Project type:	SA
Project title:	**Data–Driven Feedback Dissipativity of Discrete–Time LTI Systems**
Semester:	Spring 2023
Start date - end date:	01.05.2023 - 01.09.2023
Office space:	ETL

Project description

This project aims at understanding and advancing theories at the intersection between dissipativity theory and data-driven control for linear time-invariant (LTI) discrete-time (DT) systems.

Dissipativity theory, as introduced by Jan C. Willems in the 1970's, entails how a dynamical system stores and exchanges energy over time. In general, when the internal accumulation is less than the energy supplied, the system is said to be dissipative. In particular, when the supplied energy is described by a quadratic form in the input-output variables, we refer to it as (Q, S, R)–dissipativity. Due to its relation with Lyapunov stability and its compositional capability, dissipativity can be a desirable property for a dynamical system. Indeed, there are many examples of successful applications in the context of decentralized control.

A recent and thriving research direction is the one revolving around behavioural theory and Willem's Fundamental Lemma, stating that the information contained in a sufficiently long trajectory of a linear system is, under mild assumptions, enough to describe any other trajectory of the same length that can be generated by the system itself. In a model-free context, a so-called persistently exciting exploration input is often used to generate such trajectories, obtain a data-based representation of the underlying linear system and develop control techniques such as MPC or assess system's properties such as dissipativity and stabilizability/controllability.

Between the two worlds lies the recent work in [4]. The authors derive a data-driven condition to infer whether the system under observation is (Q, S, R)–dissipative or not. Within this context, we aim at expanding this idea to the problem of feedback dissipativity, that is, deriving a data-driven condition to characterize all feedback gains that render the system (Q, S, R)–dissipative with respect to an additional input-output pair (usually representing interconnection with neighbouring agents). The work in [3] could be especially relevant for our objective. In fact, the authors there derive a data-driven condition for the feedback stability problem, *i.e.*, describing all feedback gains that render the system asymptotically stable.

Numerical experiments will also be performed to validate the eventual data-driven conditions. Several extensions are of course possible in case time allows, as well as deviations from the original path in case the results are not satisfactory.

Tasks

1. Study fundamental concepts on dissipativity theory with special emphasis on DT systems, including (Q, S, R)–dissipativity and related linear matrix inequalities [2, 8, 5, 7].
2. Study fundamental concepts on data-driven analysis and control of LTI DT systems, including Willems' Fundamental Lemma, data-based system representation and related ideas [6, 3].
3. Understand how open-loop (Q, S, R)–dissipativity can be inferred from input-state (noise-free) data via a data-based linear matrix inequality [4].
4. Understand how all stabilizing feedback gains can be derived from a data-based linear matrix inequality [3, Theorem 3].
5. Derive a data-based condition for the feedback dissipativity problem, taking inspiration from and mixing the results in [3, 4]. In this phase, the work in [1] might be handful since it provides a similar proof in model-based fashion.
6. Experiments: validate the data-based conditions by means of numerical experiments on MATLAB or Python.
7. Extension: noise-corrupted case, for instance by assuming a bound on the noise signal as in [4].
8. Extension: embed the data-driven feedback dissipativity conditions into a decentralized control framework.

Timeline

- Month 1: Tasks 1 – 4
- Month 2: Task 5
- Month 3: Tasks 5 – 6
- Month 4: Writing final report and prepare final presentation, Tasks 7 – 8 if time permits

References

[1] Ahmed Aboudonia, Andrea Martinelli, and John Lygeros. Passivity-based decentralized control for discrete-time large-scale systems. *IEEE Control Systems Letters*, 5(6):2072–2077, 2021.

[2] Bernard Brogliato, Bernhard Maschke, Rogelio Lozano, and Olav Egeland. *Dissipative Systems Analysis and Control*. Springer Cham, 3rd edition, 2020.

[3] C. De Persis and P. Tesi. Formulas for data-driven control: Stabilization, optimality, and robustness. *IEEE Transactions on Automatic Control*, 65(3):909–924, 2020.

[4] Anne Koch, Julian Berberich, and Frank Allgower. Verifying dissipativity properties from noise-corrupted input-state data. In *59th IEEE Conference on Decision and Control (CDC)*, pages 616–621, 2020.

[5] Nicholas Kottenstette, Michael J. McCourt, Meng Xia, Vijay Gupta, and Panos J. Antsaklis. On relationships among passivity, positive realness, and dissipativity in linear systems. *Automatica*, 50(4):1003–1016, 2014.

[6] Ivan Markovsky, Linbin Huang, and Florian Dörfler. Data-driven control based on the behavioral approach: From theory to applications in power systems. *IEEE Control Systems Magazine*, 2023. To appear. Preprint version at `https://imarkovs.github.io/tutorial.pdf`.

[7] Andrea Martinelli, Ahmed Aboudonia, and John Lygeros. Stability of interconnected discrete-time dissipative systems without feedthrough. 2023. Notes.

[8] Eva María Navarro-López, Hebertt Sira-Ramírez, and Enric Fossas-Colet. Dissipativity and feedback dissipativity properties of general nonlinear discrete-time systems. *European Journal of Control*, 8(3):265–274, 2002.