq \mathbf{0} \quad ; \text{ i: all possible contingencies} \tag{45}$$

In summary, by formulating contingency constraints the following points are significant:

- The number of variables is increased to (n = number of possible contingencies) n times the number of variables of the 'actual network', i.e. the problem is tremendously increased.
- The absolute values for the difference of control variables of the 'actual network' and corresponding control variables of the outage cases must be less or equal to a numerically given value.
- A network state valid after the contingency has occured, must satisfy all operational equality and inequality constraints. The difference in the symbolic formulation to the actual network equations are as follows:
  - A new set of variables is used for each contingency.
  - The outaged network element must not be modelled in the equality constraints
  - If there is an inequality constraint for the outaged element this inequality constraint must not be formulated for the contingency network state.

4.4 Operational policy based constraints

4.4.1 Introduction

The constraints formulated in the preceding sections represent important restrictions on the power system model used in the OPF calculation. Without including these constraints the model will result in non-practical results.

One of the mathematically challenging problems is the fact that a power system operator cannot change too many controls during a given time period. This is the case if the result of the OPF optimization is not transferred automatically to the power system control. The automatic control meachanism is called closed loop control. The mathematical formulation for this problem is given in the following subsection.

4.4.2 Limited number of controller movements

In a preceding subsection the control variables $\mathbf{u}$ have been defined as functions of the OPF variables $\mathbf{x}$. Control variable movements are limited by the maximum physically possible change during a given time period.

In the case of non-closed loop OPF operation further constraints must be formulated, since the total number of control variable movements must be limited to a certain numerically given value:
The problem can be stated in words as follows:
Given: The actual power system state is given in such a way that all equality constraints (2) .. (12) are satisfied. The numeric values of the control variables for this state called 'Base Case'(BC) state are represented with the vector $\mathbf{u}_{BC}$. They are computed with the equations (31) .. (37).
Goal: Do not move more than a given number of control variables to some other network state which is operationally more convenient. This new state and the associated control variables are called $\mathbf{u}$. A typical value for this number of maximum movable control variables is e.g. 10 % of all control variables.

This problem of limiting the number of control variable movements is given not only from the base case to an optimized base case network state, but also for each outage case i:

Especially when a contingency occurs the operator does not want to deal with too many control variable movements which are needed to bring the network state back to an acceptable new transmission constrained network state after the outage has occured. By the value τ_{short}, representing the time during which the power system must have been moved back to a transmission contrained state, the operator is under heavy pressure and is interested to have a limited number of movable control variables.

Outage related control variables movements refer the movement starting point to the state represented by the vector $\mathbf{u}$. Note that the number of movable control variables for each individual outage case can differ from the number of control variable movements for the base case.

The control variable movements for the individual outage cases $\mathbf{u}^{(i)}$ are derived from the outage-related variables $\underline{\mathbf{x}}^{(i)}$ in analogy to (38) by simple transformations of the complex outage case variables $\underline{\mathbf{x}}^{(i)}$ to real outage case related control variables $\mathbf{u}^{(i)}$.

$$\mathbf{u}^{(i)} = \mathbf{g}_u^{(i)}(\underline{\mathbf{x}}^{(i)}) \;\; ; \text{i: all possible contingencies} \tag{46}$$

The problem is that it is not known beforehand which subset of the control variables to move, both base case and also outage case related. This means that the status of each control variable must be a variable and this is formulated mathematically as follows:

Each of the control variables can have a status of moved (1) or not moved (0). This status is represented by the variable vector $\mathbf{w}$ which refers to the control variables of the base case. For outage case i the corresponding status variable vectors are called $\mathbf{w}^{(i)}$.

$$\mathbf{w}, \mathbf{w}^{(i)} = \begin{bmatrix} \mathbf{0} \\ \mathbf{1} \end{bmatrix} \;\; ; \text{i: all possible contingencies} \tag{47}$$

(47) shows that for each element of the vectors $\mathbf{w}$ and $\mathbf{w}^{(i)}$ the value can only be either 0 or 1.

With these additional, discrete variable type constraints the control variables $\mathbf{u}$ and $\mathbf{u}^{(i)}$ must satisfy the following conditions:

$$\mathbf{diag}(\mathbf{u} - \mathbf{u}_{BC}) \cdot (\mathbf{w} - \mathbf{1}) = \mathbf{0} \tag{48}$$

and for the outage cases:

$$\mathbf{diag}(\mathbf{u}^{(i)} - \mathbf{u}) \cdot \left(\mathbf{w}^{(\mathbf{i})} - \mathbf{1}\right) = \mathbf{0} \;\; ; \text{i: all possible contingencies} \tag{49}$$

(47) together with (48) and (49) guarantee that the individual control variables are either moved or not moved at all.

The number of control variable movements is limited with the following simple inequality constraints representing the sum all individual variables of the vectors $\mathbf{w}$ and $\mathbf{w}^{(\mathbf{i})}$:

For the base case:

$$\mathbf{w}^T \cdot \mathbf{1} \leq CV^{max} \tag{50}$$

CV^{max} stands for the maximum number of base case related control variable movements (a scalar, numerically given value).

For the individual outage cases:

$$\mathbf{w}^{(\mathbf{i})^T} \cdot \mathbf{1} \leq CV^{(i)^{max}} \;\; ; \text{i: all possible contingencies} \tag{51}$$

$CV^{(i)^{max}}$ stands for the maximum number of outage case i related control variable movements (i.e. per outage element i, a scalar, numerically given value).

(48) .. (51) represent the mathematical formulation for the operational problem of the limited number of control variable movements.

4.5 Overview: Network type, network state and constraint set related term definitions

Important goals of the power system control are the reliable and economic operation. These goals are translated into OPF problem parts: Reliability is translated into the operational constraints, economy into the objective function (discussed later in this paper). These operational constraints can be grouped and important terms have been introduced in this section. They are summarized in the following table:

Term	Description
Actual network	Network with given branch, shunt and generator statuses. I.e. it is known which of these elements are switched in and which ones are switched out.
Contingency i network	This is the network where one element i is outaged as compared to the 'actual network'. If the outage is lasting for a long time this new network will be a new 'actual network'.
Base case network state	This is any network state (i.e. voltages, currents, powers) computed or measured at an 'actual network'. Only the power flow equality constraints of the 'actual network' are satisfied. There can be violations on all kinds of operational constraints. It is the most general state from which an OPF optimization can be started. One of the goals of the operator is to shift the network from this network state via the 'transmission constrained network state' to the 'n-1-secure network state'.
Transmission constrained network state	An 'actual network' is in this state if all transmission constraints are satisfied (some of the contingency constraints are violated).
n-1-secure network state	This is a computed state of the 'actual network' where all operational constraints including those for contingencies are satisfied.

5 OPF objectives and objective functions

5.1 Introduction

An important part in any mathematical optimization problem is the objective function which allows to make a distinct and often unique, optimal selection out of the solution region defined by the equality and inequality constraints. Also, the objective function is needed to drive a mathematical optimization process towards an optimal solution.

As already discussed before utilities can have different goals or power system operation objectives. Some goals are clearly defined like minimum active power losses in the resistive parts of the transmission system branches or minimum total cost for the active power generation of the generators. These objectives are of economical nature and can thus easily be justified.

On the other side less clear goals exist which can be of the following types and often depend on operational utility policies: If the power system is monitored and if the power system is found to be in a state where either transmission or contingency constraints are violated, operate the system in such a way that the violations are eliminated as quickly as possibly.

It is obvious and mathematicians can prove that objectives often are exchangeable with equivalent inequality constraints, i.e. putting harder inequality constraints limits can also limit the value of an objective function or the other way around: Formulating inequality constraints as part of the objective function can give valid solutions by enforcing so-called soft limits.

This paper concentrates on the formulation and not on the solution of the OPF problem by giving equality and inequality constraints. In this section possible objectives and the mathemical objective function formulations are discussed. However, one must always keep in mind that the possibility to partially exchange inequality constraints and objective function formulations is given. The solution process actually decides how to combine the constraint and objective function formulations.

5.2 Mathematical formulation of various OPF objective functions

5.2.1 Objective: Minimum active power losses

The active power losses (called 'losses' from now on) represent a quantity whose minimization can easily be justified by an electric utility if the effort to actually operate in a minimum loss mode is not too expensive. Losses are generated in the resistances of the transmission lines and are a measure of the difference of the generated total active power to the total active load at any time.

Two different loss-related cases exist: If the network model comprises only network parts of the own utility controlled area the losses are computed as follows:
Loss case A:

$$\text{Minimize } P_{Loss} = \sum_{i=1}^{N} \left(Real(\underline{S}_{Gi}) - Real(\underline{S}_{Li}) \right) \tag{52}$$

Thus in this case, the losses are a simple function of elements of the vector $\underline{\mathbf{x}}_E$ representing the generator and load related nodal complex powers.
Loss case B:

Here the modelled network also comprises network parts of the neighbored utilities. Reasons for this have been discussed in previous section and are often the case if the highest voltage levels of the network must be modelled. For this case B the losses are the sum of the active powers of each branch of the utility controlled area. For each branch the active power flow from node i to node j must be added to the branch flow from node j to node i. This results in the active power losses per branch. Thus the losses for a defined area 'a' are computed as follows:

$$\text{Minimize } P_{Loss}^{\text{area 'a'}} = \sum_{i-j \in \text{area 'a'}} \Big(Real(\underline{S}_{el-i-j_i}) + Real(\underline{S}_{el-i-j_j}) \Big) \tag{53}$$

Thus in this case, the losses are a simple function of elements of the vector $\underline{\mathbf{x}}_F$ representing the complex power of branches.

5.2.2 Objective: Minimum total active power operating cost

Each utility has control over generation of different kind, e.g. hydro, hydro-thermal or thermal power generation. Each type of generation has cost associated with it: Hydro power is, if available, usually the cheapest power (water does not cost much in mountain areas) and thermal power generation is more expensive (often oil, gas or nuclear material is used as primary energy resource. These resources have to be bought at market prices).

In addition to own power resources the utility can buy or sell power from or to neighbouring utilitities. Sometimes it can be cheaper to buy power than to produce it within the power system control area.

It is in the interest of the utilities and also of the paying power consumers to minimize the cost associated with active power generation. It can be assumed that the cost of each generator can be represented as a distinct curve of relating cost to the active power which the generator delivers. Usually this curve is given for the full range of the operating capability of the generator. The general type of a cost curve can be written as follows:

$$C_{Gi} = c_i(Real(\underline{S}_{Gi})) \tag{54}$$

where c_i is a general function of the active power of the generator i. A typical cost curve is shown in Fig. 3.

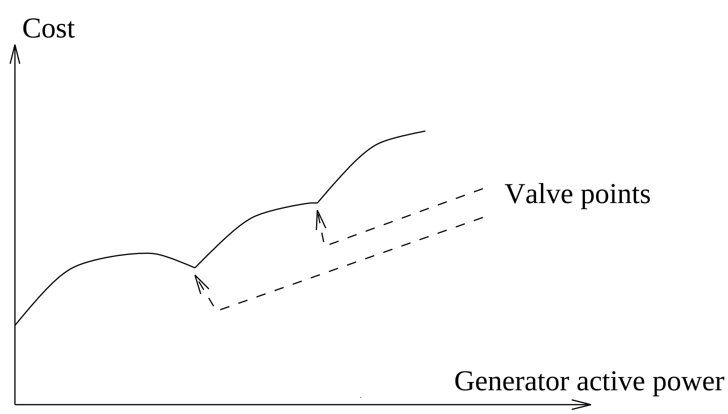


Abbildung 3: Typical true cost curve for hydro-thermal power generation

As seen in Fig. 3 the general cost curve type is very complex: Non-convex cost curves are possible. Also, it is important to note that the cost curves can be assumed to be separable with respect to the active power generation of the generatorsm, i.e. the cost of each generator is only dependent on the cost of its own active power generation and not on the cost of another generator power.

The cost curve associated with buying or selling power is not as clear. Sometimes it is step-wise linear, sometimes linear over the full range. Note that selling power leads to a cost curve with negative cost values.

The problem is the shape of the cost curves. Optimization algorithm usually cannot deal with cost curve shapes as shown in Fig. 3. Thus, the cost curves are usually modified and a convex, smooth cost curve as shown in Fig. 4 can be assumed to be the cost curve model for the OPF.

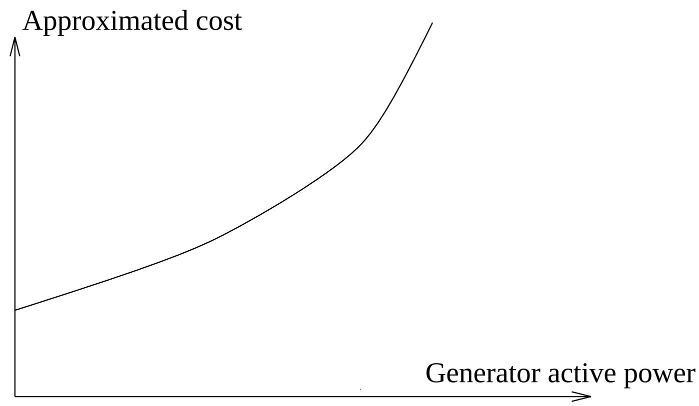


Abbildung 4: Corrected smooth and convex power generation cost curve for OPF model

Often these cost curves are assumed to be piecewise quadratic with smooth transition at the cost curve break points. Also, exponential cost curve representations are seen in OPF models. Usually the approximated smooth and convex cost curves (one per active power generation) is given as the exact reference along which the OPF optimum solution must be found. Whether this makes sense or not is not the issue of this paper. However, it should be kept in mind that the cost curve issue is open and needs to be resolved the more accurate the remaining power system model is chosen. An inaccurate cost curve model can prevent an accurate optimization result, even if the remaining model parts are model with very high accuracy or in other words, the optimization result is not more accurate than the chosen accuracy of the model components.

The total cost of all generators and transaction interchange active power can be represented as objective function as follows:

$$\begin{aligned}\text{Minimize } C \quad &= \sum_{i=1}^{N} c_{approximated_i}(Real(\underline{S}_{Gi})) \\ &+ \sum_{l \in k-l} c_{int-approximated_{k-l}}(P_{interchange-k-l}) \\ &\text{k-l: all areas l connected to area k}\end{aligned} \tag{55}$$

5.2.3 Objective: Fast transition from a violated to a non-violated network state

For many reasons a power system can suddenly move into a state where important constraints are violated. If this happens a very important objective of a utility can be, to move the power system as quickly as possible back to a network state without violating constraints, including transmission, contingency and utility policy constraints.

The problem to get a mathematical objective function is the term 'quick' which is time related. The OPF, however, does not have time as variable in the formulation. Thus another measure for the quick operation must be found. One measure can be to determine a feasible state (a state where no inequality constraints are violated) with a minimum number of control variables movements. Provided a solution for the minimum number of control variables can be found in reasonable time, this is probably one of the fastest ways for the utility to achieve a feasible operation. Thus the objective function would be:

$$\text{Minimize } F = \mathbf{w}^T \cdot \mathbf{1} \tag{56}$$

There might be other objective function formulations for this special objective. The main problem is to translate the operating philosophies into the objective function. Once it is given the mathematical process tries to solve the problem as accurately as possible. The optimization result could be such that although satisfying the mathematical model, it does not conform to the often rule-based operational policies of the utility. This will, especially in the first implementation phase of the OPF, lead to some type of iterative process for defining the objective function and the resulting mathematical optimum. At the end of this iterative process the optimization result must both satisfy the mathematical optimum criteria and also the operational, often rule based philosophies of the utility.

6 The complete OPF formulation

6.1 Mathematical OPF formulation - summary

The OPF formulation can be split in three main parts: The equality constraints, the inequality constraints and the objective function. Each of these three parts has its own properties, as discussed in the preceding sections. The following three tables summarize the OPF problem:

Equality constraints:

Description	Mathematical formulation	Equations
Actual network state	$\mathbf{g}(\underline{\mathbf{x}}, \mathbf{u}) = \mathbf{0}$	(40)
Contingency i network state	$\mathbf{g}^{(\mathbf{i})}(\underline{\mathbf{x}}^{(i)}, \mathbf{u}^{(i)}) = \mathbf{0}$	(44) ∀ i
Control variable movement status	$\mathbf{w}, \mathbf{w}^{(i)} = \left[\mathbf{0}^T \ \ \mathbf{1}^T\right]^T$	(47) ∀ i
Actual network control variable movement	$\mathbf{diag}(\mathbf{u} - \mathbf{u}_{BC}) \cdot (\mathbf{w} - \mathbf{1}) = \mathbf{0}$	(48)
Post-contingency case i variable movement	$\mathbf{diag}(\mathbf{u}^{(i)} - \mathbf{u}) \cdot \left(\mathbf{w}^{(\mathbf{i})} - \mathbf{1}\right) = \mathbf{0}$	(49) ∀ i

Inequality constraints:

Description	Mathematical formulation	Equations
Actual network state	$\mathbf{h}(\underline{\mathbf{x}}, \mathbf{u}) \leq \mathbf{0}$	(41)
Contingency i network state	$\mathbf{h}^{(\mathbf{i})}(\underline{\mathbf{x}}^{(i)}, \mathbf{u}^{(i)}) \leq \mathbf{0}$	(45) ∀ i
Post-contingency i control movement	$\mathbf{u} - \mathbf{\Delta u}^{(i)^{max}} \leq \mathbf{u}^{(\mathbf{i})} \leq \mathbf{u} + \mathbf{\Delta u}^{(i)^{max}}$	(43) ∀ i
Actual network control movement status	$\mathbf{w}^T \cdot \mathbf{1} \leq CV^{max}$	(50)
Contingency case i control movement status	$\mathbf{w}^{(\mathbf{i})^T} \cdot \mathbf{1} \leq CV^{(i)^{max}}$	(51) ∀ i

Typical OPF objective functions:

Description	Equations
Active power losses	(52), (53)
Total active power operating cost	(55)
Fast transition from a violated to a non-violated network state	(56)

Note that only one of the possible objective functions together with all equality and inequality constraints can be chosen and can be solved for one optimal solution variable set.

Here another problem with the present OPF formulations and solutions becomes obvious: Although a utility would like to satisfy more than one objective at the same time, the classical mathematical optimization algorithms allow only one objective function at a time. The problem of getting a solution which considers more than one objective function (a multi-objective function problem) at a time exists and should be addressed in the future to make the OPF result more practically acceptable and applicable.

6.2 Sensitivity of the OPF solution with respect to parameter changes

When using the OPF with any of the above mentioned objectives and the corresponding mathematical objective functions, the OPF formulation is today such that parameters like the network data, the inequality constraint limit values and the measured or predicted load values are numerically precisely given. The operator, however, has a different view of these parameters, especially with respect to the limit values of the inequality constraints: They do not represent precisely given values but are often derived based on experience and on heuristic assumptions. The operator would like to know how much the OPF result varies when the parameters are slightly varied. The idea is to give the operator a feeling for example how much the violation of a maximum branch MVA flow reduces the chosen objective function. From such a sensitivity analysis the operator could derive important operating policies and it would provide deeper insight into the OPF optimum.

Mathematically the problem is often formulated under the assumption that the binding inequality constraint set of the OPF solution remains identical even with a slight change of the parameters, mentioned before. Thus the problem is as follows: Assume that the OPF result (solution process to obtain an OPF result see paper by H. Glavitsch) has been obtained by some solution method. The resulting, numerically given values for this optimum are as follows (denoted with index ^{opt}):

$$\begin{array}{ll} \mathbf{u}^{opt}, \underline{\mathbf{x}}^{opt} & \text{(optimum state for base case variables)} \\ \mathbf{u}^{(i)^{opt}}, \underline{\mathbf{x}}^{(i)^{opt}} & \text{(optimum state for outage case i related variables)} \\ \mathbf{w}^{opt} & \text{(optimum base case control variables statuses)} \\ \mathbf{w}^{(i)^{opt}} & \text{(optimum control variable movement status for contingency i)} \end{array} \quad (57)$$

The problem is now a parametric equality constrained optimization problem and can be formulated as follows, assuming that all variables of the OPF problem are summarized into $\mathbf{X}$, all equality constraints into $\mathbf{G}(\mathbf{X})$, all binding inequality constraints into $\mathbf{H}_{binding}(\mathbf{X})$ and the objective function is $F(\mathbf{X})$:

$$\begin{array}{ll} \text{Minimize} & F(\mathbf{X}^{opt} + \delta\mathbf{x}) \\ \text{subject to} & \mathbf{G}(\mathbf{X}^{opt} + \delta\mathbf{x}) = \delta\mathbf{g} \\ \text{and} & \mathbf{H}_{\mathbf{binding}}(\mathbf{X}^{opt} + \delta\mathbf{x}) = \delta\mathbf{h} \end{array} \tag{58}$$

In (58) the values of the vectors $\delta\mathbf{g}$ and $\delta\mathbf{h}$ are slight variations to the right hand sides of the equality and binding inequality constraints. The problem is to get a solution of the type

$$\delta\mathbf{x} = \text{function of } (\mathbf{X}^{opt}, \delta\mathbf{g}, \delta\mathbf{h}) \tag{59}$$

where the term 'function of' can in the simplest case be a numeric matrix.

6.3 Discussion and future outlook

The OPF formulation as shown in section 6.1 has several characteristics which are summarized in the following points.

- The OPF formulation comprises a huge number of variables and equality / inequality constraints. Typical OPF problem sizes are summarized in the following table:

Number of nodes	Number of actual network related constraints		Number of contingency network related constraints	
	equality	inequality	equality	inequality
80	160	640	32000	120000
500	1000	4000	$1.4 \cdot 10^6$	$5 \cdot 10^6$
1000	2000	8000	$2.8 \cdot 10^6$	$20 \cdot 10^6$
2000	4000	16000	$5.6 \cdot 10^6$	$40 \cdot 10^6$

  Note that in this table the equality constraints are assumed to be formulated in the most compact form (this includes elimination of variables).

- The variable sets of the actual and contingency case related networks are almost perfectly decoupled. Each variable set is related to a different network with distinct operational constraints. The coupling between the variable sets is 'simple' in that only maximum difference values between a subset of the variable sets (the control variables) for each different contingency case to the base case must be satisfied.

- The variables are either real, complex or discrete. Either the polar or rectangular coordinate system must be used to represent the complex variables in real variable form. The polar form introduces sin and cos - type functions into the OPF equations. The rectangular form is responsible for square or square root - type and also arctan - type equations.

- Today the complete OPF formulation as given before cannot be solved in closed form. Typically, steps are undertaken to make the OPF problem solvable with classical optimization methods:

- The discrete variables of the vector $\mathbf{w}$, $\mathbf{w}^{(i)}$ are assumed to be known, i.e. the set of moveable control variables is known. Doing this leads to an optimization problem with continuous variables only.
- The maximum possible moves of the control variables within the time τ_{short} after the contingency occurs is often assumed to be zero, meaning that the control variable values do not change from the n-1-secure state to any post contingency state. This leads to an optimization problem with much less control variables.
- The contingency based equations $\mathbf{g}^{(i)}$ and $\mathbf{h}^{(i)}$ are often linearized around the n-1 secure state variables. The equations resulting from this linearization are seen as the exact equation set which the OPF optimum has to satisfy.

Today (1992), the OPF problem as stated in this paper is not solvable in real-time with computers presently available in Energy Management Centers. Faster computers, new optimization algorithms and the feedback obtained from practical OPF usage will change the OPF formulation and its real-time usage in the near future. Modelling the OPF as realistically as possible together with a robust, fast OPF execution will lead to high acceptance of this powerful power system control tool.

For the author of this paper, it is clear that the OPF will be more and more important for the electric power industry due to its potential in increasing both power system reliability and also power system economy without much investment in power system hardware.

A APPENDIX

A.1 Symbols

The following notations are used througout this text:

- Symbols representing complex variables are underlined.
- Matrices are shown in capital boldface letters.
- Vectors are shown in small boldface letters.

*	Conjugate complex
^{opt}	Associated variable is optimum variable
^{T}	Transposed
^{low}	Low limit
^{high}	Upper (high) limit
Δ	Change
$\lvert(.)\rvert$	Absolute value of a variable (.) (variable can be complex)
$\angle(.)$	Angle value of a complex variable (.)
$Real(.)$	Real part of a complex variable (.)
$Imag(.)$	Imaginary part of a complex variable (.)
(i)	Related to contingency case i
$diag(.)$	Diagonal matrix

N	Total number of electrical nodes
n	Total number of network elements
$\underline{I}_{el-i-j_i}$	Complex current of element from node i to node j, computed at node i
$\underline{y}_{ii}^{el-i-j}$	Primitive Y-matrix element (i,i) of branch element between nodes i and j
$\underline{y}_{ij}^{el-i-j}$	Primitive Y-matrix element (i,j) of branch element between nodes i and j
$\underline{y}_{ji}^{el-i-j}$	Primitive Y-matrix element (j,i) of branch element between nodes i and j
$\underline{y}_{jj}^{el-i-j}$	Primitive Y-matrix element (j,j) of branch element between nodes i and j
$\underline{V}_i$	Complex voltage at node i
$\underline{Y}_{ii}^{el-i-j}$	Two-port admittance element (i,i) of branch between nodes i and j
$\underline{Y}_{ij}^{el-i-j}$	Two-port admittance element (i,j) of branch between nodes i and j
$\underline{Y}_{ji}^{el-i-j}$	Two-port admittance element (j,i) of branch between nodes i and j
$\underline{Y}_{jj}^{el-i-j}$	Two-port admittance element (j,j) of branch between nodes i and j
$\underline{t}^{el-i-j}$	Complex tap of transformer branch between nodes i and j
$\underline{I}_{el-i-o}$	Complex current of shunt element at node i
$\underline{y}_{ii}^{el-i-o}$	Admittance element of shunt at node i
s^{el-i-o}	Shunt tap value (corresponding to the number of switched in shunts) at node i
$\underline{Y}_{iio}^{el-i-o}$	Admittance element of total shunt at node i

$\underline{I}_{Gi}$	Complex generator current at node i
$\underline{I}_{Li}$	Complex load current at node i
$\underline{S}_{Gi}$	Complex power of generator at node i
$\underline{S}_{Li}$	Complex power of load at node i
$\underline{S}_{el-i-j_i}$	Complex power of branch from node i to node j computed at node i
$\underline{S}_{el-i-j_j}$	Complex power of branch from node i to node j computed at node j
$\underline{S}_{el-i-o}$	Complex power of shunt element at node i
$\{\underline{x}_{A1}\}$	$= \{\underline{Y}_{ii}^{el-i-j}, \underline{Y}_{ij}^{el-i-j}, \underline{Y}_{ji}^{el-i-j}, \underline{Y}_{jj}^{el-i-j}, \underline{Y}_{iio}^{el-i-o}\}$ (numerically given parameters of all passive network elements)
$\{\underline{x}_{A2}\}$	$= \{\underline{t}^{el-i-j}, s^{el-i-o}\}$ (transformer and shunt related variables)
$\{\underline{x}_{B}\}$	$= \{\underline{I}_{el-i-j_i}, \underline{I}_{el-i-j_j}, \underline{I}_{el-i-o}\}$ (branch and shunt related current variables)
$\{\underline{x}_{C}\}$	$= \{\underline{I}_{Gi}, \underline{I}_{Li}\}$ (generator and load related current variables)
$\{\underline{x}_{D}\}$	$= \{\underline{V}_{i}\}$ (nodal voltage variables)
$\{\underline{x}_{E}\}$	$= \{\underline{S}_{Gi}, \underline{S}_{Li}\}$ (generator and load related complex power variables)
$\{\underline{x}_{F}\}$	$= \{\underline{S}_{el-i-j_i}, \underline{S}_{el-i-j_j}, \underline{S}_{el-i-o}\}$ (branch and shunt related complex power variables)
$\mathbf{g}_A$	Equation set mainly related to variables $\underline{\mathbf{x}}_{A2}$
$\mathbf{g}_B$	Equation set mainly related to variables $\underline{\mathbf{x}}_{B}$
$\mathbf{g}_E$	Equation set mainly related to variables $\underline{\mathbf{x}}_{E}$
$\mathbf{g}_F$	Equation set mainly related to variables $\underline{\mathbf{x}}_{F}$
$\underline{\mathbf{x}}$	Variable set which include $\underline{\mathbf{x}}_{A2}$, $\underline{\mathbf{x}}_{B}$, $\underline{\mathbf{x}}_{E}$ and $\underline{\mathbf{x}}_{F}$
$\mathbf{g}(\underline{\mathbf{x}})$	Equation set which includes $\mathbf{g}_A$, $\mathbf{g}_B$, $\mathbf{g}_E$, $\mathbf{g}_F$

$\underline{S}_{Li}^{o}$	Given complex power value of load at node i
I_{el-i-j}^{max}	Maximum current of branch between nodes i and j
S_{el-i-j}^{max}	Maximum MVA-power of branch between nodes i and j
V_i^{min}	Minimum voltage magnitude at node i
V_i^{max}	Maximum voltage magnitude at node i
P_{Gi}^{min}	Minimum active power of generator at node i
P_{Gi}^{max}	Maximum active power of generator at node i
Q_{Gi}^{min}	Minimum reactive power of generator at node i
Q_{Gi}^{max}	Maximum reactive power of generator at node i
t_{el-i-j}^{min}	Minimum tap position of in-phase transformer between nodes i and j
t_{el-i-j}^{max}	Maximum tap position of in-phase transformer between nodes i and j
δt_{el-i-j}^{min}	Minimum tap position of phase shifter tap transformer between nodes i and j
δt_{el-i-j}^{max}	Maximum tap position of phase shifter tap transformer between nodes i and j
s_{el-i-o}^{min}	Minimum tap for shunt bank at node i
s_{el-i-o}^{max}	Maximum tap for shunt bank at node i
δ_{el-i-j}^{min}	Minimum angle value between nodes i and j
δ_{el-i-j}^{max}	Maximum angle value between nodes i and j
$P_{reserve}^{min}$	Minimum active generator power spinning reserve
$P_{BS_k}^{min}$	Minimum active power value for transfer at branch set k
$P_{BS_k}^{max}$	Maximum active power value for transfer at branch set k
P_{Gi}	Active power of generator at node i (a control variable)
Q_{Gi}	Reactive power of generator at node i (a control variable)
V_{Gi}	Voltage magnitude of generator at node i (a control variable)
δt_{el-i-j}	Phase shift transformer tap magnitude of transformer between nodes i and j (a control variable)
t_{el-i-j}	In-phase transformer tap magnitude of transformer between nodes i and j (a control variable)
s_{el-i-o}	Shunt tap (a control variable)
$P_{interchange-k-l}$	Active power transaction interchange between area k and area l (a control variable)

$\mathbf{u}$	Control variable vector of the actual network
$\mathbf{u}^{(i)}$	Control variable vector of contingency case i
$\mathbf{x}$	OPF state variable vector of the actual network
$\mathbf{x}^{(i)}$	OPF state variable vector of contingency case i
$\mathbf{u}^{min}$	Control variable minimum values
$\mathbf{u}^{max}$	Control variable maximum values
$\mathbf{g}(\underline{\mathbf{x}}, \mathbf{u})$	OPF equality constraint set represented in OPF variables and control variables (actual network)
$\mathbf{h}(\underline{\mathbf{x}}, \mathbf{u})$	OPF inequality constraint set represented in OPF variables and control variables (actual network)
$\boldsymbol{\Delta}\mathbf{u}^{max}$	Maximum move of the control variables from base case state to some other (optimized) state (actual network)
$\boldsymbol{\Delta}\mathbf{u}^{(i)^{max}}$	Maximum move of the control variables from (optimized) base case state to some state after contingency i occurs (contingency case i)
τ_{short}	Short time (in seconds) during which the operator has to move the control variables to some optimal state
$\Delta\mathbf{u}/\mathbf{sec.}^{max}$	Maximum move of the control variables per second (actual network)
$\Delta\mathbf{u}/\mathbf{sec.}^{(i)^{max}}$	Maximum move of the control variables per second (contingency i network)
$\mathbf{g}^{(\mathbf{i})}(\underline{\mathbf{x}}^{(\mathbf{i})}, \mathbf{u}^{(\mathbf{i})})$	OPF equality constraint set represented in OPF variables and control variables (contingency case i network)
$\mathbf{h}^{(\mathbf{i})}(\underline{\mathbf{x}}^{(\mathbf{i})}, \mathbf{u}^{(\mathbf{i})})$	OPF inequality constraint set represented in OPF variables and control variables (contingency case i network)
$\mathbf{g}_u^{(i)}(\underline{\mathbf{x}}^{(i)})$	Functions relating the OPF variables $\mathbf{x}^{(i)}$ to the control variable $\mathbf{u}^{(i)}$ of the contingency case i
$\mathbf{w}$	OPF control variable movement status variables (actual network)
$\mathbf{w}^{(i)}$	OPF control variable movement status variables (contingency i network)
$\mathbf{u}_{BC}$	Given base case (BC) control variable values at any time t describing a network state with possible violated operational constraints
CV^{max}	Maximum number of moveable control variables (actual network)
$CV^{(i)^{max}}$	Maximum number of moveable control variables (contingency i network)

P_{Loss}	Loss function of the whole network
$P_{Loss}^{\text{area 'a'}}$	Loss function of a part of the network
C_{Gi}	Cost function of the active power of the generator at node i
$c_{approximated_i}$	Approximated, smooth active power cost curve for generator i
$c_{int-approximated_{k-l}}$	Approximated, smooth active transaction interchange power cost curve for between area k and area l
C	Total cost of all generators
F	Special objective function related to the quick move of control variables

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