Secondary Control Strategies for DC Islanded Microgrids Operation

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Islanded DC microgrids

**Microgrid**
Electrical network composed by loads, accumulators, and distributed generation units (DGUs)

**Islanded**
Not connected to the main electrical grid

**Key problems**
- *Voltage stability* at the primary level of each DGU
- *Reference value management* at the secondary level of each DGU

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**Microgrid**: set of dynamical systems (DGUs) that interact over a graph. Topology is captured by

\[ \mathcal{L} = \begin{pmatrix} \deg(v_1) & \{0, -1\} \\ \{0, -1\} & \deg(v_N) \end{pmatrix} \in \mathbb{R}^{N \times N} \]
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Example: Decentralized scheme based on local PI regulators. Reference tracking is guaranteed if control gains are chosen according to passivity-based considerations.
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Stability guaranteed as long as limits on duty cycle and internal current are satisfied. Need for higher layer management
Centralized architecture (1/3)

**Strategy:** update $V \rightarrow V^*$ without invalidating primary stability. A sufficiently long updating window is required

- Which are the global desired performances?
- How to design the CP so to take them into account?
Intuition: write every relevant quantity as a function of $V$

\[
\text{CP: } \min_V \quad \alpha J(V) + \beta N(V) + \gamma D(V) \\
\text{s. t. } \underline{V} \leq V \leq \overline{V} \\
I \leq I(V) \leq \overline{I} \\
0 \leq d(V) \leq 1 \\
\alpha, \beta, \gamma \geq 0,
\]
Intuition: write every relevant quantity as a function of $\mathbf{V}$

Joule losses

Deviation from nominal values

“Current sharing”

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s. t.:
- $V \leq V \leq \bar{V}$
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Voltage limits: $V \leq V \leq \bar{V}$

Deviation from nominal values: $I \leq I(V) \leq \bar{I}$

“Current sharing”: $0 \leq d(V) \leq 1$

Internal current limits: $\alpha, \beta, \gamma \geq 0$

Duty cycle saturation: $0 \leq d(V) \leq 1$
The CP has a quadratic cost function in $V$ and $3N$ linear constraints, which is computationally cheap.

Benchmark for the following distributed approach.
Let’s introduce **Multiagent Constrained Optimization Problems (MCP)** \(^3\)

\[
\text{MCP} = \langle X, D, F \rangle
\]

\[
X = \{x_1, \ldots, x_n\} \quad \text{optimization variables}
\]

\[
D = \{D_1, \ldots, D_n\} \quad \text{finite domains}
\]

\[
f_i : D_h \times \cdots \times D_j \rightarrow \mathbb{R}^+ \cup \{\bot\} \quad \text{local utility functions}
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\(^3\) F. Fioretto et al., *Journal of Artificial Intelligence Research* (2018)
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A complete assignment \(\sigma\) is a value assignment \(\bar{X} \in D\) such that

\[f_i(\sigma) \neq \{\bot\}, \quad \forall f_i \in F\]

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A **complete assignment** \(\sigma\) is a value assignment \(\bar{X} \in D\) such that

\[f_i(\sigma) \neq \{\perp\}, \quad \forall f_i \in F\]

The **solution** of MCP is the complete assignment that minimizes

\[\sigma^* = \arg\min_{\sigma} \sum_{f_i \in F} f_i(\sigma)\]

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Idea: the CP can be equivalently formulated as an MCP

- Define $X = \{V_1, \ldots, V_n\}$
- Decompose global functions of CP into a set of local functions $f_i$
  (we can represent both the cost function and the constraints)

$$\implies \sigma^* = V^* \quad (1)$$
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- Define $X = \{V_1, \ldots, V_n\}$
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$$\implies \sigma^* = V^* \quad (1)$$

Notice that (1) only holds when $D_i$ are dense enough to contain the optimal solution! More generally, MCP is equivalent to solve a discretized version of CP

$$\implies \text{Quality of the solution depends on the density of } D$$
The advantage of MCP lies in the existence of efficient solution search protocols, such as the Distributed Pseudotree Optimization Procedure⁴:
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\[\text{[UTIL]}\]

\(<\text{Send utility message}>\)

The advantage of MCP lies in the existence of efficient solution search protocols, such as the Distributed Pseudotree Optimization Procedure\textsuperscript{4}:

\begin{itemize}
\item \texttt{<Compute local utility>}
\end{itemize}

\textsuperscript{4} A. Petcu and B. Faltings, \textit{International Joint Conference on Artificial Intelligence} (2005)
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\[\text{<Evaluate optimal aggregate utility>}\]

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- Fully distributed procedure
- Trade-off between solution quality and computation time \(\mathcal{O}(d^w)\)

Strategy (event-based): Identify a surrounding cluster which can internally adsorb the disturbance without affecting outer nodes.
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Cluster exploration
Many ways, most straightforward is $k$-step reachability set
Structural condition

We need to impose a structural condition on the cluster

\[
\mathcal{L} = \left( \begin{array}{c|c}
\mathcal{C} & \mathcal{Y} \\
\hline
\mathcal{Y}' & \mathcal{\overline{C}} \\
\end{array} \right)
\]

**Proposition:** A necessary condition for current redistribution is:

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\text{rank}(\mathcal{Y}) < |\mathcal{C}|
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How to adsorb the disturbance
Local version of CP, with additional constraint to avoid disturbance propagation outside the cluster
Example: secondary management

Overloading node

(a) Load $I_{l,1}$.

(b) Output voltage $V_1$.

(c) Internal current $I_1$.

(d) Duty cycle $d_1$. 
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$\mathbb{C}$
Thank you for your attention!