Control of networked power systems: from energy to clusters

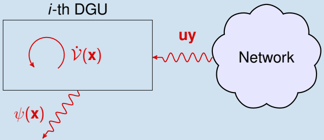
Andrea Martinelli
Automatic Control Laboratory • ETH Zurich
IfA Coffee Talk Series • November 28, 2019

Overview

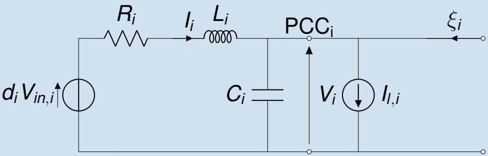
Overview

PRIMARY

Energy-based control



Application to DC microgrids



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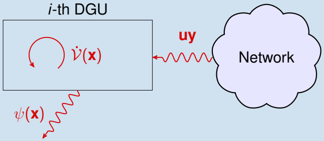

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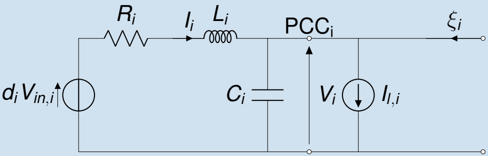
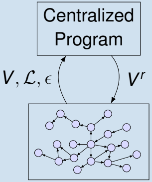
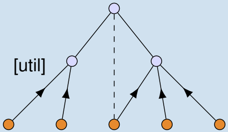


SECONDARY

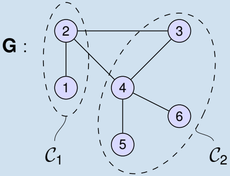
Centralized



Multiagent



Cluster-based



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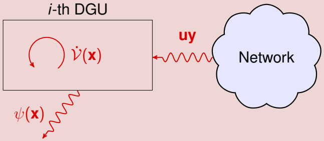

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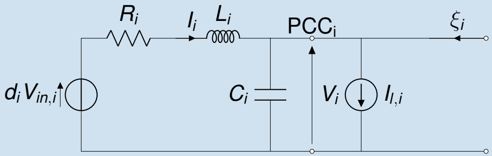

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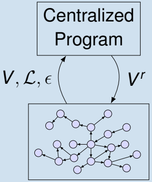

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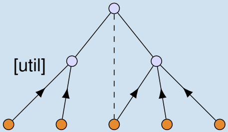

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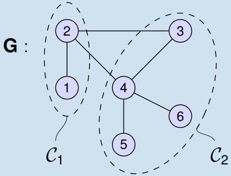


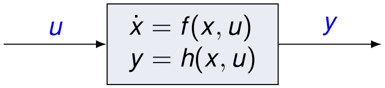
Passivity theory

Definition (I/O passivity)

A dynamical system is passive if

$$u^T y = \frac{\partial V}{\partial x}(f(x,u)) + \psi(x) \quad \forall (x,u)$$

for some $V(x) > 0$ and $\psi(x) \geq 0$.



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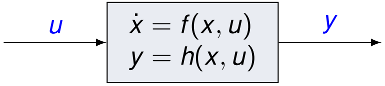
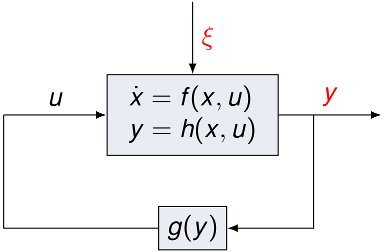


Definition (passivating regulator)

A map $u = g(y)$ passivates the system if

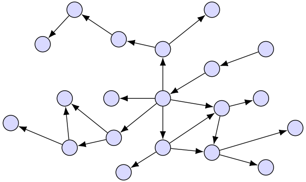
$$\xi^T y = \frac{\partial V}{\partial x}(f(x,g(y),\xi)) + \psi(x) \quad \forall\,(x,\xi)$$

for some $V(x) > 0$ and $\psi(x) \geq 0$.



Why passivity to control networked systems?

- Compositional framework to analyse global passivity
- Linked to Lyapunov stability
- Control actions have (sometimes) interpretation in terms of energy



[1] Arcak (2007), *IEEE Transactions on Automatic Control*.

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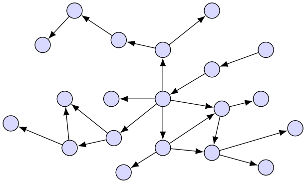

Definition[1] (skew-symmetric coupling)

A set of subsystems is interconnected in a skew-symmetric fashion if local couplings $\xi_i = c_i(y_i, y_{j\in\mathcal{N}_i})$ can be written globally as

$$\xi = Sy, \quad S^T = -S.$$

[1] Arcak (2007), *IEEE Transactions on Automatic Control*.

Stability over skew-symmetric interconnections

Theorem[2] (stability of skew-symmetric passive systems)

A set of passive systems interconnected in a skew-symmetric fashion,

$$\text{i.e.}\quad \xi_i^T y_i = \dot{V}_i(x_i) + \psi_i(x_i) \quad \text{and} \quad \xi = Sy,$$

is stable, and the state x converges to the largest invariant set in $E = \{x : \psi_i(x_i) = 0\ \forall i\}$.

[2] Nahata, Soloperto, Tucci, Martinelli, Ferrari-Trecate (2019), *Automatica*, to appear.

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Proof: Consider $V(x) = \sum_i V_i(x_i)$ as global Lyapunov function. Then

$$\begin{aligned} \dot{V}(x) &= -\sum_i \psi_i(x_i) + y^T \xi \\ &= -\sum_i \psi_i(x_i) + \cancelto{0}{y^T S y} \leq 0. \end{aligned}$$

[2]Nahata, Soloperto, Tucci, Martinelli, Ferrari-Trecate (2019), *Automatica*, to appear.

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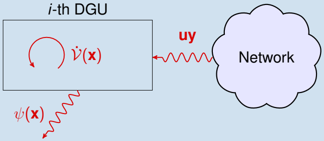

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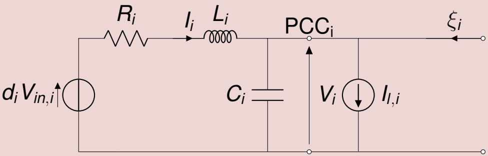

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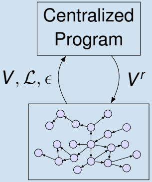
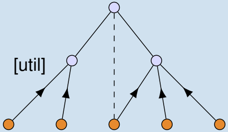
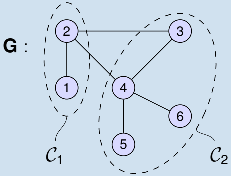


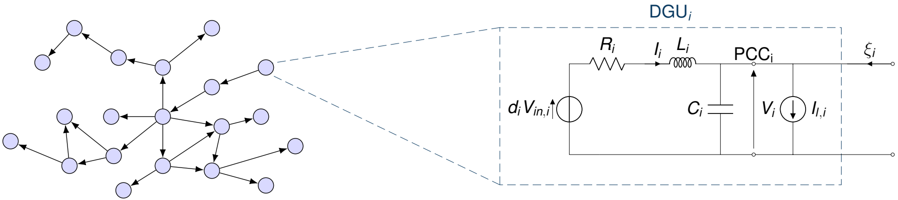
Multiagent



Cluster-based

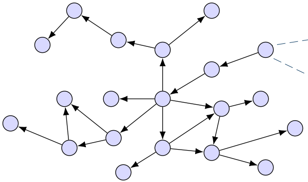


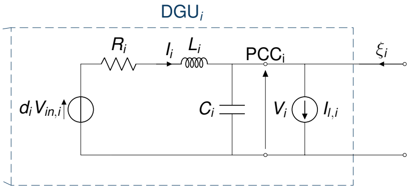
Stability of DC microgrids (1/3)



$$\mathrm{DGU}_i : \begin{cases} \dot{x}_i = A_i x_i + B_i u_i + F_i \xi_i \\ y_i = V_i \end{cases}$$

Stability of DC microgrids (1/3)





$$\xi_i = \sum_{j\in\mathcal{N}_i} \xi_{ij}$$
$$= \sum_{j\in\mathcal{N}_i^+} R_{ij}^{-1}(y_j - y_i) - \sum_{j\in\mathcal{N}_i^-} R_{ij}^{-1}(y_j - y_i)$$

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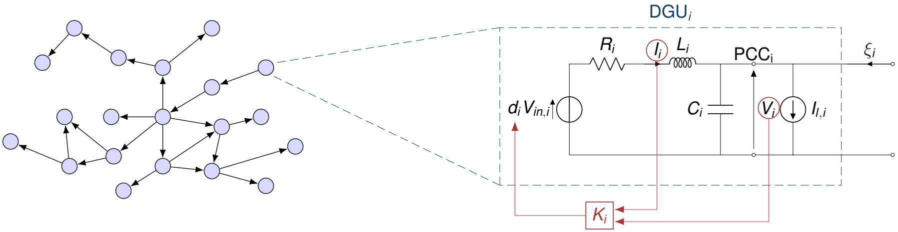

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PI regulator : $u_i = K_i \tilde{y}_i$

Stability of DC microgrids (2/3)

Theorem[2]

Any linear map $u_i = K_i \tilde{y}_i$ with

$$K_i \in \mathcal{Z}_i = \left\{ \left(k_{1i} < 1 \right) \wedge \left(k_{2i} < R_i \right) \wedge \left(0 < k_{3i} < L_i^{-1}(k_{1i} - 1)(k_{2i} - R_i) \right) \right\}$$

passivates subsystem i with respect to the couplings (ξ_i, y_i), and the global state x converges to the origin.

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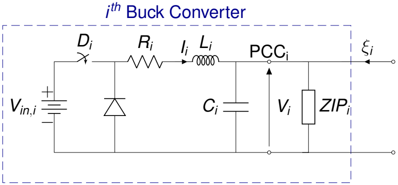
passivates subsystem i with respect to the couplings (ξ_i, y_i), and the global state x converges to the origin.

Proof. (i) $\xi = (\mathcal{A}_w - \mathcal{A}_w^T)y \to$ couplings are skew-symmetric
(ii) Parametrized storage functions $V_i(x_i) = x_i^T \begin{pmatrix} * & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix} x_i \to$ passive dynamics
(iii) LaSalle argumentations $\to$ the origin is the largest invariant set in E

[2] Nahata, Soloperto, Tucci, Martinelli, Ferrari-Trecate (2019), *Automatica*, to appear.

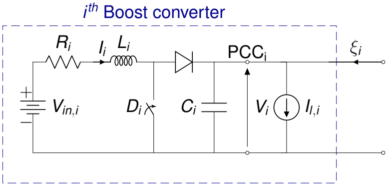
Stability of DC microgrids (3/3): Extensions

Low-voltage microgrids with ZIP loads



- Nonlinear average model
- Passivating PI controllers
- Local passivity $\Rightarrow$ reduced basin of attraction

Medium-voltage microgrids[3]



- Nonlinear average model
- Passivating dynamic laws

$$d_i = \mathrm{sat}(\overline{d}_i + \phi_i)$$
$$\dot{\phi}_i = (V_i^r I_i - V_i I_i) - \gamma_i(d_i - \overline{d}_i)$$

[3]Martinelli, Nahata, Ferrari-Trecate (2018), *Conference on Decision and Control*.

Energetic interpretation of network stability

The passivity condition is expressed, for each node, by an equation of the form

$$\xi^T y = \dot{V}(x) + \psi(x), \qquad \psi(x) \geq 0,$$

that represents the power balance for a controlled DGU.

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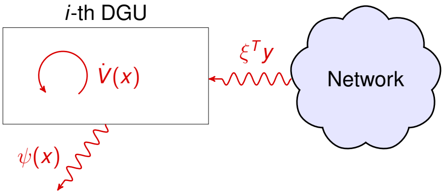

Accumulated energy
$\leq$
Injected energy

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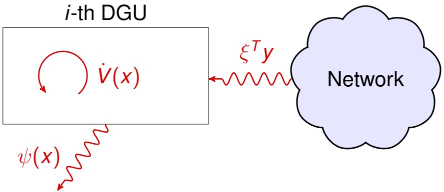

Accumulated energy
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Injected energy

Warning: stability guaranteed as long as physical limits are satisfied

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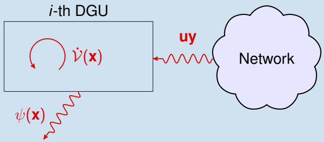

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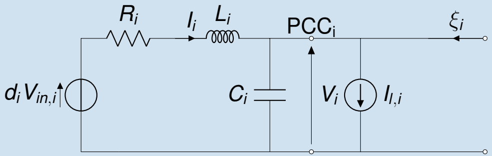

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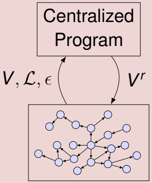
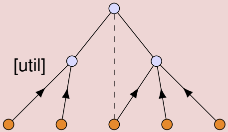
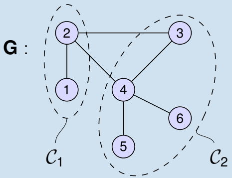
Centralized



Multiagent

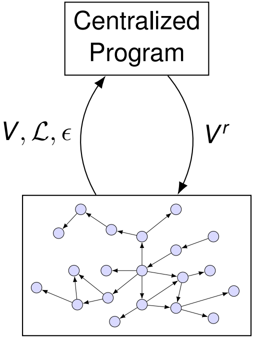


Cluster-based



Centralized approach[4]

Update V^r with a fixed time window. Assumption: time-scale separation.



[4]Martinelli, La Bella, Scattolini (2019), *European Control Conference*

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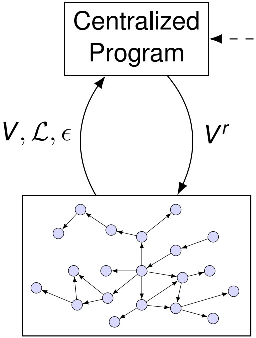

$$
\begin{aligned}
\min_{V} \quad & \alpha \mathcal{J}(V) + \beta \mathcal{N}(V) + \gamma \mathcal{D}(V) \\
\text{s. t.} \quad & \underline{V} \leq V \leq \overline{V} \\
& \underline{I} \leq I(V) \leq \bar{I} \\
& 0 \leq d(V) \leq 1 \\
& \alpha, \beta, \gamma \geq 0,
\end{aligned}
$$

Quadratic in V and $3N$ linear constraints.

[4] Martinelli, La Bella, Scattolini (2019), *European Control Conference*

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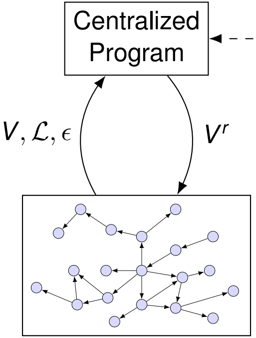

Joule losses + Deviations + Current sharing

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Multiagent constraint optimization

$$\mathrm{MP}^3 = \langle X, D, F \rangle \quad \begin{cases} X = \{x_1, \ldots, x_N\} \ \text{optimization variables} \\ D = \{D_1, \ldots, D_N\} \ \text{finite domains} \\ F = \{f_1, \ldots, f_k\}, \ f_i : D_h \times \cdots \times D_j \to \mathbb{R}^+ \cup \{\perp\} \ \text{local utility functions} \end{cases}$$

[3] Fioretto, Pontelli, Yeoh (2018), *Journal of Artificial Intelligence Research*

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A complete assignment σ is a value assignment $\bar{X} \in D$ such that

$$f_i(\sigma) \neq \{\perp\} \quad \forall f_i \in F.$$

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A complete assignment σ is a value assignment $\bar{X} \in D$ such that

$$f_i(\sigma) \neq \{\perp\} \quad \forall f_i \in F.$$

The solution to MP is then

$$\sigma^* = \arg\min_{\sigma} \sum_{f_i \in F} f_i(\sigma).$$

[3] Fioretto, Pontelli, Yeoh (2018), *Journal of Artificial Intelligence Research*

Distributed approach

Idea: the CP can be equivalently formulated as an MP[4]

- define $X = \{V_1, \ldots, V_N\}$
- Decompose global functions of CP into a set of local functions f_i

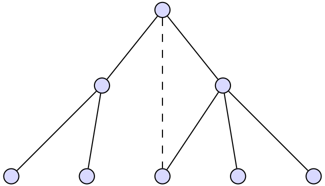
$$\Rightarrow \quad \lim_{|D| \to \infty} \sigma^* = V^r.$$

In general, MP is equivalent to a discretized version of the CP $\rightarrow$ solution quality depends on the density of D.

[4] Martinelli, La Bella, Scattolini (2019), *European Control Conference*

Distributed pseudotree optimization procedure

DPOP[4]: agents agree on the optimal value assignment σ by exploiting a message exchange protocol among neighbouring nodes.



[4] Petcu & Faltings (2005), *International Joint Conference on Artificial Intelligence*

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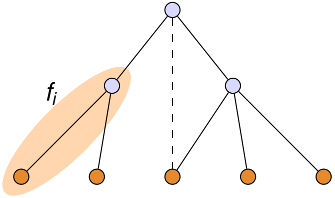

<compute local utility>

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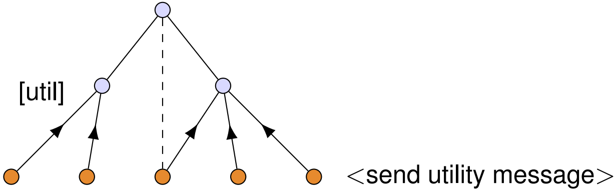

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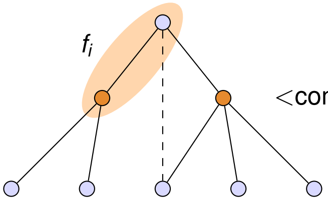

<compute local utility>

[4] Petcu & Faltings (2005), *International Joint Conference on Artificial Intelligence*

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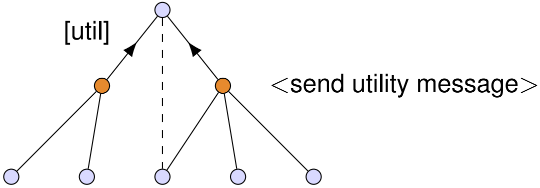

[4] Petcu & Faltings (2005), *International Joint Conference on Artificial Intelligence*

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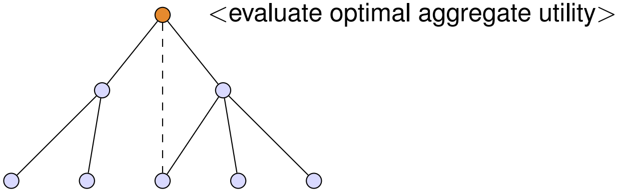

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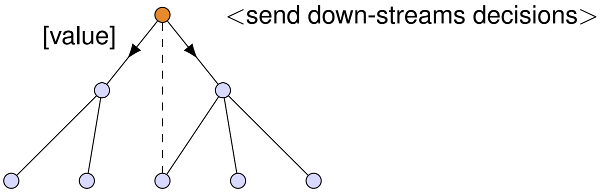

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Distributed pseudotree optimization procedure

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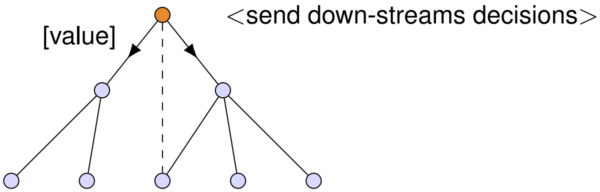


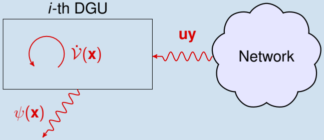
- Fully distributed algorithm
- Trade-off between solution quality and computation time $\mathcal{O}(d^w)$

[4] Petcu & Faltings (2005), *International Joint Conference on Artificial Intelligence*

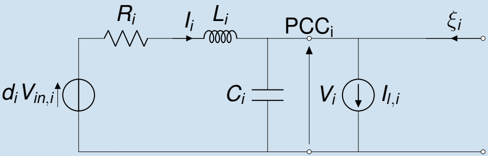
Overview

PRIMARY

Energy-based control

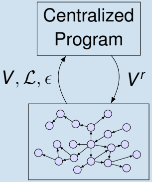


Application to DC microgrids

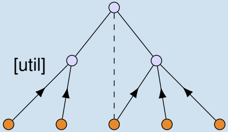


SECONDARY

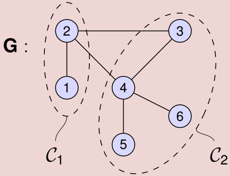
Centralized



Multiagent

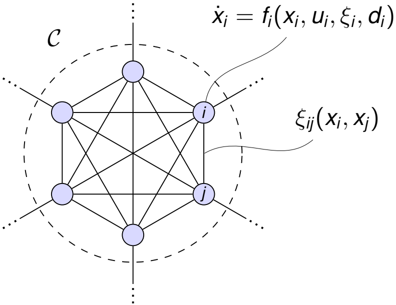


Cluster-based



Cluster-based approach

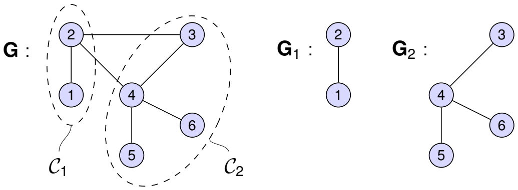
Goal: identify a cluster of nodes that can adsorb a local disturbance without affecting external nodes $\rightarrow$ disturbance containment



Traditional clustering methods: modularity, persistence probability, spectral analysis,...
do not capture the structural condition we require.

$\rightarrow$ Need for a new clustering measure!

Cluster-based approach

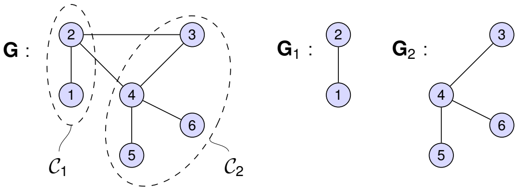


$$L = \left(\begin{array}{c|c|c|c} C_1 & Y_{12} & \cdots & Y_{1m} \\ \hline Y_{12}' & C_2 & & \\ \hline \vdots & & \ddots & \\ \hline Y_{1m}' & & & C_m \end{array}\right)$$

Assumption 1. The graph **G** is strongly connected

Assumption 2. The induced subgraphs identified by the clusters in the partition $\mathcal{P} = \{\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_m\}$ are strongly connected. No trivial clusters.

Cluster-based approach



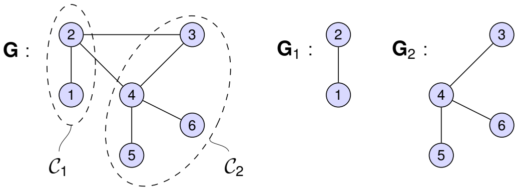
$$L = \left(\begin{array}{c|c|c|c} C_1 & Y_{12} & \cdots & Y_{1m} \\ \hline Y_{12}^{'} & C_2 & & \\ \hline \vdots & & \ddots & \\ \hline Y_{1m}^{'} & & & C_m \end{array}\right)$$

Theorem[5]

If Assumptions 1 and 2 are satisfied, then any diagonal block of L associated with a cluster in $\mathcal{P}$ is nonsingular and has all eigenvalues with positive real part.

[5]Martinelli & Lygeros, submitted

Cluster-based approach



$$L = \left(\begin{array}{c|c|c|c} C_1 & Y_{12} & \cdots & Y_{1m} \\ \hline Y'_{12} & C_2 & & \\ \hline \vdots & & \ddots & \\ \hline Y'_{1m} & & & C_m \end{array}\right)$$

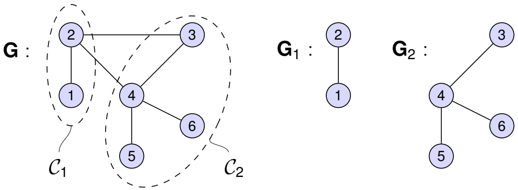
Theorem[5]

If Assumptions 1 and 2 are satisfied, then any diagonal block of L associated with a cluster in $\mathcal{P}$ is nonsingular and has all eigenvalues with positive real part.

Proof. Decompose $C_i = L_i + D_i$, where L_i is the Laplacian of graph $\mathbf{G}_i$. (i) C_i is strictly diagonally dominant. (ii) L_i is irreducible, and $L_i + D_i$ as well because D_i is diagonal. Strictly diagonal dominance + irreducibility $\iff$ nonsingularity.

[5] Martinelli & Lygeros, submitted

Cluster-based approach



$$L = \left(\begin{array}{c|c|c|c} C_1 & Y_{12} & \cdots & Y_{1m} \\ \hline Y'_{12} & C_2 & & \\ \hline \vdots & & \ddots & \\ \hline Y'_{1m} & & & C_m \end{array}\right)$$

Let's introduce a rank difference function $\delta(M_1, M_2) = \text{rank}(M_1) - \text{rank}(M_2)$.

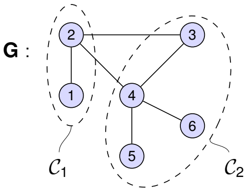
Definition[5]

We say that a cluster C_i has $d \in \mathbb{N}$ degrees of freedom if $\delta(C_i, Y_i) = |C_i| - \text{rank}(Y_i) = d$.

We know that $0 \leq \delta(C_i, Y_i) < |C_i|$. In fact, C_i is full-rank (Theorem) and C_i and Y_i share the same number of rows.

[5] Martinelli & Lygeros, submitted

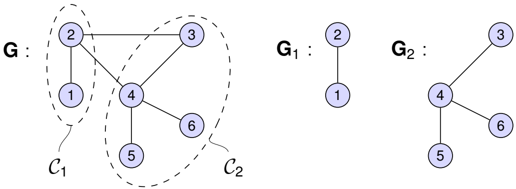
Cluster-based approach



$$L = \left(\begin{array}{c|c|c|c} C_1 & Y_{12} & \cdots & Y_{1m} \\ \hline Y_{12}' & C_2 & & \\ \hline \vdots & & \ddots & \\ \hline Y_{1m}' & & & C_m \end{array}\right)$$

What does d mean? Number of nodes that we can modify without incurring in a net flux variation between C_i and the rest of the network.

Cluster-based approach



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What does d mean? Number of nodes that we can modify without incurring in a net flux variation between C_i and the rest of the network.

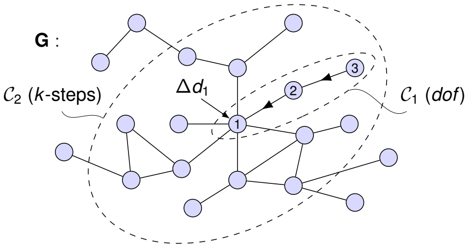
How can d be used? Fitness measure to generate optimal partitions.

The possible partitions of a set scale according to the Bell numbers (e.g. $B_{20} \approx 10^{12}$)
$\rightarrow$ heuristic algorithms are needed.

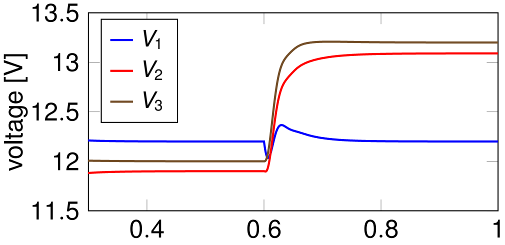
Greedy exploration strategy

Algorithm 1

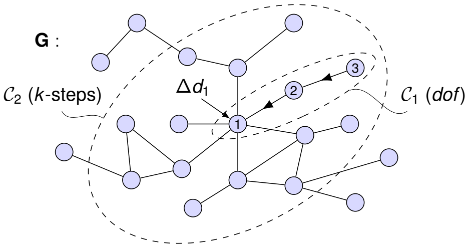
given overloading node i with set of neighbors $\mathcal{N}_i$
initialize $\mathcal{C} = \{i\}, \mathcal{N}_\mathcal{C} = \mathcal{N}_i, \delta_\mathcal{C} = 0$
repeat
  $\mathcal{H} = \{j \in \mathcal{N}_\mathcal{C} \ : \ \delta_{\mathcal{C},j} > \delta_\mathcal{C}\}$
  if $\mathcal{H} \neq \emptyset$ **then**
    $\mathcal{C} \leftarrow \mathcal{C} \cup \left\{ j = \arg\max_{j \in \mathcal{H}} (\Psi_j) \right\}$
  else
    $\mathcal{C} \leftarrow \mathcal{C} \cup \left\{ j = \arg\max_{j \in \mathcal{N}_\mathcal{C}} (\deg(j)) \right\}$
  end if
  $\mathcal{N}_\mathcal{C} \leftarrow (\mathcal{N}_\mathcal{C} \cup \mathcal{N}_j) \setminus (j \cup (\mathcal{C} \cap \mathcal{N}_j))$
  solve optimization problem for nodes in $\mathcal{C}$
until a feasible solution is found
return $x^r_\mathcal{C}$



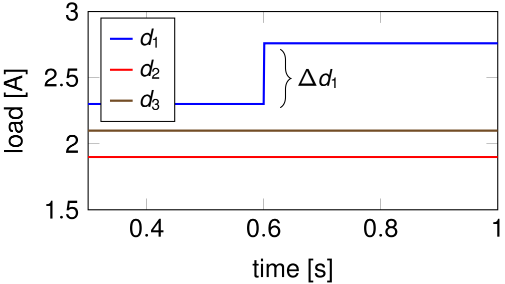
Simulation



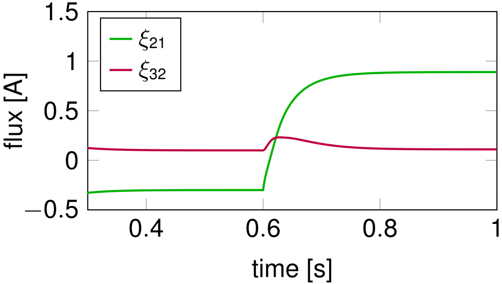
13.5
13
12.5
12
11.5
voltage [V]
V_1
V_2
V_3
0.4
0.6
0.8
1



G :
$\mathcal{C}_2$ (k-steps)
Δd_1
$\mathcal{C}_1$ (dof)
1
2
3



3
2.5
2
1.5
load [A]
d_1
d_2
d_3
Δd_1
0.4
0.6
0.8
1
time [s]



1.5
1
0.5
0
−0.5
flux [A]
ξ_{21}
ξ_{32}
0.4
0.6
0.8
1
time [s]

Q&A

Thank you for the attention!